Math 110Q FINAL EXAM Fall 2015

No graphing calculators Name Key

Section P_____

PART 1: TRUE or FALSE

Circle TRUE or FALSE, whichever is correct. (2 points each)

1. $2^{-3} = -6$ TRUE (FALSE)

$\frac{1}{2^3} = \frac{1}{8}$

2. $x^2(x+1) + (x+1) = (x+1)x^2$ for all real numbers x. TRUE (FALSE)

$= (x+1)(x^2+1)$

3. If $-7x \leq 21$, then $x \geq -3$. (TRUE) FALSE

4. The relation $\{(2, 6), (5, 8), (3, 4), (7, 4)\}$ represents a function. (TRUE) FALSE

5. $\log_{10} x = \log x$ (TRUE) FALSE

PART 2: FILL-IN-THE-BLANK (2 points each)

6. Perform the indicated operation and simplify. $5x^4(x^3 - 4x^2 + 2)$ Answer $5x^7 - 20x^6 + 10x^4$

7. To solve the equation $x^2 + 4x = -2$ by completing the square, the first step is to add 4 to both sides. $(\frac{4}{2})^2 = (2)^2 = 4$

8. Write the following inequality using interval notation. $x \leq -7$ Answer $(-\infty, -7]$

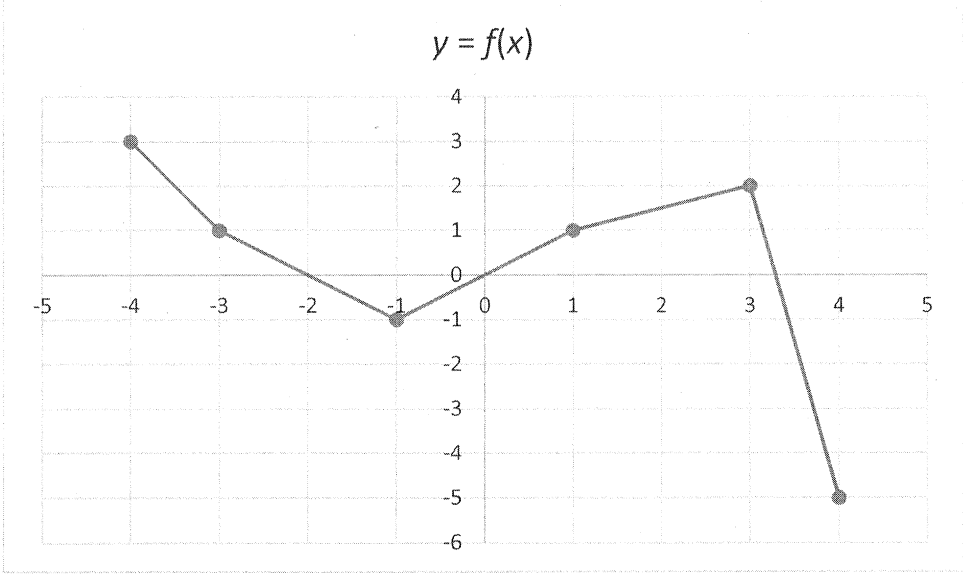
9. The slope of a vertical line is Undefined.

10. If the graph of a function is odd, then it is symmetric with respect to the origin.

11. Consider the following line: $3x + 6y = 10$. Find the slope of a line that is perpendicular to the line.

$m = -\frac{3}{6} = -\frac{1}{2}$ $m_\perp = 2$ Answer 2

12. Consider the following function $y = f(x)$.



a. What is the domain of f? $[-4, 4]$

b. What is the range of f? $[-5, 3]$

c. Find the intervals on which the function is decreasing? $(-4, -1) \cup (3, 4)$

d. $f(0) =$ 0

e. For what values of x does $f(x) = 0$? $-2, 0, \approx 3\frac{1}{4}$

13. Write the function whose graph is the graph of $y = \sqrt{x}$, but is reflected about the x-axis.

Answer $f(x) = -\sqrt{x}$

14. Write the function whose graph is the graph of $y = x^3$, but is shifted down four units.

Answer $g(x) = x^3 - 4$

15. Write the function whose graph is the graph of $y = x^2$, but is shifted left five units.

Answer $h(x) = (x+5)^2$

16. Consider the following rational function.

$$R(x) = \frac{3x^2}{x^2 - 36}$$

$(x-6)(x+6)$

a. Find the horizontal asymptotes.

Answer $y = 3$

b. Find the vertical asymptotes.

