Numerical Summary Measures

- Sample Mean:

$$\bar{x} = \frac{\sum x_i}{n}$$

- Sample Median: The middle number when the observations are arranged in order from smallest to largest.
- Sample Variance:

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = \frac{\sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}}{n-1}$$

- Chebyshev's Rule: Let $k > 1$. For any set of observations, the percent of observations within k standard deviations of the mean is at least $100(1 - \frac{1}{k^2})\%$.

Probability

- Complement Rule: $P(A') = 1 - P(A)$
- Addition Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- Addition Rule for Two Disjoint Events: $P(A \cup B) = P(A) + P(B)$
- Conditional Probability: If $P(B) > 0$ then

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- Multiplication Rule: $P(A \cap B) = P(A|B)P(B)$
- Multiplication Rule for Independent Events: $P(A \cap B) = P(A)P(B)$

Random Variables and Discrete & Continuous Probability Distributions

- Mean, or expected value, of a discrete random variable X:

$$\mu = E(X) = \sum_{\text{all } x} x p(x)$$

- Variance of a discrete random variable X:

$$\sigma^2 = \text{Var}(X) = \sum_{\text{all } x} x^2 p(x) - \mu^2$$

- If X is a binomial random variable based on n trials, each with success probability equal to p, then

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, 3, \ldots, n \qquad \binom{n}{x} = \frac{n!}{x!(n-x)!}$$

  where $\mu = np$, $\sigma^2 = np(1-p)$, and $\sigma = \sqrt{np(1-p)}$.

- If X is a normal random variable with mean μ and standard deviation σ, then a standard normal random variable is given by

$$Z = \frac{X - \mu}{\sigma}$$

Sampling Distributions

- Properties of the sample mean $\bar{X}$:

$$E(\bar{X}) = \mu_{\bar{X}} = \mu \qquad \text{and} \qquad V(\bar{X}) = \sigma^2_{\bar{X}} = \frac{\sigma^2}{n} \qquad \text{and} \qquad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

Confidence Intervals Based on a Single Sample

- Confidence Interval for μ when n is large or the population is normal, and σ is known:

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

- Confidence Interval for μ when the population is normal and σ is unknown:

$$\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}$$

- Confidence interval for a population proportion p when n is large enough so that $n\hat{p} \geq 5$ and $n(1-\hat{p}) \geq 5$:

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

- The minimum sample size required for a $100(1-\alpha)\%$ confidence interval for a population mean μ to have margin of error bound B when the population is normal and σ is known:

$$n = \left(\frac{\sigma z_{\alpha/2}}{B}\right)^2$$

- The minimum sample size required for a $100(1-\alpha)\%$ confidence interval for a population proportion p to have margin of error bound B is

$$n = \hat{p}(1-\hat{p})\left[\frac{z_{\alpha/2}}{B}\right]^2$$

  where $\hat{p}$ is either a reasonable estimate of p based on past experience, or $\hat{p}$ is set to 0.5 to obtain a conservative estimate of n.

Hypothesis Tests Based on a Single Sample

- Hypothesis test for μ when n is large or the population is normal, and σ is known:
  - Null Hypothesis: $H_0 : \mu = \mu_0$
  - Test Statistic:

$$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

  - Alternative hypotheses, rejection regions, and p-values:

Alternative Hypothesis	Rejection Region	p-value
$H_a : \mu > \mu_0$	$Z \geq z_\alpha$	$P(Z \geq z)$
$H_a : \mu < \mu_0$	$Z \leq -z_\alpha$	$P(Z \leq z)$
$H_a : \mu \neq \mu_0$	$\lvert Z\rvert \geq z_{\alpha/2}$	$2P(Z \geq z)$ if $z \geq 0$
		$2P(Z \leq z)$ if $z < 0$

- Hypothesis test for μ when the population is normal and σ is unknown:
  - Null Hypothesis: $H_0 : \mu = \mu_0$
  - Test Statistic:

$$T = \frac{\bar{x} - \mu_0}{S/\sqrt{n}}$$

  - Alternative hypotheses, rejection regions, and p-values:

