Instructions - Part 1

This qualifying exam is scored based on the answers to 25 problems. Each problem is worth 4 points, and partial credit will be awarded for work done on a problem.

The problems are divided into six areas of physics, and three areas will be covered in each two hour period. In each area there is a choice of problems to solve. If additional problems are attempted the best results will be selected for the score on the test.

Part 1 (2 hours)

Special Relativity (4 points): solve 1 of 2 problems.

Classical Mechanics (24 points): solve 6 of 8 problems.

Electromagnetism and Electronics (24 points): solve 6 of 8 problems.

You may use a calculator to find specific values required for some problems.

Useful constants for part 1:

$g = 9.8$ m/s^2

$G = 6.67 \times 10^{-11}$ N m^2 / kg^2

$R = 8.134$ J/mol-K

$c = 3.00 \times 10^8$ m/s

$e = 1.60 \times 10^{-19}$ C

$\varepsilon_0 = 8.85 \times 10^{-12}$ C^2 / N.m^2

$\mu_0 = 4\pi \times 10^{-7}$ T m / A

$m_e = 0.511$ MeV/c^2, $m_p = 938$ MeV/c^2

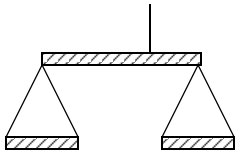
Special Relativity: complete 1 problem

1. An observer in reference frame S' sees an object traveling at $0.8c$ at right angles to the x' axis. Reference frame S is oriented with its x-axis parallel to x'. An observer in reference frame S sees S' have a velocity of $0.8c$ in the x-direction. Find the angle of the velocity (in degrees) of the object from the x-axis as observed in S.

2. The rapidity ϕ is a hyperbolic angle in Minkowski space linking the time and space coordinates, where $\phi = \text{arctanh}(v/c)$. Write down the transformation matrix for one-dimensional motion from (x, ct) to (x', ct') in terms of ϕ.

Classical Mechanics: complete 6 problems

1. A balance is made of a rigid rod free to rotate at a point not at the center of the rod and is balanced with unequal weight pans at each end as shown at right. When an unknown mass m is placed in the left pan it is balanced by a mass m_1 in the right pan. When mass m is placed in the right pan it is balanced by a mass m_2 in the left pan. Find m from the values of m_1 and m_2.



2. A mass is hanging from a string to form a pendulum. The mass is released when the string is horizontal. Find the magnitude of the acceleration of the mass as a function of θ, the angle of the string from the initial horizontal position.

3. Ball A with mass 1 kg is moving at a speed of 5 m/s. Ball A strikes a glancing blow in an elastic collision with ball B of unknown mass at rest. After the collision ball A moves at right angles to its initial direction at 4 m/s. Find the magnitude of the momentum of the second ball immediately after the collision.

4. A mass M is hung from a spring of non-negligible mass m and spring constant k. Find the period T of the motion of the mass.

5. Two particles of mass m and M are initially at rest and infinitely separated from each other. The only force acting on them is gravity. Find the relative velocity v of the masses as a function of their separation d.

6. The degrees of freedom describe the number of independent variables needed to describe the motion of each point in a rigid body. Consider a disk constrained to roll without slipping on a planar surface such that the plane of the disk remains perpendicular to the surface. Describe the number of degrees of freedom for the velocity of the disk along with a brief explanation.

7. A rigid circular disk in the xy-plane rotates about its symmetry axis parallel to the z-axis at an angular velocity equal to ω. Find the magnitude of the curl of the velocity vector field at a point located a distance r from the axis of rotation.

8. A particle of mass m is constrained to move along a straight line and is attracted towards a point on the line with a force proportional to the distance x from the point. Find the Lagrangian function for this system.

Electromagnetism and Electronics: complete 6 problems

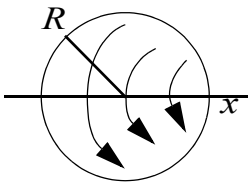
1. Assume classical dynamics were true even at high velocities. Find the electric potential in volts needed to accelerate an electron from rest to the speed of light.

2. A conducting sphere of radius a is surrounded by a dielectric shell of inner radius b and outer radius c. A total charge Q is placed on the conducting sphere. Find an expression for the magnitude of the electric field within the dielectric at a distance r from the center of the conducting sphere (that is, $b < r < c$) in mks units.A resistor in a circuit dissipates 0.25 W of energy. What is the rate of energy dissipation if the voltage across the resistor is increased from 5.0 V to 7.5 V?