$x-6=0$ $\quad x+6=0$

$+6 \quad +6 \quad -6 \quad -6$

$x=6 \quad x=-6$

Answer ~~$\{x \mid x \neq \pm 6\}$~~ $x = 6, -6$

17. Change the exponential statement to an equivalent statement involving a logarithm.

$$\frac{1}{64} = 4^{-3}$$

Answer $\log_4 \frac{1}{64} = -3$

18. List the potential rational zeros of f.

$$f(x) = x^3 - 8x^2 + 16x - 8$$

Answer $\pm 1, \pm 2, \pm 4, \pm 8$

PART 3: MULTIPLE CHOICE

Circle the letter of the correct answer. (2 points each)

19. Consider the following quadratic function.

$$f(x) = 3x^2 - 12x + 17$$

Determine whether there is a maximum or minimum value, and find that maximum or minimum value.

a. minimum value; 5

b. maximum value; 2

c. minimum value; 2

d. maximum value; 5

e. none of these

$x = -\frac{b}{2a} = \frac{-(-12)}{2(3)} = \frac{12}{6} = 2$

$f(2) = 5$

20. Consider the point (– 5, 4). Find the point that is symmetric to it with respect to the origin.

a. (– 5, 4) b. (– 5, – 4) c. (5, 4) d. (5, – 4) e. none of these

PART 4: OPEN-ENDED

Show all of your work clearly on this exam for full credit. ***You must circle your final answer***!!

21. (4 points) Reduce the following expression to lowest terms. $\frac{x^2 - 9x + 18}{2x - 6} = \frac{(x-3)(x-6)}{2(x-3)}$

$\frac{x-6}{2}$

22. Simplify each expression so that all exponents are positive. Whenever an exponent is negative or zero, assume that the base does not equal zero.

a. (4 points) $\dfrac{12xy^5z^3}{20xy^{-4}z^8} = \dfrac{3y^5y^4}{5z^3}$

$\dfrac{3y^9}{5z^3}$

b. (5 points) $\left(27x^{\frac{27}{2}}y^{\frac{-9}{2}}\right)^{\frac{2}{3}} = \sqrt[3]{27}\,x^9y^{-3} = \dfrac{3^2x^9}{y^3}$

$\dfrac{9x^9}{y^3}$

23. (4 points) Find the quotient and remainder when $x^3 - 6x^2 + 11$ is divided by $x - 2$.

$$\begin{array}{r|rrrr} 2 & 1 & -6 & 0 & 11 \\ & & 2 & -8 & -16 \\ \hline & 1 & -4 & -8 & -5 \end{array}$$

quotient: $x^2 - 4x - 8$

remainder: -5

24. (5 points each) Perform the indicated operations and simplify. Leave your answer in factored form.

a. $\dfrac{\dfrac{3x^2+5x+2}{x^2-3x-4}}{\dfrac{3x+2}{3x-15}}$

$\dfrac{3x^2+5x+2}{x^2-3x-4} \cdot \dfrac{3x-15}{3x+2}$

$\dfrac{(3x+2)(x+1)}{(x-4)(x+1)} \cdot \dfrac{3(x-5)}{(3x+2)}$

$\dfrac{3(x-5)}{x-4}$

b. $\dfrac{\dfrac{3}{x-5}-\dfrac{2}{x+4}}{(x+4)(x-5)}$

$\dfrac{3(x+4)-2(x-5)}{(x+4)(x-5)} = \dfrac{3x+12-2x+10}{(x+4)(x-5)}$

$\dfrac{x+22}{(x+4)(x-5)}$

25. (4 points) Perform the indicated operation and simplify. Leave you answer in radical form.

$2\sqrt{48}-\sqrt{75}$

$6 \cdot 8 \quad 25 \cdot 3$

$3 \cdot 2 \cdot 2 \cdot 2$

$2 \cdot 2 \cdot 2\sqrt{3} - 5\sqrt{3} = 8\sqrt{3} - 5\sqrt{3}$

$3\sqrt{3}$

26. (5 points) Solve the following equation. $\left(\frac{x}{x-2}\right)=\left(\frac{2}{x-2}-1\right)$ $(x-2)$

$x = 2-(x-2)$

$x = 2-x+2$

$2x = 4$

$\frac{2x}{2} = \frac{4}{2}$

$x = 2$

$\frac{2}{2-2} = \frac{2}{2-2} - 1$

$\frac{2}{0} = \frac{2}{0} - 1$

$\therefore$

$\emptyset$

27. (5 points) President Baker has \$3.20 in his pocket, all in dimes and quarters. He has a total of twenty coins. How many dimes and how many quarters does President Baker have in his pocket? **Write an equation and use it to answer the question.**

let d = # of dimes

q = # of quarters

$d+q=20$

$.10d+.25q=3.20$

$-.10(d+q) = -.10(20)$

$-.1d-.1q=-2$

$-.1d-.1q=-2$

$.1d+.25q=3.2$

$.15q = 1.2$

$\frac{.15q}{.15} = \frac{1.2}{.15}$

$q = 8$

$d+8=20$

$-8 \quad -8$

$d = 12$

There are 12 dimes and 8 quarters in President Baker's packet.