Alternative Hypothesis	Rejection Region
$H_a : \mu > \mu_0$	$T \geq t_\alpha$
$H_a : \mu < \mu_0$	$T \leq -t_\alpha$
$H_a : \mu \neq \mu_0$	$\lvert T \rvert \geq t_{\alpha/2}$

- Hypothesis test for a population proportion p when n is large enough so that $np_0 \geq 5$ and $n(1 - p_0) \geq 5$:
  - Null Hypothesis: $H_0 : p = p_0$
  - Test Statistic:

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

  - Alternative hypotheses, rejection regions, and p-values:

Alternative Hypothesis	Rejection Region	p-value
$H_a : p > p_0$	$Z \geq z_\alpha$	$P(Z \geq z)$
$H_a : p < p_0$	$Z \leq -z_\alpha$	$P(Z \leq z)$
$H_a : p \neq p_0$	$\lvert Z \rvert \geq z_{\alpha/2}$	$2P(Z \geq z)$ if $z \geq 0$
		$2P(Z \leq z)$ if $z < 0$

Hypothesis Tests and Confidence Intervals Based on Two Samples

- Confidence interval for $\mu_1 - \mu_2$ based on two independent samples where σ_1 and σ_2 are known, and the populations are normal or n_1 and n_2 are both large:

$$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

- Confidence interval for $\mu_1 - \mu_2$ based on two independent samples where σ_1 and σ_2 are uknown but equal, and the populations are normal:

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

  where

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

  and the degrees of freedom are $\text{df} = n_1 + n_2 - 2$.

- Confidence interval for $\mu_1 - \mu_2$ based on paired observations from a normal population:

$$\bar{d} \pm t_{\alpha/2}\frac{s_D}{\sqrt{n}}$$

  where the degrees of freedom are $\text{df} = n - 1$.

- Confidence interval for $p_1 - p_2$ based on two independent samples when n_1 and n_2 are large enough so that $n_1\hat{p}_1 \geq 5$, $n_1(1 - \hat{p}_1) \geq 5$, $n_2\hat{p}_2 \geq 5$, and $n_2(1 - \hat{p}_2) \geq 5$:

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1 - \hat{p}_1)}{n_1} + \frac{\hat{p}_2(1 - \hat{p}_2)}{n_2}}$$

- Hypothesis test for $\mu_1 - \mu_2$ based on two independent samples where σ_1 and σ_2 are known, and the populations are normal or n_1 and n_2 are both large:
  - Null Hypothesis: $H_0 : \mu_1 - \mu_2 = \Delta_0$
  - Test Statistic:

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

  - Alternative hypotheses, rejection regions, and p-values:

Alternative Hypothesis	Rejection Region	p-value
$H_a : \mu_1 - \mu_2 > \Delta_0$	$Z \geq z_\alpha$	$P(Z \geq z)$
$H_a : \mu_1 - \mu_2 < \Delta_0$	$Z \leq -z_\alpha$	$P(Z \leq z)$
$H_a : \mu_1 - \mu_2 \neq \Delta_0$	$\lvert Z\rvert \geq z_{\alpha/2}$	$2P(Z \geq z)$ if $z \geq 0$
		$2P(Z \leq z)$ if $z < 0$

- Hypothesis test for $\mu_1 - \mu_2$ based on two independent samples where σ_1 and σ_2 are uknown but equal, and the populations are normal:
  - Null Hypothesis: $H_0 : \mu_1 - \mu_2 = \Delta_0$
  - Test Statistic:

$$T = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

    where

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

    and the degrees of freedom are $\text{df} = n_1 + n_2 - 2$.
  - Alternative hypotheses and rejection regions:

Alternative Hypothesis	Rejection Region
$H_a : \mu_1 - \mu_2 > \Delta_0$	$T \geq t_\alpha$
$H_a : \mu_1 - \mu_2 < \Delta_0$	$T \leq -t_\alpha$
$H_a : \mu_1 - \mu_2 \neq \Delta_0$	$\lvert T\rvert \geq t_{\alpha/2}$

- Hypothesis test for $\mu_1 - \mu_2$ based on paired observations from a normal population:
  - Null Hypothesis: $H_0 : \mu_D = \Delta_0$
  - Test Statistic:

$$T = \frac{\bar{D} - \Delta_0}{S_D / \sqrt{n}}$$

  - Alternative hypotheses and rejection regions:

Alternative Hypothesis	Rejection Region
$H_a : \mu_D > \Delta_0$	$T \geq t_\alpha$
$H_a : \mu_D < \Delta_0$	$T \leq -t_\alpha$
$H_a : \mu_D \neq \Delta_0$	$\lvert T\rvert \geq t_{\alpha/2}$

- Hypothesis tests for $p_1 - p_2$ based on two independent samples when n_1 and n_2 are large enough so that $n_1\hat{p}_1 \geq 5$, $n_1(1-\hat{p}_1) \geq 5$, $n_2\hat{p}_2 \geq 5$, and $n_2(1-\hat{p}_2) \geq 5$:
  - Null Hypothesis: $H_0 : p_1 - p_2 = 0$
  - Test Statistic:

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_c(1-\hat{p}_c)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

    where

$$\hat{p}_c = \frac{X_1 + X_2}{n_1 + n_2}$$

  - Alternative hypotheses, rejection regions, and p-values:

Alternative Hypothesis	Rejection Region	p-value
$H_a : p_1 - p_2 > 0$	$Z \geq z_\alpha$	$P(Z \geq z)$
$H_a : p_1 - p_2 < 0$	$Z \leq -z_\alpha$	$P(Z \leq z)$
$H_a : p_1 - p_2 \neq 0$	$\lvert Z\rvert \geq z_{\alpha/2}$	$2P(Z \geq z)$ if $z \geq 0$
		$2P(Z \leq z)$ if $z < 0$

  - Null Hypothesis: $H_0 : p_1 - p_2 = \Delta_0$ (where $\Delta_0 \neq 0$)
  - Test Statistic:

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - \Delta_0}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}}$$

  - Alternative hypotheses, rejection regions, and p-values:

Alternative Hypothesis	Rejection Region	p-value
$H_a : p_1 - p_2 > \Delta_0$	$Z \geq z_\alpha$	$P(Z \geq z)$
$H_a : p_1 - p_2 < \Delta_0$	$Z \leq -z_\alpha$	$P(Z \leq z)$
$H_a : p_1 - p_2 \neq \Delta_0$	$\lvert Z\rvert \geq z_{\alpha/2}$	$2P(Z \geq z)$ if $z \geq 0$
		$2P(Z \leq z)$ if $z < 0$

The Analysis of Variance

ANOVA Table Summary

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p Value
Factor	SSA	$k-1$	$\text{MSA} = \frac{\text{SSA}}{k-1}$	$\frac{\text{MSA}}{\text{MSE}}$	p
Error	SSE	$n-k$	$\text{MSE} = \frac{\text{SSE}}{n-k}$		
Total	SST	$n-1$			

Correlation and Linear Regression

- The least squares estimates of β_0 and β_1 are given by

$$\hat{\beta}_1 = \frac{n \sum x_i y_i - \left(\sum x_i\right)\left(\sum y_i\right)}{n \sum x_i^2 - \left(\sum x_i\right)^2}$$

  and

$$\hat{\beta}_0 = \frac{\sum y_i - \hat{\beta}_1 \sum x_i}{n} = \bar{y} - \hat{\beta}_1 \bar{x}$$

- The estimated regression line is given by $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$.
- ANOVA Table Summary:

