Practice Exam 3

1. (*0 points*) Find the directional derivative of $f(x, y) = \sec(xy)$ at the point $(1, \frac{\pi}{4})$ in the direction $\mathbf{v}$ $= < 1, -2 >$

2. (*0 points*) Find all local extrema and saddle points of the function $f(x, y) = x^3 - 3xy + y^3$.

3. (*0 points*) Use the method of Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = 2xy$ on the circle $x^2 + y^2 = 18$

4. (*0 points*) Sketch the solid whose volume is given by

$$\int_0^5 \int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} (5 - \sqrt{x^2 + y^2}) dy dx$$

5. (*0 points*) Evaluate each of the following:

a) $\int_0^1 \int_y^1 e^{x^2+1} dx dy$

b) $\int_0^{\pi/2} \int_0^{\sin x} \sin(\cos x) dy dx$

c) $\int_{-2}^2 \int_0^{\sqrt{4-x^2}} \sin^2 \sqrt{x^2 + y^2} dy dx$

6. (*0 points*) Find the volume of the tetrahedron bounded between the planes $x = 1, y = 1, z = 0$ and $x + y + z = 1$ using a triple integral, then write this triple integral 6 different ways.

7. (*0 points*) Set up, but do not evaluate a triple integral in cylindrical coordinates for the volume of the solid that lies above the paraboloid $z = x^2 + 2y^2 - 2$ and below the paraboloid $z = -3x^2 - 2y^2 + 2$.

8. (*0 points*) Set up, but do not evaluate a triple integral in spherical coordinates for the volume of the solid that lies inside the sphere $x^2 + y^2 + z^2 = 4$, bounded by the cones $z = \sqrt{x^2 + y^2}$, $z = -\sqrt{x^2 + y^2}$ and the planes $x = 0$, $y = 0$.

9. (*0 points*)

a) Find the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ for the transformation $x = u^2v + v^2$, $y = uv^2 - u^2$.

b) Evaluate $\iint_R (3x - y)^{3/2}(x + y)^5 dA$ where R is the parallelogram bounded by $y = -x$, $y = -x + 1$, $y = 3x$ and $y = 3x - 1$. Use the change of variables $u = 3x - y$, $v = x + y$.