3. An electric generator dissipates energy at a rate of 20 W when it establishes a potential difference of 120 V with a current of 2 A. Find the emf of the generator.

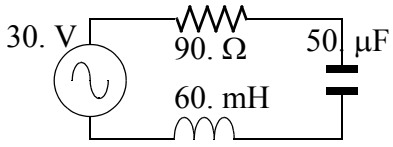
4. The collector current in an npn transistor in a circuit is initially 10 mA with a collector-emitter voltage of 5 V and base current of 200 μA. The collector voltage is increased by 1 V and the collector current increases to 11 mA. Starting from the initial conditions an increase of the base current by 10 μA also increases the collector current to 11 mA, Find the gain of the transistor in the circuit.

5. A conducting spherical shell of radius R has a uniform surface current per unit length of J. The sphere is centered at the origin and the current flows around the x-axis as shown at right. Find the magnitude of the B-field at the center of the sphere.



6. A circular coil with a radius of $a = 5.0$ cm has $N = 100$ turns. A current of 0.1 A flows through the coil. The coil is turned in a magnetic field of 1.5 Wb/m^2 through 180^o starting from a position where the plane of the coil is perpendicular to the field. Find the work in J needed to turn the coil.

7. In the circuit at right a 30. V ac source at 60 Hz is connected to a 90. Ω resistor, 50. μF capacitor, and 60. mH inductor in series. Find the tangent of the phase of the current through the circuit with respect to the voltage across the source.



8. A monochromatic electromagnetic wave in a vacuum is incident normally on a dielectric material with a relative permittivity of 2.25 and a relative permeability of 1. Find the ratio of the magnitude of the electric vector of the transmitted wave to the incident wave.

Instructions - Part 2

This qualifying exam is scored based on the answers to 25 problems. Each problem is worth 4 points, and partial credit will be awarded for work done on a problem.

The problems are divided into six areas of physics, and three areas will be covered in each two hour period. In each area there is a choice of problems to solve. If additional problems are attempted the best results will be selected for the score on the test.

Part 2 (2 hours)

Quantum Mechanics (12 points): solve 3 of 4 problems.

Atomic, Nuclear, and Particle Physics (12 points): solve 3 of 4 problems.

Waves and Optics (12 points): solve 3 of 4 problems.

Thermodynamics and Statistical Physics (12 points): solve 3 of 4 problems.

You may use a calculator to find specific values required for some problems.

Useful constants for part 2:

$c = 3.00 \times 10^{8}$ m/s

$h = 6.63 \times 10^{-34}$ J s $= 1.24 \times 10^{-6}$ eV m/c; $hc = 1240$ eV nm

$m_e = 0.511$ MeV/c^2, $m_p = 938$ MeV/c^2

$N_A = 6.02 \times 10^{23}$ mol^{-1}

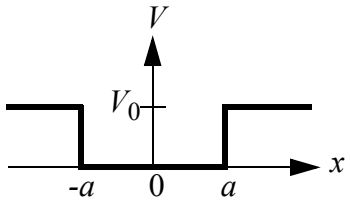
$k = 1.38 \times 10^{-23}$ J/K

$R = 8.314$ J / mol K

$\sigma = 5.67 \times 10^{-8}$ W / m^2 K^4

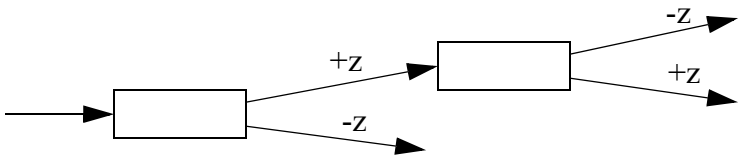
Quantum Mechanics: complete 3 problems

1. Consider the one-dimensional potential well shown at right. The sides of the well are changed from a to b, where $b < a$. Describe what happens to energy levels due to the change (ie how do the overall levels change and how do the gaps between adjacent levels change).



2. A neutron $m_n = 940.$ MeV/c^2 is at thermal equilibrium with matter at 300. K. Find the deBroglie wavelength in nm.

3. The following figure shows an unpolarized beam of electrons being sent through two Stern-Gerlach analyzers where the second is rotated 180° compared to the first. The first analyzer separates the beams into spins in the +z and -z direction, and the second splits the +z beam into -z and +z beams. Find the fraction of electrons from the initial beam that end up in the final +z beam.