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p-value
Regression	SSR	1	$\text{MSR} = \frac{\text{SSR}}{1}$	$\frac{\text{MSR}}{\text{MSE}}$	p
Error	SSE	$n-2$	$\text{MSE} = \frac{\text{SSE}}{n-2}$		
Total	SST	$n-1$			

- Coefficient of Determination:

$$r^2 = \frac{\text{SSR}}{\text{SST}}$$

- Correlation Coefficient:

$$r = \frac{\sum x_i y_i - \frac{1}{n}\left(\sum x_i\right)\left(\sum y_i\right)}{\sqrt{\left[\sum x_i^2 - \frac{1}{n}\left(\sum x_i\right)^2\right]\left[\sum y_i^2 - \frac{1}{n}\left(\sum y_i\right)^2\right]}}$$

- Alternate formulas for correlation and regression:

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x \qquad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$S_{xy} = \sum xy - \frac{\left(\sum x\right)\left(\sum y\right)}{n} \qquad S_{xx} = \sum x^2 - \frac{\left(\sum x\right)^2}{n} \qquad S_{yy} = \sum y^2 - \frac{\left(\sum y\right)^2}{n}$$

$$r = \frac{S_{xy}}{\sqrt{S_{xx}}\sqrt{S_{yy}}}$$

Table I: Binomial Distribution Cumulative Probabilities

Let X be a binomial random variable with parameters n and p. This table contains the cumulative probabilities

$$P(X \leq x) = \sum_{k=0}^{x} P(X = k) = P(X = 0) + P(X = 1) + P(X = 2) + \cdots + P(X = x)$$

$n = 5$								p							
x	0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
0	0.951	0.774	0.59	0.328	0.237	0.168	0.078	0.031	0.010	0.002	0.001	0.000			
1	0.999	0.977	0.918	0.737	0.633	0.528	0.337	0.187	0.087	0.031	0.016	0.007	0.001	0.000	
2	1.000	0.999	0.991	0.942	0.896	0.837	0.683	0.500	0.317	0.163	0.104	0.058	0.009	0.001	0.000
3		1.000	1.000	0.993	0.984	0.969	0.913	0.812	0.663	0.472	0.367	0.263	0.082	0.023	0.001
4				1.000	0.999	0.998	0.990	0.969	0.922	0.832	0.763	0.672	0.409	0.226	0.049

$n = 10$								p							
x	0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
0	0.904	0.599	0.349	0.107	0.056	0.028	0.006	0.001	0.000						
1	0.996	0.914	0.736	0.376	0.244	0.149	0.046	0.011	0.002	0.000					
2	1.000	0.988	0.930	0.678	0.526	0.383	0.167	0.055	0.012	0.002	0.000	0.000			
3		0.999	0.987	0.879	0.776	0.650	0.382	0.172	0.055	0.011	0.004	0.001			
4		1.000	0.998	0.967	0.922	0.850	0.633	0.377	0.166	0.047	0.020	0.006	0.000		
5			1.000	0.994	0.980	0.953	0.834	0.623	0.367	0.150	0.078	0.033	0.002	0.000	
6				0.999	0.996	0.989	0.945	0.828	0.618	0.350	0.224	0.121	0.013	0.001	
7				1.000	1.000	0.998	0.988	0.945	0.833	0.617	0.474	0.322	0.070	0.012	0.000
8						1.000	0.998	0.989	0.954	0.851	0.756	0.624	0.264	0.086	0.004
9							1.000	0.999	0.994	0.972	0.944	0.893	0.651	0.401	0.096

