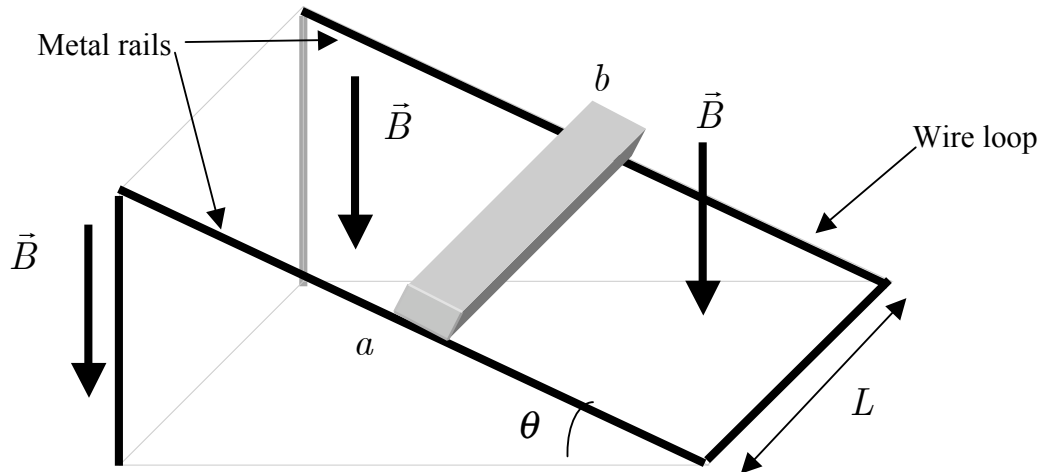


NIU Ph.D. candidacy examination Fall 2010 (9/25/2010)

Electricity and Magnetism

Solve 3 out of 4 problems.

Problem 1: [40 points]



A metal bar of length L , mass m , and resistance R is placed on frictionless rails that are inclined at an angle θ above the horizontal. The rails have negligible resistance. A uniform magnetic field is directed downward as shown above. The bar is released from rest and slides down the rail. The acceleration due to gravity is g .

- (a) Is the direction of the induced current in the bar from a to b or from b to a ? [10 points]
- (b) What is the terminal speed of the bar? [10 points]
- (c) What is the velocity of the bar as a function of time? [20 points]

Problem 2: [40 points]

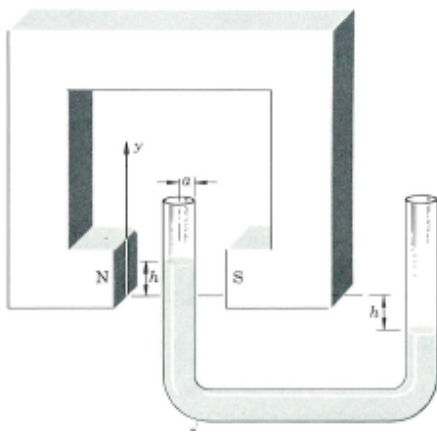
The plane $z = 0$ in a Cartesian coordinate system carries a surface charge density of the form $\sigma = \sigma_0 (1 - \cos(\alpha x) \cos(\beta y))$. Here, α and β are real (as opposed to complex) constants.

- a) Find throughout space the electrostatic potential due to this charge distribution. [20 points]
- b) Check that your solution satisfies the Poisson equation for $z = 0$. [15 points]
- c) Discuss the potential at large distance from the xy -plane. [5 points]

Problem 3: [40 points]

A liquid of permeability μ and mass density ρ is contained in a thin U-tube of circular cross-section. Consider the liquid as a linear paramagnet. When the tube is inserted between the flat poles of a permanent magnet, the liquid between the poles rises by a height h (see the figure). You may neglect all end effects. The initial field between the poles is H_0 , the permeability of the tube material is $\mu = 1$, and the inner radius of the tube is a .

- a) Find an expression for h in terms of the magnetization M in the liquid and H_0 and the acceleration due to gravity g . [13 points]
- b) By solving the appropriate magneto-static boundary value problem express M in terms of H_0 and derive an expression for h in terms of H_0 . [20 points]
- c) A paramagnetic fluid has mass density ρ of 2 g/cm^3 and a permeability of 99. How high (approximately) will the fluid rise if $\mu_0 H_0 = 0.1 \text{ T}$? [7 points]



Problem 4: An infinitely long solid insulating cylinder [40 points]

An infinitely long solid insulating cylinder has radius a and a uniform constant charge density ρ . It is rotating around the $\hat{z}$ axis, which is its axis of symmetry, in the $+\phi$ direction with angular velocity ω . The cylinder has the same permittivity and permeability as vacuum.

- (a) Find the magnetic field $\vec{B}$ everywhere. [12 points]

Now assume that $\omega = \omega_0 + \alpha t$ is a slowly increasing function of time.

