MATH 336 **Final Exam** **Fall 2014**

1. (18 pts.) Determine the solution $y(x)$ of the initial value problem $xy' = \sqrt{x} + 4y,\ y(1) = 2$.

2. (18 pts.) Find the solution $y(x)$ of the initial value problem $(4x + 1)y' = y^3,\ y(0) = 2$.

3. (18 pts.) Find the general solution $y(x)$ of $\quad xy^2 \dfrac{dy}{dx} = x^3 + 2y^3$.

4. (17 pts.) Find the general solution of

$$\left(\cos 2x + y^{-2} + 2x^{-2} \ln y\right) + \left(y^{-1} - 2x^{-1}y^{-1} - 2xy^{-3}\right) \frac{dy}{dx} = 0.$$

5. (18 pts.) Use **the method of undetermined coefficients** to find the general solution $y(x)$ of $\quad y'' - 3y' + 2y = 2e^x + e^{3x}$.

6. (10 pts.) Determine the **form** of the particular solution $y_p(x)$ of the following differential equation. (Do not solve for the unknown coefficients.)

$$y^{(3)} + 4y' = x^2 e^x + x + \cos 2x.$$

7. (10 pts.) Use the Improved Euler Method with step size $h = 1/2$ to approximate $y(2)$, where $y(x)$ is the solution of the initial-value problem $\dfrac{dy}{dx} = \dfrac{-y}{x}; \quad y(1) = 3$.

8. (17 pts.) Use the variation of parameters method to find a particular solution $y_p(x)$ of

$$x^2 y'' - 4xy' + 6y = x^{-2},$$

given that $y_c(x) = c_1 x^2 + c_2 x^3$ is the general solution of the associated homogeneous linear equation.

9. (18 pts.) Use Laplace transforms to find the solution $x(t)$ of the initial value problem

$$x'' + 4x = 2,\ x(0) = 1,\ x'(0) = 3.$$

10. (18 pts.) Find the general solution of the following system of differential equations. (Here, $x = x(t)$ and $y = y(t)$ are both functions of the independent variable t).

$$x' = -x - 3y, \quad y' = x - 5y$$

11. (10 pts.) Find the inverse Laplace transform $f(t) = \mathcal{L}^{-1}\{F(s)\}$ of $F(s) = \dfrac{s}{(s+3)^3}$.

12. (10 pts.) At time $t = 0$, a large tank contains 100 gallons of brine containing 50 lbs. of salt. Brine containing 3 pounds of salt per gallon is pumped into the tank at a rate of 6 gallons per minute and the well-mixed solution is pumped out of the tank at a rate of 4 gallons per minute. Explicitly display the initial-value problem for the amount $x(t)$ of salt in the tank at time t. (Only set up the initial-value problem; do not solve it.)

13. (18 pts.) A mass of 2 kg is attached to a spring that is stretched 1/2 m by a force of 4 N. At time $t = 0$, the mass is 1 m to the right of the equilibrium position, and is moving left with speed 1 m/s. The position of the mass at time t can be expressed as $x(t) = C\cos(\omega_0 t - \alpha)$. Set up and solve an initial-value problem to determine C, ω_0, and α.