$n = 15$								p							
x	0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
0	0.860	0.463	0.206	0.035	0.013	0.005	0.000								
1	0.990	0.829	0.549	0.167	0.080	0.035	0.005	0.000							
2	1.000	0.964	0.816	0.398	0.236	0.127	0.027	0.004	0.000						
3		0.995	0.944	0.648	0.461	0.297	0.091	0.018	0.002	0.000					
4		0.999	0.987	0.836	0.686	0.515	0.217	0.059	0.009	0.001	0.000				
5		1.000	0.998	0.939	0.852	0.722	0.403	0.151	0.034	0.004	0.001	0.000			
6			1.000	0.982	0.943	0.869	0.610	0.304	0.095	0.015	0.004	0.001			
7				0.996	0.983	0.950	0.787	0.500	0.213	0.050	0.017	0.004			
8				0.999	0.996	0.985	0.905	0.696	0.390	0.131	0.057	0.018	0.000		
9				1.000	0.999	0.996	0.966	0.849	0.597	0.278	0.148	0.061	0.002	0.000	
10					1.000	0.999	0.991	0.941	0.783	0.485	0.314	0.164	0.013	0.001	
11						1.000	0.998	0.982	0.909	0.703	0.539	0.352	0.056	0.005	
12							1.000	0.996	0.973	0.873	0.764	0.602	0.184	0.036	0.000
13								1.000	0.995	0.965	0.920	0.833	0.451	0.171	0.010
14									1.000	0.995	0.987	0.965	0.794	0.537	0.140

Table I: Binomial Distribution Cumulative Probabilities (Continued)

$n = 20$								p							
x	0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
0	0.818	0.358	0.122	0.012	0.003	0.001	0.000								
1	0.983	0.736	0.392	0.069	0.024	0.008	0.001								
2	0.999	0.925	0.677	0.206	0.091	0.035	0.004	0.000							
3	1.000	0.984	0.867	0.411	0.225	0.107	0.016	0.001							
4		0.997	0.957	0.630	0.415	0.238	0.051	0.006	0.000						
5		1.000	0.989	0.804	0.617	0.416	0.126	0.021	0.002						
6			0.998	0.913	0.786	0.608	0.250	0.058	0.006	0.000					
7			1.000	0.968	0.898	0.772	0.416	0.132	0.021	0.001	0.000				
8				0.990	0.959	0.887	0.596	0.252	0.057	0.005	0.001	0.000			
9				0.997	0.986	0.952	0.755	0.412	0.128	0.017	0.004	0.001			
10				0.999	0.996	0.983	0.872	0.588	0.245	0.048	0.014	0.003			
11				1.000	0.999	0.995	0.943	0.748	0.404	0.113	0.041	0.01			
12					1.000	0.999	0.979	0.868	0.584	0.228	0.102	0.032	0.000		
13						1.000	0.994	0.942	0.75	0.392	0.214	0.087	0.002		
14							0.998	0.979	0.874	0.584	0.383	0.196	0.011	0.000	
15							1.000	0.994	0.949	0.762	0.585	0.37	0.043	0.003	
16								0.999	0.984	0.893	0.775	0.589	0.133	0.016	0.000
17								1.000	0.996	0.965	0.909	0.794	0.323	0.075	0.001
18									0.999	0.992	0.976	0.931	0.608	0.264	0.017
19									1.000	0.999	0.997	0.988	0.878	0.642	0.182