- (b) Find the **total** electric field $\vec{E}$ everywhere. [16 points]

- (c) Suppose that the cylinder is surrounded by a thin circular loop of wire of radius b and resistance R . Find the current induced in the wire. [12 points]

NIU Physics Ph.D. candidacy examination Spring 2010 (2/20/2010)

Electricity and Magnetism

Solve 3 out of 4 problems.

Problem 1: Motion of a point charge under the uniform static magnetic field [40 points]

Consider a point positive charge with mass m and charge e (>0) moving in a uniform static magnetic field $\vec{B}$ parallel to the z -axis. The initial conditions for the velocity in Cartesian coordinates is at $t = 0$ are $v_x(0) = v_0$, $v_y(0) = 0$, $v_z(0) = 0$.

- Express the equation of motion in vector form as well as its components in Cartesian coordinates. [10 points]
- Using the initial conditions, find the components of the velocity of the point charge as a function of time. Evaluate the change of its kinetic energy and explain why. [15 points]
- Given the initial conditions in the $z = 0$ plane are $x(0) = y(0) = z(0) = 0$. Express the trajectory of the point charge, its radius and periodicity. [15 points]

Problem 2: A very long, metallic, grounded waveguide [40 points]

Consider a very long, metallic and grounded waveguide with a square shaped inner cross section of side D . A point charge Q is suspended on the axis of this guide at a location far from either end.

- Find the electrostatic potential everywhere inside the guide. [25 points]
- What is the asymptotic form of the potential at locations far from the point charge? [10 points]
- Make a sketch of the electric field lines in a region far from the point charge. [5 points]

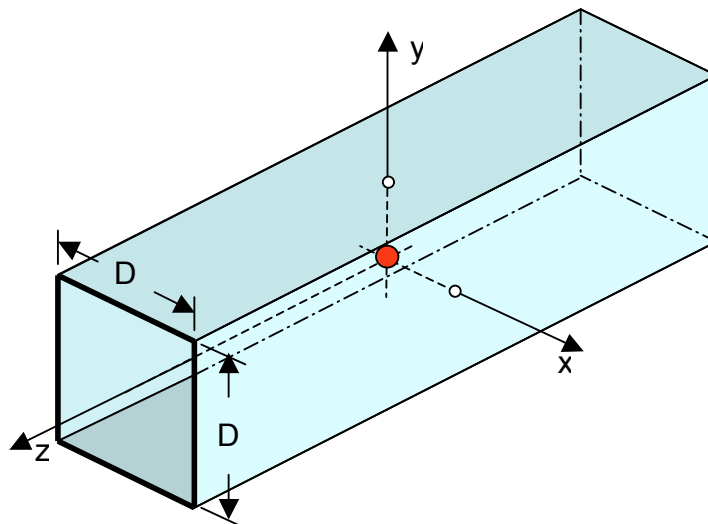


Figure 1

Problem 3: Magnetic field induced between circular parallel capacitor plates [40 points].

When a *charged* circular parallel plate capacitor is discharged by connecting a conducting wire as shown in Figure 3, a magnetic field $\vec{B}$ is induced. The radius of the parallel plates is R and the distance between the parallel plates is d ($d \ll R$). $I(t)$ is the electric current on the conducting wire. We neglect the effects of stray fields at the edges of the plates and of radiation.

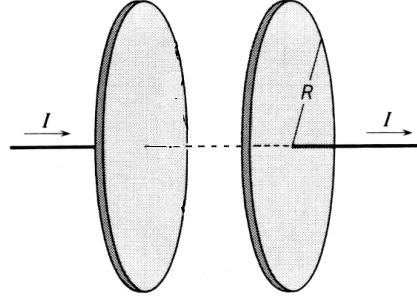


Figure 2

- Express the displacement current density in terms of the electric current $I(t)$ on the conducting wire [15 points]
- Find $\vec{B}(\vec{r}, t)$ between the plates, far from the edges. [25 points]

Problem 4: Long coaxial cable [40 points]

Consider a long coaxial cable consisting of an inner cylinder with radius a and an outer cylinder of radius b . Suppose that the electric field is given in cylindrical coordinates (r, ϕ, z) in the region $a < r < b$ by

$$\vec{E}(r, \phi, z) = Nr^n \cos(kz - \omega t) \hat{r}$$

where N , k , and ω are positive constants and n is a rational number. The region between the cylinders has the same permittivity and permeability as vacuum.

- Solve for n and determine the magnetic field for $a < r < b$. [16 points]
- Find the charge density and the current density on the inner cylinder. [8 points]
- Find an expression for the vector potential $\vec{A}$ in the region $a < r < b$. [8 points]
- Find the total time-averaged electromagnetic power transmitted by the cable. [8 points]

NIU Ph.D. candidacy examination Fall 2009 (9/19/2009)

Electricity and Magnetism

Solve 3 out of 4 problems.

