Adapting Growth Models to Uncover Psychological Trends

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What is Growth Modeling?

► Modeling change over time
  • Overall growth pattern across individuals
  • Variance in growth pattern across individuals
► Growth parameters are predicted by key covariates
► Different trajectory shapes:
  • Linear/piecewise linear
  • Polynomial
  • Nonlinear

Advantages of Random Coefficient Growth Models

Multilevel Models
1. Times of measurement may vary across individuals (time unstructured)
2. All observations are used (Do not need same # of observations/ person = unbalanced)
3. Can model change over time within-individuals and differences across individuals

Modeling Linear Growth

Level 1
\[ NWF_{i} = \pi_{0i} + \pi_{1i}(\text{term})_{i} + e_{ii}, \quad e_{ii} \approx N(0, \sigma^2) \]

Level 2
\[ \pi_{0i} \approx N(0, \tau_{00}), \quad \pi_{1i} \approx N(0, \tau_{11}) \]

NWF = Nonsense Word Fluency
► NWF is part of the Dynamic Indicators of Basic Early Literacy Skills (DIBELS) measurement system
► Measures alphabetic principle - A foundational reading skill for beginning readers
### Centering Options

<table>
<thead>
<tr>
<th>Term</th>
<th>Term – 1 Initial Status</th>
<th>Term – 3 Midpoint</th>
<th>Term – 5 Final Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) K- May</td>
<td>0</td>
<td>-2</td>
<td>-4</td>
</tr>
<tr>
<td>(2) 1st Sept.</td>
<td>1</td>
<td>-1</td>
<td>-3</td>
</tr>
<tr>
<td>(3) 1st Jan.</td>
<td>2</td>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>(4) 1st May</td>
<td>3</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>(5) 2nd Sept.</td>
<td>4</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

### Example of Conditional Quadratic Growth Model

**Level 1**

\[ NWF_i = \pi_{0i} + \pi_{1i}(\text{term} - \text{term}_0) + \pi_{2i}(\text{term} - \text{term}_0)^2 + e_i \]

**Level 2**

\[ \pi_{0i} = \beta_{00} + \beta_{01}(\text{Attend K})_i + r_{0i} \]

\[ \pi_{1i} = \beta_{10} + \beta_{11}(\text{Attend K})_i + r_{1i} \]

\[ \pi_{2i} = \beta_{20} + \beta_{21}(\text{Attend K})_i + r_{2i} \]

where **Attend K** is a binary variable of whether the child attended kindergarten in the treatment school or not.
Growth Model for Onset of Tense Productivity

Level-1 Model
\[ TAP_i = \pi_1 (\text{age}_{ti} - 21) + \pi_2 (\text{age}_{ti} - 21)^2 + \epsilon_i \]

Level-2 Model
\[ \pi_1 = \beta_{10} + r_{1i} \]
\[ \pi_2 = \beta_{20} + r_{2i} \]

\( TAP = \text{tense and agreement productivity} \)

- from Rispoli, Hadley, & Holt, 2009

Residuals as predictors of later Accuracy

- Does linear and quadratic growth predict later accuracy?
- Output level-2 residuals
- Use to predict accuracy at 33 months in regression analyses
- Both linear trends at 21 months and quadratic trends between 21 and 30 months significantly predicted later accuracy in tense and agreement at 33 months.

Incorporating Covariates

- Covariates at Level 1 are time-varying
  - Relates covariate over time to outcome over time.
  - Can include level-1 interactions
- Covariates at Level 2 are time invariant
  - Model cross-level interactions
  - Can use different covariates for different level-2 equations
Growth Model for Onset of Tense Productivity

Level-1 Model
\[ Y_{ti} = \pi_{0i} + \pi_{1i}(\text{age}_{ti} - 30) + \pi_{2i}(\text{age}_{ti} - 30)^2 + \pi_{3i}(\text{MLU}_{ti}) + \pi_{4i}((\text{age}_{ti} - 30)\times\text{MLU}_{ti}) + e_{ti} \]

Level-2 Model
\[ \pi_{0i} = \beta_{00} + \beta_{01}(\text{gender})_i + \beta_{02}(\text{education})_i + \beta_{03}(\text{history})_i + \beta_{04}(\text{comprehension})_i + r_{0i} \]
\[ \pi_{1i} = \beta_{10} + \beta_{11}(\text{gender})_i + \beta_{12}(\text{education})_i + \beta_{13}(\text{history})_i + \beta_{14}(\text{comprehension})_i + r_{1i} \]

From: Hadley & Holt, 2006

Discontinuous Growth Model

- Multiphase Growth
- Change in growth trajectory between phases
- Could be due to:
  - internal factors (e.g., transitioning to a new physical or developmental phase)
  - external factors (e.g., relocating, moving to next grade level)
- characterized by an abrupt change in the growth trajectory at the time that the transition occurs

Discontinuous Models

- Discontinuity may occur at same time for all persons (fixed transition point) or at different times for different persons (varying transition points).
- Multilevel models can accommodate both fixed and varying transition points.
Fixed Transition

- Transition same for all individuals
  - ex. transition across grade levels
  - ex. transition from medical school to medical residency
1. How does growth change from before the transition to after the transition? or
2. How does growth before the transition predict outcomes after the transition?