$n = 25$								p							
x	0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
0	0.778	0.277	0.072	0.004	0.001	0.000									
1	0.974	0.642	0.271	0.027	0.007	0.002									
2	0.998	0.873	0.537	0.098	0.032	0.009	0.000								
3	1.000	0.966	0.764	0.234	0.096	0.033	0.002								
4		0.993	0.902	0.421	0.214	0.09	0.009	0.000							
5		0.999	0.967	0.617	0.378	0.193	0.029	0.002							
6		1.000	0.991	0.780	0.561	0.341	0.074	0.007	0.000						
7			0.998	0.891	0.727	0.512	0.154	0.022	0.001						
8			1.000	0.953	0.851	0.677	0.274	0.054	0.004						
9				0.983	0.929	0.811	0.425	0.115	0.013	0.000					
10				0.994	0.970	0.902	0.586	0.212	0.034	0.002	0.000				
11				0.998	0.989	0.956	0.732	0.345	0.078	0.006	0.001				
12				1.000	0.997	0.983	0.846	0.500	0.154	0.017	0.003	0.000			
13					0.999	0.994	0.922	0.655	0.268	0.044	0.011	0.002			
14					1.000	0.998	0.966	0.788	0.414	0.098	0.030	0.006			
15						1.000	0.987	0.885	0.575	0.189	0.071	0.017			
16							0.996	0.946	0.726	0.323	0.149	0.047	0.000		
17							0.999	0.978	0.846	0.488	0.273	0.109	0.002		
18							1.000	0.993	0.926	0.659	0.439	0.220	0.009	0.000	
19								0.998	0.971	0.807	0.622	0.383	0.033	0.001	
20								1.000	0.991	0.910	0.786	0.579	0.098	0.007	
21									0.998	0.967	0.904	0.766	0.236	0.034	0.000
22									1.000	0.991	0.968	0.902	0.463	0.127	0.002
23										0.998	0.993	0.973	0.729	0.358	0.026
24										1.000	0.999	0.996	0.928	0.723	0.222

Table III Standard Normal Distribution Cumulative Probabilities

Let Z be a standard normal random variable: $\mu = 0$ and $\sigma = 1$.
This table contains cumulative probabilities: $P(Z \leq z)$.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table III Standard Normal Distribution Cumulative Probabilities (Continued)

Let Z be a standard normal random variable: $\mu = 0$ and $\sigma = 1$.
This table contains cumulative probabilities: $P(Z \leq z)$.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Special Critical Values: $P(Z \geq z_\alpha) = \alpha$

α	0.10	0.05	0.025	0.01	0.005	0.001	0.0005	0.0001	
z_α	1.2816	1.6449	1.9600	2.3263	2.5758	3.0902	3.2905	3.7190	
α	0.00009	0.00008	0.00007	0.00006	0.00005	0.00004	0.00003	0.00002	0.00001
z_α	3.7455	3.7750	3.8082	3.8461	3.8906	3.9444	4.0128	4.1075	4.2649

Table V Critical Values for the t Distribution

This table contains critical values associated with the t distribution, t_α, defined by the degrees of freedom and α.