Problem 1: Electrostatic potential [40 points]

Consider an electrostatic potential $\phi(\mathbf{r}) = \frac{Ae^{-\kappa r}}{r}$:

- Using the Poisson equation, find the charge density distribution in the space except at the origin. (15 points)
- The fact that the potential becomes infinity at the origin means the existence of a point charge at the origin. First obtain the electric field from the potential $\phi(\mathbf{r})$, then find the magnitude of the point charge and its sign. (15 points)
- Find the total charge distributed other than the origin, also indicate its sign. (10 points)

Problem 2: A uniformly polarized ferroelectric slab is inserted into a parallel plate capacitor [40 points]

A uniformly polarized ferroelectric slab is inserted into a parallel plate capacitor as shown in the Figure. The capacitor area is A , and the lateral sizes are much larger than a , b . The two plates of the capacitor are connected through a thin wire.

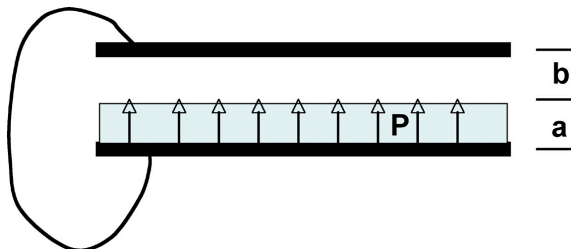


Figure 1

- Neglecting edge effects, determine the electric field everywhere in the capacitor. [16 points]
- What is the electric field inside the ferroelectric plate for $b \gg a$, and for vanishing b ? [8 points]
- Determine the force vector acting on the upper capacitor plate. [16 points]

Problem 3: A spherical shell with inner radius a and outer radius b with a uniform, fixed magnetization $\mathbf{M}$ [40 points].

A spherical shell with inner radius a and outer radius b carries a uniform, fixed magnetization $\mathbf{M}$ as shown in the figure. Using the method of separation of variables, find the vectors of the *magnetic field* and of the *magnetic induction* at:

- a) $r < a$, [8 points]
- b) $r > b$, [16 points]
- c) $a < r < b$. [16 points]

Hint: a solution for the Laplace's equation in spherical coordinates with azimuthal symmetry and z-axis along the magnetization direction.

Then the general solution (magnetic scalar potential)

$$\text{is } \Phi_m(r, \theta) = \frac{U(r)}{r} P(\theta) = \sum_{l=0}^{\infty} \left[A_l r^l + \frac{B_l}{r^{l+1}} \right] P_l(\cos(\theta)).$$

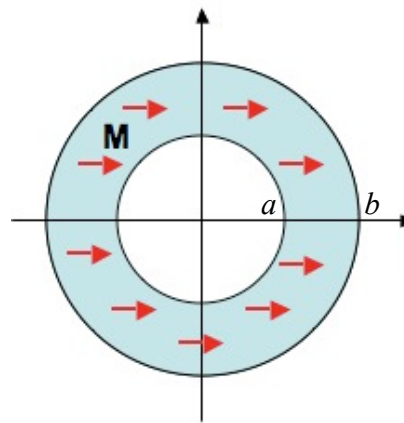


Figure 2

Problem 4: Long concentric conducting tubes [40 points]

Consider a set of long concentric conducting tubes (length l) sharing the same axis along the tubes shown in Figure 3. The radii of the tubes are a_1 and a_2 , respectively. At one end these tubes are connected via a resistor while the other ends are connected to a power supply, $\Delta\phi$.

- (a) Find the magnetic field induced by the currents with the same magnitude but with the opposite directions.
- (b) Find the self-inductance of this circuit. [10 points]

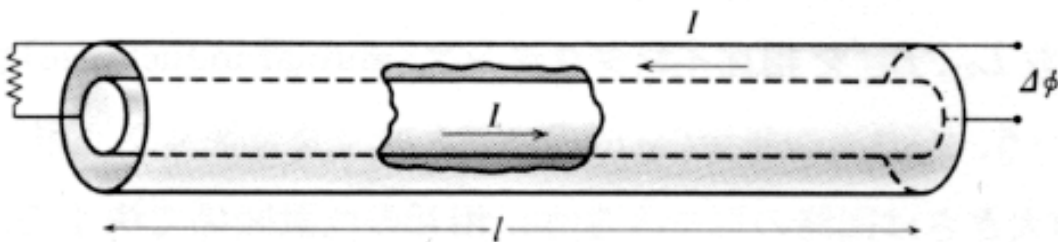


Figure 3

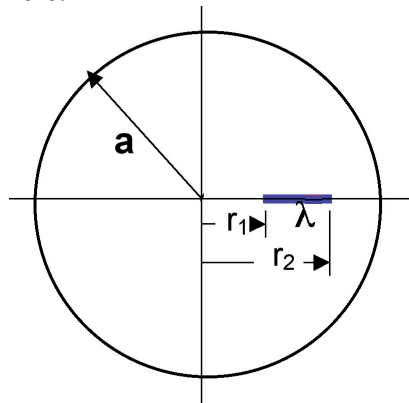
NIU Ph.D. candidacy examination Spring 2009 (2/21/2009)

Electricity and Magnetism

Solve 3 out of 4 problems.

