MATH 336 **Final Exam** **Fall 2012**

Part A: Do any 8 of the following 10 problems.

A1. Determine the solution $y(x)$ of the initial-value problem

$$x^2y' - 2xy = 3, \quad y(2) = 1.$$

A2. Find the solution $y(x)$ of the initial-value problem

$$\frac{dy}{dx} = y^2 \sin 4x, \qquad y(0) = 1$$

A3. Find the general solution of

$$(2xy^{-1} + 2ye^{2x} - 1) + (e^{2x} + y^2 - x^2y^{-2})\frac{dy}{dx} = 0.$$

A4. Find the general solution $y(x)$ of $\quad xy\dfrac{dy}{dx} = x^2 + 2y^2.$

A5. Use the method of undetermined coefficients to find the general solution $y(x)$ of the differential equation

$$y'' - 2y' - 3y = 10\cos 3x.$$

A6. Use the method of undetermined coefficients to find the general solution $y(x)$ of

$$y'' + 6y' + 8y = e^x + e^{-2x}.$$

A7. Use the variation of parameters method to find the general solution $y(x)$ of

$$y'' + 4y' + 4y = \frac{e^{-2x}}{x},$$

given that $y_c(x) = c_1e^{-2x} + c_2xe^{-2x}$ is the general solution of the associated homogeneous equation.

A8. Use Laplace transforms to find the solution $x(t)$ of the initial value problem

$$x'' + 9x = 4e^{-2t}, \; x(0) = 1, \; x'(0) = -2.$$

A9. (a) Let $f(t) = \begin{cases} 2t & \text{for } 0 \le t \le 2 \\ 4 & \text{for } t > 2. \end{cases}$ Determine the Laplace transform $F(s) = \mathcal{L}\{f(t)\}$ using any valid method.

(b) Find the inverse Laplace transform $f(t) = \mathcal{L}^{-1}\{F(s)\}$, where $F(s) = \dfrac{2s - 26}{s^2 - 4s + 20}$.

A10. Find the general solution of the following system of differential equations. ($x = x(t)$ and $y = y(t)$ are both functions of the independent variable t.)

$$x' = 3x - 6y, \quad y' = 6x - 9y.$$

Part B: Do any 2 of the following 3 problems.

B1. An object is shot upward from the ground with an initial velocity of 640 ft/sec, and experiences a constant deceleration of 32 ft/sec^2 due to gravity as well as a deceleration of $\frac{v(t)}{10}$ ft/sec^2 due to air resistance, where $v(t)$ is the object's velocity in ft/sec.

(a) Set up and solve an initial-value problem to determine the object's velocity $v(t)$ at time t.

(b) At what time does the object reach its highest point?

B2. A mass of 4 kg is attached to a spring and moves without friction. Assume that a force of 6 N stretches the spring 1/2 meter. At time $t = 0$, the mass is pulled 3 meters from the equilibrium position and set in motion toward the equilibrium position with a speed of 3 meters per second. The position $x(t)$ of the mass at time t can be expressed as $x(t) = C\cos(\omega_0 t - \alpha)$. Determine C, ω_0, and α.

B3. A large tank initially contains 20 gallons of pure water. Brine containing 2 lb. of salt per gallon enters the tank at the rate of 6 gallons per minute, and the well-mixed brine in the tank flows out at the rate of 4 gallons per minute. Set up and solve an initial-value problem to determine the amount $x(t)$ of salt in the tank at time t.