28. (5 points each) Solve each equation.

a. $x(x+8) = 20$

$x^2+8x=20$
$-20\ -20$

$x^2+8x-20=0$

$(x+10)(x-2)=0$

$x+10=0$ $\quad x-2=0$
$-10\ -10$ $\quad +2\ +2$
$x=-10$ $\quad x=2$

$\{-10, 2\}$

b. $x^2 - 6x + 4 = 0$

$x = \frac{-(-6) \pm \sqrt{(-6)^2-4(1)(4)}}{2(1)}$

$= \frac{6 \pm \sqrt{36-16}}{2}$

$= \frac{6 \pm \sqrt{20}}{2} < \frac{4}{5}$

$= \frac{6}{2} \pm \frac{2\sqrt{5}}{2}$

$= 3 \pm \sqrt{5}$

$x^2-6x+4=0$
$-4\ -4$

$\left(\frac{-6}{2}\right)^2$
$(-3)^2$
9

$x^2-6x+9 = -4+9$

$\sqrt{(x-3)^2} = \pm\sqrt{5}$

$x-3 = \pm\sqrt{5}$
$+3\ +3$

$x = 3 \pm \sqrt{5}$

c. $\sqrt{24-2x} = x$ (both sides squared)

$24-2x = x^2$
$-24\ +2x$

$0 = x^2+2x-24 = (x+6)(x-4)$

$x+6=0$ $\quad x-4=0$
$-6\ -6$ $\quad +4\ +4$

$x = -6, 4$

$\sqrt{24-2(-6)} = -6$
$\sqrt{24+12} = -6$
$\sqrt{36} = -6$
$6 = -6$ ✗

$\sqrt{24-2(4)} = 4$
$\sqrt{24-8} = 4$
$\sqrt{16} = 4$
$4 = 4$ ✓

$\{4\}$

d. $|3x - 6| = 12$

$3x-6 = \pm 12$
$+6\ +6$

$\frac{3x}{3} = \frac{6 \pm 12}{3}$

$\frac{6+12}{3} = \frac{18}{3} = 6$

$\frac{6-12}{3} = \frac{-6}{3} = -2$

$\{6, -2\}$

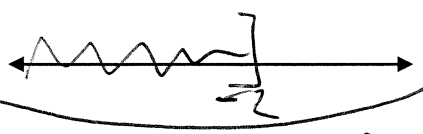
29. (5 points each) **Solve** the inequality, **graph** the solution set and write the solution in **interval notation**.

a. $-5x - 4 \geq 6$

$+4 \quad +4$

$\frac{-5x}{-5} \geq \frac{10}{-5}$

$x \leq -2$ $(-\infty, -2]$



b. $x(x+5) \geq 6$

$x(x+5) = 6$

$x^2 + 5x = 6$

$-6 \quad -6$

$x^2 + 5x - 6 = 0$

$(x+6)(x-1) = 0$

$x+6=0 \quad x-1=0$

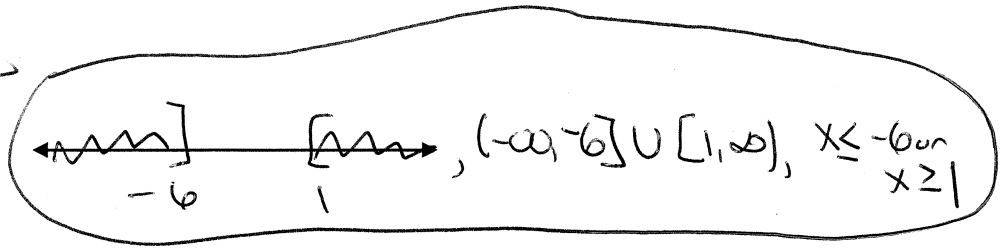
$-6 \quad -6 \quad +1 \quad +1$

$x = -6, 1$

$(x)^2 + 5(x) - 6$ $- \cdot -$ $+ \cdot -$ $+ \cdot +$

$(x+6)(x-1)$ (+) (−) (+)