Problem 1: The potential inside and outside of a conducting sphere containing a line charge
[40 points]

A line charge of uniform density λ is placed radially inside a grounded conducting spherical shell of inner radius a , the two ends of the line charge being at distance r_1 and r_2 from the center of the sphere.

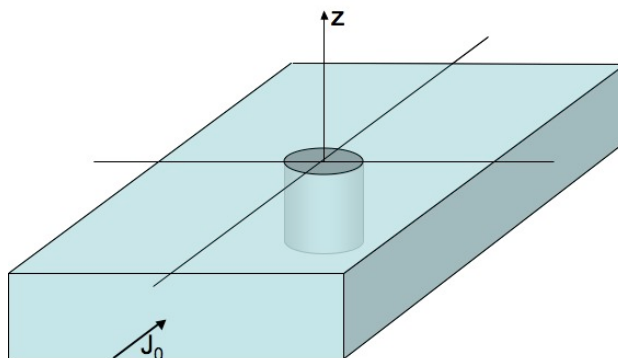


- (20 pts) Determine the image charge.
- (20 pts) What is the potential ϕ outside and inside the sphere?

Hint: $\int dx / \sqrt{1 + x^2 - 2cx} = \ln(-c + x + \sqrt{1 + x^2 - 2cx})$

Problem 2: A large conducting slab with a small hole [40 points]

A small cylindrical hole of radius a is made near the center of a large conducting slab of uniform conductivity σ carrying - prior to the introduction of the hole - a current of uniform density J_0 .



The lateral size of the slab is considered large enough that effects due to its sides can be neglected. Furthermore, its thickness is large enough that any z -axis dependence at the top and bottom faces can be neglected as well.

- (20 pts) Determine the electrostatic potential distributions in the slab and in the hole, respectively.
- (10 pts) Find the electric field in the hole.
- (10 pts) The space of the hole is filled with a material of conductivity σ' . Determine the current density in this space.

Problem 3: A cylinder filled with the uniform magnetization [40 points]

Consider a cylinder of length L and radius R centered at the origin and aligned the z -axis, filled with the uniform magnetization $\vec{M} = M\vec{z}$.

- (15 pts) Show that the magnetic scalar potential along the z -axis is

$$\Phi_M = \frac{1}{2} M \left[\sqrt{\left(z - \frac{1}{2}L\right)^2 + R^2} - \sqrt{\left(z + \frac{1}{2}L\right)^2 + R^2} + 2z \right]$$

inside the cylinder $(-\frac{1}{2}L < z < \frac{1}{2}L)$

- (5 pts) What is the magnetic scalar potential along the z -axis outside the cylinder?
- (15 pts) What is the $\mathbf{B}$ field along the z -axis inside the cylinder?
- (5 pts) What is the $\mathbf{B}$ field along the z -axis outside the cylinder?

Problem 4: The propagation of an electromagnetic wave [40 points]

Consider the propagation of an electromagnetic wave with peak electric field E_0 and angular frequency ω between two parallel perfectly conducting infinite plates spaced a distance d apart. One plate occupies the $x = 0$ plane and the other the $x = d$ plane. Take the wave to propagate in the $\hat{z}$ direction, and consider only the case that the electric field $\vec{E}$ is everywhere parallel to the plates.

- Find expressions for the electric and magnetic fields $\vec{E}$ and $\vec{B}$ between the plates. [25 points]
- Find the minimum angular frequency $\omega_{\min}$ below which waves of this type will not propagate without attenuation. [10 points]
- Find the exponential attenuation length for waves with angular frequency below $\omega_{\min}$. [5 points]

NIU Ph.D. candidacy examination Fall 2008 (9/20/2008)

Electricity and Magnetism

Solve 3 out of 4 problems.

Problem 1: The electric field inside a dielectric sphere [40 points]

A dielectric sphere, of radius a , has a uniform polarization $\mathbf{P}$, find the electric field inside the sphere $\vec{\mathbf{E}}_{polar}(\mathbf{r})$ with the following steps. Consider a model that the dielectric sphere for the positive and negative charge.

- (a) Find the electric field $\vec{\mathbf{E}}_{general}(\mathbf{r})$ inside a sphere, of radius a with a uniform charge density ρ . Take the center of the sphere as the origin. [20 points]
- (b) Consider two spheres of radius a , one with its center at $+\mathbf{u}/2$ from the origin with charge density $+\rho$, the other with its center at $-\mathbf{u}/2$ from the origin with charge density $-\rho$. Find the electric field $\vec{\mathbf{E}}_+(\mathbf{r})$ and $\vec{\mathbf{E}}_-(\mathbf{r})$ for each sphere. [10 points]
- (c) The electric field $\vec{\mathbf{E}}_{polar}(\mathbf{r})$ from the above $\vec{\mathbf{E}}_+(\mathbf{r})$ and $\vec{\mathbf{E}}_-(\mathbf{r})$, and then express $\vec{\mathbf{E}}_{polar}(\mathbf{r})$ with the polarization vector $\mathbf{P}$, assuming $|\mathbf{u}| \ll a$. [10 points]

Problem 2: A very long solenoid [40 points]

A very long solenoid, of radius a , has its axis of symmetry along the z -axis. The solenoid has N turns of wire and total length l . The wire carries a slowly time-varying current I_s , so that $dI_s/dt \neq 0$. (You may consider only regions in the solenoid but very far from the ends of the solenoid. Give both magnitude and direction for vector quantities.)

