NIU Ph.D. Candidacy Examination Spring 2019 (1/8/2019)
Electricity and Magnetism

You may solve ALL FOUR problems if you choose. The points of the best three problems will be counted towards your final score of this part of the examination. (40 points each. Total of 120 points)

Do not just quote a result. Show your work clearly step by step.

1. [40 points] The method of images. Two infinite parallel grounded conducting planes are held a distance \( a \) apart. A point charge \( q \) is placed in the region between them, a distance \( x \) from one plane.
   
   (a) Find the force \( F \) on the point charge \( q \). [20 points]
   
   (b) What is the force \( F \) on \( q \) when distance \( a \) goes to the infinity? [10 points]
   
   (c) What is the force \( F \) on \( q \) at \( x = a/2 \)? [10 points]

2. [40 points] A very long coaxial cylindrical cable of length \( d \) has an inner conducting cylinder inner radius \( a \) and an outer conducting cylinder with radius \( b \), with \( a < b \ll d \). The material between the conducting cylinders is an insulator with a slightly non-uniform electric permittivity \( \epsilon(r) = \epsilon_1 + r\epsilon'_1 \) and a slightly non-uniform magnetic permeability \( \mu(r) = \mu_1 + r\mu'_1 \), where \( \epsilon_1, \epsilon'_1, \mu_1 \) and \( \mu'_1 \) are constants, and \( r \) is the cylindrical radial coordinate (the distance from the axis of symmetry of the cylinders).

   (a) Suppose the inner cylinder carries charge \( Q \) and the outer cylinder carries charge \( -Q \). Find, in whatever order is most convenient: the electric field and the electric potential everywhere between the cylinders, the total energy stored, and the capacitance per unit length of the cable. [20 points]

   (b) Suppose instead that the inner cylinder carries current \( I \) and the outer cylinder carries the same current back in the other direction. Find, in whatever order is most convenient: the magnetic field \( B \) and the vector potential everywhere between the cylinders, the total energy stored, and the inductance per unit length of the cable. [20 points]
3. [40 points] Collection of time-dependent charged particles at rest. We consider a set of point particles fixed in space with time-dependent charge $q_k(t)$ so that the total charge density is given by $\rho(r, t) = \sum_k q_k(t)\delta(r - r_k)$. We assume that $E(r, t = 0) = B(r, t = 0) = 0$, and suppose that $E(r, t) = \frac{1}{4\pi\varepsilon_0} \sum_k q_k(t) \frac{r-r_k}{|r-r_k|^3}$ for $t > 0$.

(a) Construct a simple current density which satisfies the continuity equation. [12 points]

(b) Describe the flow of charge predicted by the current density computed in part (a). [10 points]

(c) Find $B(r, t)$ and check that this field together with $E(r, t)$ do satisfy all four Maxwell’s equations. [18 points]

4. [40 points] Transverse-Magnetic (TM) wave guided by a planar conductor. A monochromatic plane wave in vacuum ($x > 0$) with $E_x > 0$ and $E_x < 0$ impinges on a perfect conductor ($x < 0$) at an angle of incidence $\theta$; see Fig. 1.

(a) Show that, in the steady state (i.e. once the reflected wave is established), a non-uniform TM wave occupies the vacuum space above the conductor. [20 points]

(b) Calculate the time-averaged Poynting vector everywhere ($x > 0$). [10 points]

(c) What is the charge density induced on the surface of the conductor? [10 points]

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{geometry for problem 4}
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