$(-\infty) \quad -6 \quad (0) \quad 1 \quad (\infty)$



$(-\infty, -6] \cup [1, \infty)$, $x \leq -6$ or $x \geq 1$

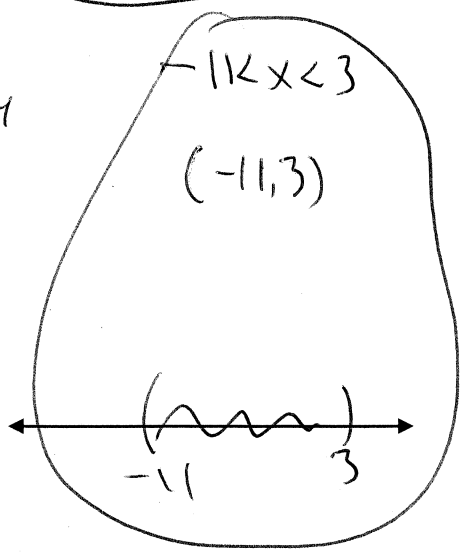
c. $|x+4| < 7$

$-7 < x+4 < 7$

$-4 \quad -4 \quad -4$

$-11 < x < 3$

$(-11, 3)$



30. (4 points) Find the exact distance between the pair of points (3, – 4) and (– 5, 3).

$d = \sqrt{(-5-3)^2+(3+4)^2} = \sqrt{(-8)^2+(7)^2} = \sqrt{64+49} = \sqrt{113}$

31. (5 points) Consider the following pair of points. (12, 5) and (– 9, – 2)

Find the equation of the line containing these points in slope-intercept form or general form.

$m = \frac{-2-5}{-9-12} = \frac{-7}{-21} = \frac{1}{3}$

$y = mx+b$

$y = \frac{1}{3}x+b$

$5 = \frac{1}{3}(12)+b$

$5 = 4+b$

$-4 \quad -4$

$1 = b$

$y = \frac{1}{3}x+1$

$Ax+By=C$

$1x-3y=C$

$1(12)-3(5)=C$

$12-15=C$

$-3=C$

$x-3y=-3$

32. (5 points) Find the center and the radius of the following circle.

$x^2+y^2+4x-12y-9=0$

$x^2+4x+\underline{4}+y^2-12y+\underline{36} = 9+\underline{4}+\underline{36}$

$+9 \quad +4$

$\left(\frac{4}{2}\right)^2 \quad \left(\frac{-12}{2}\right)^2$

$(2)^2 \quad (-6)^2$

$4 \quad 36$

$(x+2)^2+(y-6)^2 = 49$

center: $(-2, 6)$

radius: $r = 7$

33. (4 points each) Consider the following line. $3x + 6y = 10$

a. Find an equation for the line which is parallel the line containing the point $(-4, 5)$ in slope-intercept form. $m = -\frac{A}{B} = -\frac{3}{6} = -\frac{1}{2}$ $m_{\perp} = -\frac{1}{2}$

$y = mx + b$ $y = -\frac{1}{2}x + b$ $5 = -\frac{1}{2}(-4) + b$ $5 = 2 + b$

$-2 \quad -2$

$3 = b$

$y = -\frac{1}{2}x + 3$

$+\frac{1}{2}x \quad +\frac{1}{2}x$

$2\left(\frac{1}{2}x + y = 3\right)2$

b. (3 points extra credit) Write the equation you found above in a. in general form.

$x + 2y = 6$

34. (4 points each) Find the domain of each function.