- (a) Find the electric field both inside and outside of the solenoid. [15 points.]
- (b) Find the Poynting vector both inside and outside of the solenoid. [10 points]
- (c) Consider an imaginary cylinder, coaxial with the solenoid, with length d and radius r with $r < a$. Find the rate at which energy is flowing into this cylinder from the outside. [10 points]
- (d) Repeat part (c), but now with $r > a$. [5 points]

Problem 3: Radiation associated with a particle moving at low velocity ($\beta \ll 1$) [40 points]

Consider the Lienard-Wichert formula in SI units:

$$\vec{E} = \frac{q}{4\pi\epsilon_0} \left\{ \left[\frac{\hat{n} - \vec{\beta}}{\gamma^2 R^2 (1 - \vec{\beta} \cdot \hat{n})^3} \right]_{ret} + \frac{1}{c} \left[\frac{\hat{n} \times [(\hat{n} - \vec{\beta}) \times \dot{\vec{\beta}}]}{R(1 - \vec{\beta} \cdot \hat{n})^3} \right]_{ret} \right\}.$$

- Briefly explain the use of this equation: what does it compute? What are the meanings of all the symbols it incorporates? [7 points]
- Discuss the physical meaning of the two terms in the curly brackets. [5 points]
- Write down the corresponding expression for the magnetic field as a function of $\vec{E}$. [5 points]
- We now consider the limit of very small velocities ($\beta \ll 1$):
 - Give the expression of the *radiated* electric field. [6 points]
 - Give the corresponding expression for the *radiated* magnetic field. [4 points]
 - Compute the Poynting vector associated to the *radiated* fields. [5 points]
 - Deduce the total radiated power. [8 points]

Problem 4: Optics in a doubly-negative (left handed) meta-material [40 points]

In man-made dielectric and magnetic material unusual situations arise when the electric permittivity and magnetic permeability are both real and negative. We consider a plane wave with electric field $\mathbf{E} = \mathbf{E}_0 \exp[-i(\mathbf{k} \cdot \mathbf{r} - \omega t)]$ and magnetic field $\mathbf{H} = \mathbf{H}_0 \exp[-i(\mathbf{k} \cdot \mathbf{r} - \omega t)]$.

- Let first consider a “standard” material with permittivity ϵ and magnetic permeability μ both real and positive. Find the Maxwell equations satisfied by $\mathbf{E}_0$ and $\mathbf{H}_0$ in such a medium, the velocity of light in such a medium, and what combination of $\mathbf{E}_0$, $\mathbf{k}$ and $\mathbf{H}_0$ form a right-handed set. [5 points]
- From now on we consider a doubly-negative meta-material and write $\epsilon = -|\epsilon|$ and $\mu = -|\mu|$. Write the Maxwell equations satisfied by $\mathbf{E}_0$ and $\mathbf{H}_0$ in the meta-material and show that the role of $\mathbf{E}_0$ and $\mathbf{H}_0$ are interchanged compared to the “standard” case explored in (1). [10 points]
- What can you state about the set $(\mathbf{E}_0, \mathbf{H}_0, \mathbf{k})$? [5 points]
- We now specialize to a plane wave propagating along the $\mathbf{z}$ axis and take $\mathbf{E} = E_0 \exp[-i(kz - \omega t)]\mathbf{x}$ and $\mathbf{H} = H_0 \exp[-i(kz - \omega t)]\mathbf{y}$ (where $\mathbf{u}$ stands for the unit vector along the u direction). Find the Poynting vector $\mathbf{S}$ in the meta-material and compare its direction to $\mathbf{k}$. [10 points]
- Discuss the implication of (4) to the sign of the index of refraction. Consider an incoming optical ray propagating at the interface of a “standard” material (with index of refraction $n_1 > 0$) and a meta-material (with index of refraction $n_2 = -|n_2| < 0$). Take the incident and refracted angles to be respectively θ_1 and θ_2 . Write Snell’s refraction law at the interface and draw a schematic stressing the difference(s) with the usual situation of refraction at the interface between two standard materials. [10 points]

NIU Ph.D. candidacy examination Spring 2008 (2/16/2008)

Electricity and Magnetism

Solve 3 out of 4 problems.

Problem 1: A very long hollow cylindrical shell [40 points]

A very long hollow cylindrical shell of radius a is centered on the z -axis and carries surface charge density $\sigma = \sigma_0 \cos(\phi)$.