df	0.20	0.10	0.05	0.025	0.01	0.005	0.001	0.0005	0.0001
1	1.3764	3.0777	6.3138	12.7062	31.8205	63.6567	318.3088	636.6192	3183.0988
2	1.0607	1.8856	2.9200	4.3027	6.9646	9.9248	22.3271	31.5991	70.7001
3	0.9785	1.6377	2.3534	3.1824	4.5407	5.8409	10.2145	12.9240	22.2037
4	0.9410	1.5332	2.1318	2.7764	3.7469	4.6041	7.1732	8.6103	13.0337
5	0.9195	1.4759	2.0150	2.5706	3.3649	4.0321	5.8934	6.8688	9.6776
6	0.9057	1.4398	1.9432	2.4469	3.1427	3.7074	5.2076	5.9588	8.0248
7	0.8960	1.4149	1.8946	2.3646	2.9980	3.4995	4.7853	5.4079	7.0634
8	0.8889	1.3968	1.8595	2.3060	2.8965	3.3554	4.5008	5.0413	6.4420
9	0.8834	1.3830	1.8331	2.2622	2.8214	3.2498	4.2968	4.7809	6.0101
10	0.8791	1.3722	1.8125	2.2281	2.7638	3.1693	4.1437	4.5869	5.6938
11	0.8755	1.3634	1.7959	2.2010	2.7181	3.1058	4.0247	4.4370	5.4528
12	0.8726	1.3562	1.7823	2.1788	2.6810	3.0545	3.9296	4.3178	5.2633
13	0.8702	1.3502	1.7709	2.1604	2.6503	3.0123	3.8520	4.2208	5.1106
14	0.8681	1.3450	1.7613	2.1448	2.6245	2.9768	3.7874	4.1405	4.9850
15	0.8662	1.3406	1.7531	2.1314	2.6025	2.9467	3.7328	4.0728	4.8800
16	0.8647	1.3368	1.7459	2.1199	2.5835	2.9208	3.6862	4.0150	4.7909
17	0.8633	1.3334	1.7396	2.1098	2.5669	2.8982	3.6458	3.9651	4.7144
18	0.8620	1.3304	1.7341	2.1009	2.5524	2.8784	3.6105	3.9216	4.6480
19	0.8610	1.3277	1.7291	2.0930	2.5395	2.8609	3.5794	3.8834	4.5899
20	0.8600	1.3253	1.7247	2.0860	2.5280	2.8453	3.5518	3.8495	4.5385
21	0.8591	1.3232	1.7207	2.0796	2.5176	2.8314	3.5272	3.8193	4.4929
22	0.8583	1.3212	1.7171	2.0739	2.5083	2.8188	3.5050	3.7921	4.4520
23	0.8575	1.3195	1.7139	2.0687	2.4999	2.8073	3.4850	3.7676	4.4152
24	0.8569	1.3178	1.7109	2.0639	2.4922	2.7969	3.4668	3.7454	4.3819
25	0.8562	1.3163	1.7081	2.0595	2.4851	2.7874	3.4502	3.7251	4.3517
26	0.8557	1.3150	1.7056	2.0555	2.4786	2.7787	3.4350	3.7066	4.3240
27	0.8551	1.3137	1.7033	2.0518	2.4727	2.7707	3.4210	3.6896	4.2987
28	0.8546	1.3125	1.7011	2.0484	2.4671	2.7633	3.4082	3.6739	4.2754
29	0.8542	1.3114	1.6991	2.0452	2.4620	2.7564	3.3962	3.6594	4.2539
30	0.8538	1.3104	1.6973	2.0423	2.4573	2.7500	3.3852	3.6460	4.2340
40	0.8507	1.3031	1.6839	2.0211	2.4233	2.7045	3.3069	3.5510	4.0942
50	0.8489	1.2987	1.6759	2.0086	2.4033	2.6778	3.2614	3.4960	4.0140
60	0.8477	1.2958	1.6706	2.0003	2.3901	2.6603	3.2317	3.4602	3.9621
70	0.8468	1.2938	1.6669	1.9944	2.3808	2.6479	3.2108	3.4350	3.9257
80	0.8461	1.2922	1.6641	1.9901	2.3739	2.6387	3.1953	3.4163	3.8988
90	0.8456	1.2910	1.6620	1.9867	2.3685	2.6316	3.1833	3.4019	3.8780
100	0.8452	1.2901	1.6602	1.9840	2.3642	2.6259	3.1737	3.3905	3.8616
200	0.8434	1.2858	1.6525	1.9719	2.3451	2.6006	3.1315	3.3398	3.7891
200	0.8434	1.2858	1.6525	1.9719	2.3451	2.6006	3.1315	3.3398	3.7891
∞	0.8416	1.2816	1.6449	1.9600	2.3263	2.5758	3.0902	3.2905	3.7190

Table VII Critical Values for the F Distribution

This table contains critical values associated with the F distribution, F_α, defined by α and degrees of freedom ν_1 and ν_2.