a. $f(x) = \dfrac{x-2}{x^2 - 2x - 15}$ $= \dfrac{x-2}{(x-5)(x+3)}$

$x - 5 = 0 \quad x + 3 = 0$

$+5 \quad +5 \quad -3 \quad -3$

$x = 5, -3$

$\{x \mid x \neq 5, -3\}$

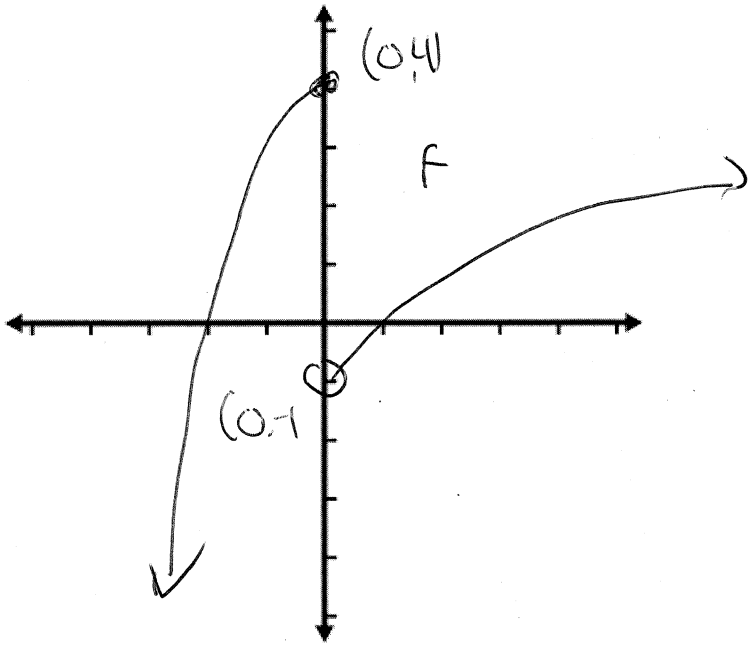
b. $g(x) = \log_6(7x - 3)$ $7x - 3 > 0$

$+3 \quad +3$

$\frac{7x}{7} > \frac{3}{7}$ $x > \frac{3}{7}$

$\left(\frac{3}{7}, \infty\right)$

35. (5 points) Graph the following function. $f(x)=\begin{cases}4-x^2 & \text{if } x\le 0\\ \sqrt{x}-1 & \text{if } x>0\end{cases}$



36. Consider the following pair of functions. $f(x) = x^2 - x + 1$; $g(x) = 4x + 3$

a. (4 points) Find $f(-3)$. $=(-3)^2-(-3)+1$

$9+3+1$

$\boxed{13}$

b. (5 points) Find $(f \circ g)(1)$. $=f(g(1)) = f(4(1)+3) = f(4+3) = f(7)$

$=(7)^2-(7)+1 \quad =49-7+1 \quad =42+1 \quad =\boxed{43}$

c. (5 points) Find $(g \circ f)(x)$. $=g(f(x)) = g(x^2-x+1) = 4(x^2-x+1)+3$

$4x^2-4x+4+3$

$\boxed{4x^2-4x+7}$

37. (5 points) The following function is one-to-one. $f(x)=\sqrt[4]{x+7}$

Find its inverse, f^{-1}.

$y=\sqrt[4]{x+7}$ $x=\sqrt[4]{y+7}$

$x^4=y+7$ $x^4-7=y$

$f^{-1}(x)=x^4-7$

38. (5 points) Solving the following equation. $3^{8x}=9^{x^2}$ $(3^2)^{x^2}=3^{2x^2}$

$8x=2x^2$

$-8x \quad -8x$

$0=2x^2-8x=2x(x-4)$

$\frac{2x=0}{2\ 2}$ $x-4=0$

$+4\ +4$

$x=0, 4$

$\{0,4\}$

39. (5 points) Write the expression as a sum and/or difference of logarithms.

Express the powers as factors.

$\log\left(\frac{y^2}{x\sqrt{z}}\right)=\log y^2-\log x-\log\sqrt{z}$

$2\log y-\log x-\log z^{1/2}$

$2\log y-\log x-\frac{1}{2}\log z$

40. (4 points) Write the following expression as a single logarithm. Distribute, where appropriate.

$$\log_5 x + \log_5(x+2) - \log_5 8$$

$\log_5\left(\frac{x(x+2)}{8}\right)$

$\log_5\left(\frac{x^2+2x}{8}\right)$

41. (5 points) Solve each equation. $\log x + \log(x-15) = 2$ $= \log[x(x-15)] = 2$

$10^2 = x(x-15)$ $100 = x^2-15x-100$ $0 = x^2-15x-100$

$0 = (x-20)(x+5)$

$\log 20 + \log(20-15) = 2$ $\log(-5)$

$x-20=0$ $x+5=0$

$+20 \quad +20 \quad -5 \quad -5$

$x = 20, -5$

$\log 20 + \log 5 = 2$

$\log(20 \cdot 5) = 2$

$\log 100 = 2$

$\{20\}$

42. (5 points) Radium is a radioactive element that decays exponentially. We currently have 80 grams of radium. The half-life of radium is approximately 1600 years. What is the decay rate of radium? $A(t) = Pe^{rt}$. Round your answer to nearest ~~hundredth~~ thousandth of a percent

$\frac{40}{40} = \frac{80e^{r \cdot 1600}}{40}$ $\frac{1}{2} = e^{1600r}$ $\frac{\ln \frac{1}{2}}{1600} = \frac{1600r}{1600}$

$r = \frac{\ln \frac{1}{2}}{1600} \approx -.04333$ per year

$-.0433$ per year