- Find the electrostatic potential inside the shell. [20 points]
- Find the electrostatic potential outside the shell. [20 points]

Problem 2: dipole located inside a grounded conducting spherical shell [40 points]

A point dipole $\mathbf{p}$ is located at position $\mathbf{r}$ on the x -axis inside a grounded conducting spherical shell of radius R as shown in the figure. We choose the coordinate system such that the dipole lies in the xz -plane.

Determine the (vector) electric field at the center of the shell for:

- $\alpha = 0$ deg. [20 points]
- $\alpha = 90$ deg. [20 points]

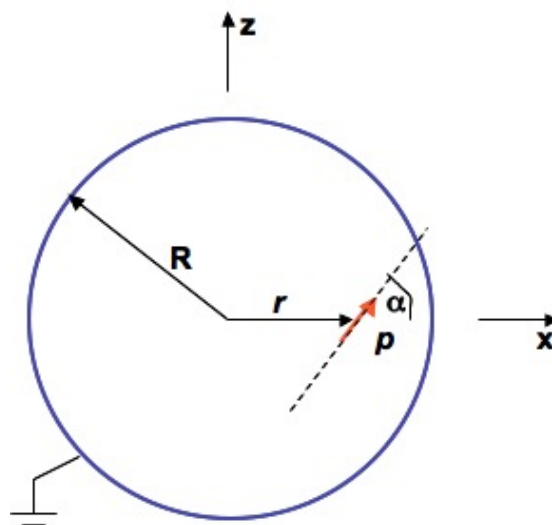


Fig. 1 A dipole located inside a grounded conducting shell

Hint: Consider the dipole $\mathbf{p}$ as composed of two charges $+q$ and $-q$ separated by a vector $\mathbf{a}$ parallel to $\mathbf{p}$ that is centered on $\mathbf{r}$. Take the limit $a \rightarrow 0$ with the condition that $\mathbf{p} = q\mathbf{a} = \text{constant}$.

Problem 3: Vector potential of an infinitely wide flat conductive plate [40 points]

The mathematical expression for the vector potential $\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{j}(\vec{r}')}{|\vec{r} - \vec{r}'|} dV'$ has a close resemblance to the expression for the electrostatic potential $\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} dV'$.

- Find the electric field of the inside of a flat plate of a thickness d with charges evenly distributed (the charge density ρ). [10 points].
- Using the above resemblance between the expressions of the vector potential and the electrostatic potential, find the vector potential under the presence of a uniform current inside an infinitely wide flat conductive plate of a thickness d . [20 points]
- Find the magnetic field $\vec{B}(\vec{r})$ for the above. [10 points]

Problem 4: Diffraction & Faraday's law [40 points]

Consider a linearly polarized (along the x -axis) plane wave of frequency ω and amplitude E_0 incident on an opaque screen with a square aperture of edge size a .

- Write down the electric and magnetic fields associated to this plane wave as a function of ω and E_0 . [10 points]
- By a simple application of Faraday's law to a loop parallel to the screen on the side away from the source (Fig. 2), show that the wave has a longitudinal magnetic field component, B_z , after it has passed through the aperture. [15 points]
- Show that the ratio B_z/B_y is a measure of the diffraction angle. [15 points]

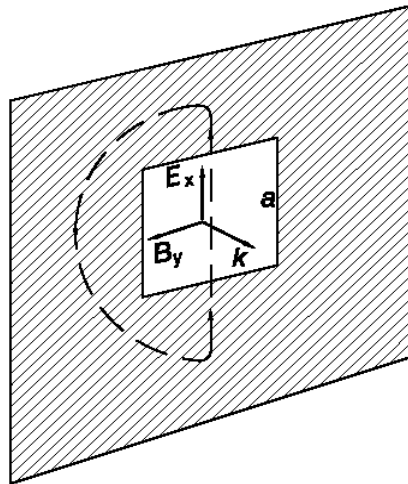


Figure 1: geometry associated to problem 1. k is the wavevector of the initial plane wave and E_x and B_y are the corresponding electric and magnetic fields. [Figure from K. T. McDonald and M. Zolotarev, *Am. J. Phys.* **68**, 674 (2000)]

NIU Ph.D. candidacy examination Fall 2007 (9/22/2007)

Electricity and Magnetism

Solve 3 out of 4 problems.

Problem 1: An infinite flat sheet of uniform charge density [40 points]

An infinite flat sheet of uniform charge density σ , located in the x, y plane, oscillates along the x -axis. The velocity of the charges is given by $\vec{v} = v_0 \hat{x} \cos(\omega_0 t)$.