4. The wavefunction for a particular hydrogen-like atom is $\psi(r, \theta, \phi) = Nr^2 e^{-Zr/3a_0} e^{2i\phi} \sin^2\theta$. Find the eigenvalue of the angular momentum operator L_z.

Atomic, Nuclear, and Particle Physics: complete 3 problems

1. A gamma ray photon from Cs^{137} is incident on a sample of U^{238} and ejects photoelectrons from the K-shell with a binding energy of 116 keV. A magnetic spectrometer (using a field to bend the photoelectrons in a semicircle) measures the photoelectrons and yields a value of $Br = 3.083 \times 10^{-3}$ T m. Find the energy of the incident photon in keV.

2. Let E_K and E_L be the energy of an electron in the K-shell and L-shell respectively and assume that the energy to remove a second electron from a shell is approximately equal to the energy required to remove the first electron. The Auger effect occurs when an electron is emitted from the L-shell as another L-shell electron fills a vacancy in the K-shell. Find the kinetic energy of the emitted Auger electron in terms of the shell energies.

3. The decay of a radioactive sample obeys the following relationship, $N = N_0 e^{-\lambda t}$. Identify all the variables in the relationship and write an expression for the half-life of the sample and for the mean lifetime of the sample.

4. A fellow student proposes the reaction $\bar{p} + \mu^+ \rightarrow n + \bar{\nu}_\mu$. Determine if the reaction is allowed, and if it is forbidden identify which conservation laws are violated.

Waves and Optics: complete 3 problems

1. A vibrating string 0.40 m long with a linear mass density of 4.0 g/m produces a fundamental pitch at a frequency of 384 Hz. Find the tension of the string when the fundamental pitch is one octave higher.

2. A ripple tank is set up with two point source wave generators operating at the same frequency. A series of nodal lines are observed on the surface of the water in the tank. Draw a diagram typical of what one would observe in the tank and describe the phase difference on a nodal line between the waves from the two sources.

3. A camera lens consists of two components in close combination. One is a converging lens of 100. mm focal length and the other is a diverging lens of 150. mm focal length. Find the focal length of this combination of lenses.

4. A plane wave solution of the electromagnetic wave equation is

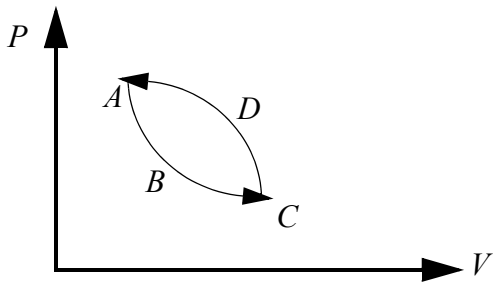
$$\vec{E} = \hat{y}E_{0y}\cos(\omega t - kx + \alpha) + \hat{z}E_{0z}\cos(\omega t - kx + \beta)$$

State the conditions on α, β, E_{0y}, and E_{0z} such that this wave is circularly polarized.

Thermodynamics and Statistical Physics: complete 3 problems

1. Two identical calorimeters A and B have equal volumes of water at 20.0 ^{o}C. A 5 g piece of Al (specific heat 0.887 J/K) at 40.0 ^{o}C is placed in A and it reaches an equilibrium temperature of 22.0 ^{o}C. A 5 g piece of an unknown metal alloy at 40.0 ^{o}C is placed in B and it reaches an equilibrium temperature of 21.5 ^{o}C. Find the specific heat of the alloy.

2. Consider the PV diagram with paths ABC and ADC at right and answer the following questions. Include a brief explanation for each answer. a) Which path if any is generally at a higher temperature? b) Which path if any involves heat lost to the environment? c) Is there a net flow of work into or out of the system during one complete cycle?



3. A cylinder with a piston has a volume V at temperature T and blackbody radiation is maintained in the cylinder at equilibrium with the walls. The piston is drawn out slowly while maintaining the same temperature of the walls until the volume is $2V$. Find the work done by the radiation against the piston.

4. A sealed and thermally insulated container of total volume V is divided into two equal volumes by an impermeable wall. The left half of the container is initially occupied by 1 mole of an ideal monatomic gas at 0 ^{o}C and the right half is a vacuum. The wall is removed and the gas expands to fill the entire volume. Find the change in internal energy of the gas in J.