Modeling Fixed Transitions

<table>
<thead>
<tr>
<th>Growth Factor</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase 1 growth</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Phase 2 growth</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

- Piecewise linear growth models
  - Create vectors that code for growth factors or
  - Multilevel growth model with separate predictors for each phase

Varied Transition Points

- Transition does not occur at same time for all individuals
  - Transition from public to private school
  - Transition from military service to civilian life
  - Transition across developmental stages
- Random coefficients models are very well suited for modeling transitions that occur at different times for different individuals

Molloy, Ram, & Gest’s (2011) Self-Concept Growth and Lability

- Expected negative declines in academic and social self-concept during the transition from elementary to middle school (has been previously shown).
- Particularly interested in the lability of growth during both elementary school and middle school. Previous studies indicate that higher lability associated with negative outcomes, e.g., depression, general intelligence, and memory.

Molloy, Ram, & Gest’s (2011) Self-Concept Growth and Lability (2)

- Sequential cohort design
- Initially in 3rd, 4th, or 5th grade, followed until end of 7th grade
- Number of waves of data differ for each cohort by design
- Multilevel models allow for varying data collection schedules

Self-Concept L1 Growth Model

\[
Self-Concept_{it} = \beta_0 + \beta_1(grade_{it}) + \beta_2(grade_{it}^2) + \beta_3(grade_{it} \times middle\ school_{it}) + \beta_4(grade_{it}^2 \times middle\ school_{it}) + \epsilon_{it}
\]

- Used multilevel modeling to examine linear and quadratic growth in self-concept
- Dichotomous indicator = 0 when child is in elementary school and = 1 when child is in middle school
- Interaction of indicator variable with growth parameters, \(\beta_3\) and \(\beta_4\) model the change in growth as the child transitions to middle school.
Results

► Quadratic trend not significant, so simpler linear model was used
► Significant MS transition by linear trend in academic SC - linear rate changed from flat to negative when transitioning to middle school
► No significant effect of MS transition on social SC growth rate
► For both academic and social SC, there was significant individual differences in trends

Self-Concept Growth and Lability Results

► Quantified lability as SD of $e_t$ - created a mean and SD of lability for each child and used as between-subject variables.
► Academic SC did not have sign. different lability from elementary to middle school
► Social SC lability did sign. decline from elementary to middle school.

► Intraindividual lability sign. predicted of youth adjustment – Lability was associated with negative academic adjustment factors, e.g., peer academic reputation and negative social adjustment, e.g., peer victimization.
► Intraindividual lability sign. negatively predicted peer and teacher ratings of competence in 7th grade above and beyond mean self-concept, e.g., teacher ratings of academic skills, social engagement.
Growth Modeling in SEM Framework

► A specialized structural model as opposed to multilevel modeling with comes from regression tradition
► With some manipulation, can generate equivalent models
► Can be used in larger model with antecedent variables and proximal and distal outcomes

McArdle & Hamagami’s (2001) Linear Dynamic Analysis

► Captures developmental processes as they evolve over time
► Captures how a developmental process is intertwined with other developmental processes
  ▪ How 2 or more developmental processes concurrently evolve over time
  ▪ How cross-lagged effects drive the concurrent evolution of the processes over time

from Dogan, Stockdale, Widaman, & Conger, 2010
In Bivariate LDS model, the true change in # of sexual partners from time 1 to 2 is a function of the growth factor in # of partners, prior status in sexual partners at time 1, and prior status alcohol usage at time 1.

Conversely, alcohol use from time 1 to 2 is a function of the growth factor in alcohol use, prior status in alcohol use at time 1 and prior status in # of sexual partners at time 1.

Linear Dynamic Model to test Sequential Relationships between Alcohol Use and # of Sexual Partners

- Does alcohol use alter cognitive functioning and behavior resulting in sexual disinhibition and increased # of sexual partners?
- Does high sexual activity, particularly with a # of partners result in increased alcohol usage, possibly to counter the negative feelings of low self-worth that emerge from this activity?

Alcohol Use and # of Sexual Partners (2)

- Is the relationship between alcohol use and # of sexual partners spurious, and due to other influences, e.g., sensation seeking or impulsivity?
- A bivariate linear dynamic model was used in which alcohol use and # of sexual partners were assessed annually or biannually 8 times over a 13 year timeframe, beginning when participants were in 9th grade.
Alcohol Use & # of Sexual Partners  

Results (1)  
► Full model (Model 1) had good fit  
► Model 2 without coupling parameters from alcohol use at prior time point to change in # of sexual partners, i.e., green line, had significantly poorer fit (and generally had poor fit)  
► Model 3 without coupling parameters from # of sexual partners to change in alcohol use, i.e., orange line, had significantly poorer fit from Model 1, but only 1 coupling parameter was significant and the fit indices were similar to Model 1  
► Deduced that model 3 was most appropriate, given model fit and parsimony

Alcohol Use & # of Sexual Partners  

Results (2)  
► Coupling parameters between alcohol use at time \( t-1 \) and \( \Delta \) in # of sexual partners at time \( t \), i.e., green line, were positive and significant between ages 15 & 21.  
► These 2 findings together show a sequential relationship in which higher previous alcohol use predicted increased # of sexual partners, supporting theory of alcohol as causative agent in # of sexual partners.

Alcohol Use & # of Sexual Partners  

Results (3)  
► In order to rule out common cause hypothesis, intercept, growth factors and change scores, i.e., \( \Delta a_2 \), were regressed on key covariates, e.g., excitement seeking, impulsive, key demographics.  
► Coupling parameters did not change with inclusion of covariates  
► The effect of alcohol use on \( \Delta \) in # of sexual partners remained sign., after controlling for covariates.

Are Growth Models becoming too Complex?  

Multivariate methods often best honor the reality about which the researcher is purportedly trying to generalize. – Fish, 1988  
► More complete view of antecedent variables, micro and macro growth patterns, parallel processes and outcomes of growth  
► Allows stronger and more nuanced inferences about developmental processes
References (1)


References (2)