- Find the electric and magnetic fields everywhere. [30 points]
- What is the average power radiated per unit area of the sheet? [10 points]

Problem 2: Meissner effect [40 points]

The magnetic scalar potential outside a spinning shell (radius R , angular velocity ω , total charge q) of charge is:

$$\Phi_{out} = -\frac{\mu_0 \omega q R^2}{12\pi r^2} P_1(\cos \theta)$$

- Show that the magnetic field outside a spinning spherical shell of charge is

$$\vec{B}(\vec{r}) = -\nabla \Phi_M(\vec{r}) = \frac{\mu_0}{4\pi} \left(\frac{3\vec{m} \cdot \vec{r}}{r^5} \vec{r} - \frac{\vec{m}}{r^3} \right)$$

where the magnetic dipole moment of the spinning sphere is

$$\vec{m} = \frac{1}{3} q R^2 \vec{\omega}. \quad [20 \text{ points}]$$

- Below the critical magnetic field H_c , type I superconductors exhibit the Meissner effect, in which all magnetic fields are excluded from the volume of the superconductor. Consider a superconducting sphere of radius a placed in an otherwise uniform magnetic induction B_0 . Since the magnetic field is excluded from the sphere, the magnetic induction B is tangent to the sphere at its surface as shown in Figure below.

- Show that the total magnetic induction outside the sphere is

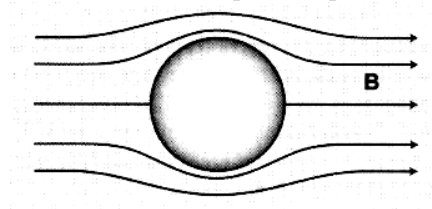
$$\vec{B}(\vec{r}) = \vec{B}_0 + \frac{\mu_0}{4\pi} \left(\frac{3\vec{m} \cdot \vec{r}}{r^5} \vec{r} - \frac{\vec{m}}{r^3} \right)$$

where the induced dipole moment is

$$\vec{m} = -\frac{2\pi a^3}{\mu_0} \vec{B}_0 \quad [10 \text{ points}]$$

- Calculate the energy required to place the sphere in the magnetic field.

[hint: the magnetic field induction exerts a pressure on the surface of the sphere. Compute δW done on the field first when the radius of the sphere expands from r to $r + \delta r$.] [10 points]



Magnetic field excluded from a superconducting sphere.

Problem 3: Space charge limited current in a diode [40 points]

Consider the space charge-limited flow of current in a planar vacuum diode. We assume that the cathode represents an unlimited source of electrons at zero velocity that are accelerated by the electric field toward the anode. The electrons in the anode-cathode gap form a space charge density

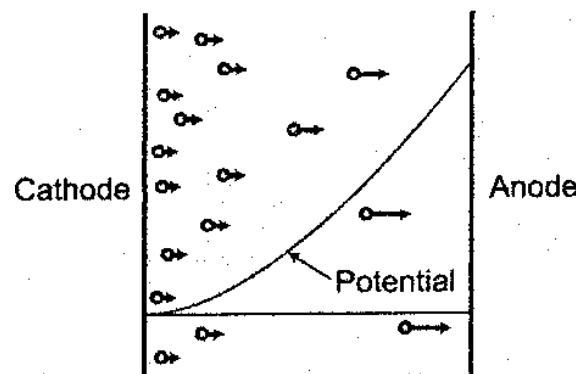
$$\rho = \frac{J}{v}$$

where J is the current density and v the electron velocity, that modifies the field and repels the electrons leaving the cathode. The current becomes limited when the electric field at the surface of the cathode vanishes.

- Express the Poisson's equation for this problem for non-relativistic motion of electrons. [20 pints]
- For nonrelativistic motion of the electrons, calculate the space charge-limited current in the diode. V is the potential of the anode relative to the cathode. The quantity

$$K = \frac{4\epsilon_0 A}{9d^2} \sqrt{\frac{2e}{m}}$$

where A is the area of the cathode, d the anode-cathode separation, $e = |q|$ the magnitude of the electron charge, and m the electron mass, is called the perveance of the diode. [20 points]

**Problem 4: Paradox?** [40 points]

Consider a point charge q moving at constant relativistic velocity v along the z -axis.

- Write down the expression for the particle's electric field in all direction perpendicular to its velocity. How does the field depend on γ the Lorentz factor? [10 points]
- Compute the electric flux Φ_E from the particle using Gauss's law. **[Hint: the integral**

$$\int_0^\pi \frac{\sin z}{[1 - \beta^2 \sin^2 z]^{3/2}} dz = 2\gamma^2 \text{ might turn out to be useful...}]. \quad [20 \text{ points}]$$

- Compare your answer from (1) and (2) and try to explain the apparent "discrepancy" in the γ -dependence. [10 points]

NIU Ph.D. candidacy examination spring 2007 (2/17/2007)

