6.1 Graphene Hamiltonian

Verify that the "Dirac Hamiltonian" for the graphene K point can be derived by the theory of invariants using our previous results for the point group C_{3v} ($\simeq \mathcal{P}_3$).

- Derive all tensor operators $\mathcal{K}^l$ up to second order in $\mathbf{k} = (k_x, k_y)$.
- Derive the basis matrices $\mathcal{X}^{l*}$.
- Derive the Hamiltonian $\mathcal{H}(\mathcal{K})$.

Remark: The full theory needs to take into account two subtleties we have ignored here [cf. Winkler and Zülicke, PRB **82**, 245313 (2010)]:

- Strictly speaking, at the K point of the graphene Brillouin zone the group of the wave vector is $D_{3h} = C_{3v}$ + inversion.
- Additional constraints arise for $\mathcal{H}(\mathcal{K})$ due to the fact that we have two non-equivalent points K and K' in the Brillouin zone which are related to each other both by spatial symmetry operations of the graphene space group and by time reversal symmetry. (For similar reasons the "Dirac Hamil;tonian" is generally forbidden to occur in spinless models at the Γ point $k = 0$ of the Brillouin zone.)

However, despite these details our simplified approach gives us the correct Hamiltonian up to second order in $\mathbf{k}$.