$\alpha = 0.05$

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	15	20	30	40	50	60	100
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.43	19.45	19.46	19.47	19.48	19.48	19.49
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.70	8.66	8.62	8.59	8.58	8.57	8.55
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.86	5.80	5.75	5.72	5.70	5.69	5.66
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.62	4.56	4.50	4.46	4.44	4.43	4.41
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	3.94	3.87	3.81	3.77	3.75	3.74	3.71
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.51	3.44	3.38	3.34	3.32	3.30	3.27
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.22	3.15	3.08	3.04	3.02	3.01	2.97
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.01	2.94	2.86	2.83	2.80	2.79	2.76
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.85	2.77	2.70	2.66	2.64	2.62	2.59
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.72	2.65	2.57	2.53	2.51	2.49	2.46
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.62	2.54	2.47	2.43	2.40	2.38	2.35
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.53	2.46	2.38	2.34	2.31	2.30	2.26
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.46	2.39	2.31	2.27	2.24	2.22	2.19
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.40	2.33	2.25	2.20	2.18	2.16	2.12
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.35	2.28	2.19	2.15	2.12	2.11	2.07
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.31	2.23	2.15	2.10	2.08	2.06	2.02
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.27	2.19	2.11	2.06	2.04	2.02	1.98
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.23	2.16	2.07	2.03	2.00	1.98	1.94
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.20	2.12	2.04	1.99	1.97	1.95	1.91
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.18	2.10	2.01	1.96	1.94	1.92	1.88
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.15	2.07	1.98	1.94	1.91	1.89	1.85
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.13	2.05	1.96	1.91	1.88	1.86	1.82
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.11	2.03	1.94	1.89	1.86	1.84	1.80
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.09	2.01	1.92	1.87	1.84	1.82	1.78
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.01	1.93	1.84	1.79	1.76	1.74	1.70
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	1.92	1.84	1.74	1.69	1.66	1.64	1.59
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.87	1.78	1.69	1.63	1.60	1.58	1.52
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.84	1.75	1.65	1.59	1.56	1.53	1.48
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.77	1.68	1.57	1.52	1.48	1.45	1.39

Table VII Critical Values for the F Distribution

This table contains critical values associated with the F distribution, F_α, defined by α and degrees of freedom ν_1 and ν_2.

$\alpha = 0.01$ ν_1

ν_2	1	2	3	4	5	6	7	8	9	10	15	20	30	40	50	60	100
2	98.50	99.00	99.17	99.25	99.3	99.33	99.36	99.37	99.39	99.40	99.43	99.45	99.47	99.47	99.48	99.48	99.49
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	26.87	26.69	26.50	26.41	26.35	26.32	26.24
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.20	14.02	13.84	13.75	13.69	13.65	13.58
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.72	9.55	9.38	9.29	9.24	9.20	9.13
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.56	7.40	7.23	7.14	7.09	7.06	6.99
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.31	6.16	5.99	5.91	5.86	5.82	5.75
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.52	5.36	5.20	5.12	5.07	5.03	4.96
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	4.96	4.81	4.65	4.57	4.52	4.48	4.41
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.56	4.41	4.25	4.17	4.12	4.08	4.01
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.25	4.10	3.94	3.86	3.81	3.78	3.71
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.01	3.86	3.70	3.62	3.57	3.54	3.47
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.82	3.66	3.51	3.43	3.38	3.34	3.27
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.66	3.51	3.35	3.27	3.22	3.18	3.11
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.52	3.37	3.21	3.13	3.08	3.05	2.98
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.41	3.26	3.10	3.02	2.97	2.93	2.86
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.31	3.16	3.00	2.92	2.87	2.83	2.76
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.23	3.08	2.92	2.84	2.78	2.75	2.68
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.15	3.00	2.84	2.76	2.71	2.67	2.60
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.09	2.94	2.78	2.69	2.64	2.61	2.54
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.03	2.88	2.72	2.64	2.58	2.55	2.48
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	2.98	2.83	2.67	2.58	2.53	2.50	2.42
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	2.93	2.78	2.62	2.54	2.48	2.45	2.37
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	2.89	2.74	2.58	2.49	2.44	2.40	2.33
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.85	2.70	2.54	2.45	2.40	2.36	2.29
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.70	2.55	2.39	2.30	2.25	2.21	2.13
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.52	2.37	2.20	2.11	2.06	2.02	1.94
50	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78	2.70	2.42	2.27	2.10	2.01	1.95	1.91	1.82
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.35	2.20	2.03	1.94	1.88	1.84	1.75
100	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59	2.50	2.22	2.07	1.89	1.80	1.74	1.69	1.60