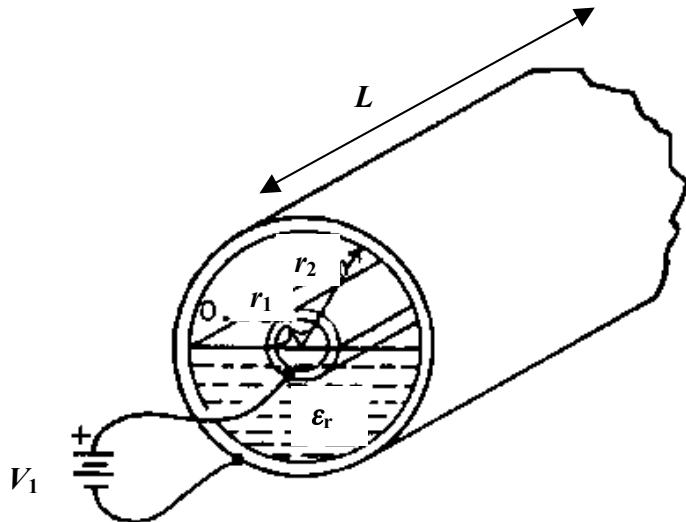
Electricity and Magnetism

Solve 3 out of 4 problems.

Problem 1: Dielectrics [40 points]

The space between two coaxial conducting cylinders of length L is half-filled with a dielectric having relative dielectric constant ϵ_r . The cylinders have radius r_1 and r_2 , as shown in fig, and are connected to a V_1 battery.

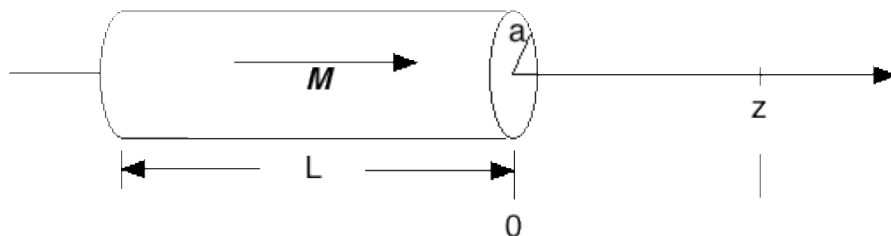
- A. Find the fields $\vec{E}$ and $\vec{D}$ in the air and in the dielectric in the space $r_1 < r < r_2$. [20 points]
- B. Find the surface charge induced on the inner conductor at points adjacent to the air, and at points adjacent to the dielectric. [10 points]
- C. Find the total charge on the inner conductor, and the capacitance. [10 points]



Problem 2: A cylindrical permanent magnet [40 points]

A cylindrical permanent magnet of length L and radius a has uniform magnetization $\vec{M}$ directed along the axis of the magnet (see Figure).

- A. Find the magnetic field outside the magnet on the z axis. [20 points]
- B. Analyze the limit $z \gg a, L$ and compare the result to the field of a magnetic dipole. [20 points]



Problem 3: Electromagnetic waves in a conductor? [40 points]

We consider a homogenous and isotropous medium with $\epsilon_r \sim 1$ and $\mu_r \sim 1$. We assume the medium to be neutral conductor which is well described by Ohm's law with conductivity σ .

- A. Write the Maxwell's equations satisfied by the electric $\vec{E}$ and magnetic $\vec{B}$ fields in the medium and deduce a second order partial derivative equation for $\vec{E}$. Show that when $\sigma \rightarrow 0$ the usual vacuum wave equation is recovered. [10 points]
- B. We now seek the solution of the latter equation by considering the electric field to be of the form $\vec{E} = E_0 \exp[i(\omega t - kx)]\hat{z}$, where E_0 is a constant, ω is a positive real number, and $\hat{z}$ is the unit vector along the z -axis.
 - a. Find the expression of k^2 as a function of $\omega, \sigma, \epsilon_0$ (the vacuum electric permittivity), and c (the velocity of light). [10 points]
 - b. Consider a Copper conductor, with a conductivity $\sigma = 5.7 \times 10^8 \Omega^{-1} \cdot \text{m}^{-1}$. Find the frequency domain where $\frac{\sigma}{\epsilon_0 \omega} \gg 1$. Is this approximation applicable for typical AC electrical frequencies and typical frequencies associated to the visible spectrum? [10 points]
 - c. Assuming $\frac{\sigma}{\epsilon_0 \omega} \gg 1$, compute k as a function of $\omega, \sigma, \epsilon_0$, and c , and deduce the general form of the electromagnetic wave in the conductor and give the solutions for the electric and magnetic fields. [10 points]

Problem 4: Electromagnetic field created by a moving charge [40 points]

The figure below describes the trajectory of a particle with charge $q > 0$ and constant velocity $(3/5)c$ (where c is the velocity of light). The change of velocity direction at $(x=0, y=0)$ is instantaneous. At $t=0$, the charge is located at $(x=0, y=a)$.

- A. At $t=0$, what is the electric field at points P ($x=0, y=a$) and Q ($x=2a, y=a$) [a simple sketch showing the E-field directions at both points along with a qualitative (and clear) discussion of your sketch are required]. [20 points]
- B. Plot the electric field lines at $t=0$ in the (x,y) plane. [10 points]
- C. Plot the magnetic field lines at $t=0$ in the (x,y) plane. [10 points]

