

STAT 300 – Formulas and Probability Tables

$$\bar{x} = \frac{\sum x}{n} \quad s^2 = \begin{cases} \frac{\sum (x - \bar{x})^2}{n-1} \\ \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1} \end{cases}$$

$$s = \sqrt{s^2}$$

$$\begin{cases} (A \cup B)^c = A^c \cap B^c \\ (A \cap B)^c = A^c \cup B^c \end{cases}$$

$$P(A) = 1 - P(A^c)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\begin{aligned} P(A \cup B \cup C) &= P(A) + P(B) + P(C) \dots \\ &\dots - P(A \cap B) - P(A \cap C) - P(B \cap C) \dots \\ &\dots + P(A \cap B \cap C) \end{aligned}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = \begin{cases} P(B)P(A|B) \\ P(A)P(B|A) \end{cases}$$

$$P(B) = \dots$$

$$\dots \begin{cases} P(A_1 \cap B) + \dots + P(A_k \cap B) \\ P(A_1)P(B|A_1) + \dots + P(A_k)P(B|A_k) \\ \sum_{i=1}^k P(A_i)P(B|A_i) \end{cases}$$

$$P(A_j|B) = \frac{P(A_j \cap B)}{P(B)} = \frac{P(A_j)P(B|A_j)}{\sum_{i=1}^k P(A_i)P(B|A_i)}$$

If events A and B are independent:
 $P(A \cap B) = P(A) \cdot P(B)$

$$p(x) = P(X = x)$$

$$F(x) = \begin{cases} P(X \leq x) \\ \sum_{y:y \leq x} p(y) \end{cases}$$

$$P(a \leq X \leq b) = F(b) - F(a-1)$$

$$P(X = a) = F(a) - F(a-1)$$

$$E(X) = \mu = \sum x \cdot p(x)$$

$$E[h(X)] = \sum h(x)p(x)$$

$$E(aX + b) = aE(X) + b$$

$$V(X) = \sigma^2 = \begin{cases} E[(X - \mu)^2] = \sum (x - \mu)^2 p(x) \\ E(X^2) - [E(X)]^2 \end{cases}$$

$$V(aX + b) = a^2V(X)$$

Binomial Distribution

$$b(x; n, p) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x} & \text{If } x = 0, 1, 2, \dots, n \\ 0 & \text{otherwise} \end{cases}$$

$$\binom{n}{x} = \frac{n!}{x!(n-x)!} \quad E(X) = np \quad V(X) = np(1-p)$$

Poisson Distribution

$$p(x; \mu) = \begin{cases} \frac{e^{-\mu} \mu^x}{x!} & \text{If } x = 0, 1, 2, \dots \\ 0 & \text{otherwise} \end{cases}$$

$$E(X) = V(X) = \mu$$

$$P(a \leq X \leq b) = \int_a^b f(x) dx$$

$$F(x) = \begin{cases} P(X \leq x) \\ \int_{-\infty}^x f(y) dy \end{cases}$$

$$P(a \leq X \leq b) = F(b) - F(a)$$

$$P(X > a) = 1 - F(a)$$

$$p = \begin{cases} F(\eta(p)) \\ \int_{-\infty}^{\eta(p)} f(x) dx \end{cases}$$

$$E(X) = \int_{-\infty}^{\infty} x \cdot f(x) dx$$

$$E[h(X)] = \int_{-\infty}^{\infty} h(x) f(x) dx$$

$$V(X) = \sigma^2 = \begin{cases} E[(X - \mu)^2] = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx \\ E(X^2) - [E(X)]^2 \end{cases}$$

Uniform Distribution

$$f(x) = \begin{cases} \frac{1}{B - A} & \text{If } A \leq x \leq B \\ 0 & \text{otherwise} \end{cases}$$

Normal Distribution

$$f(x) = \frac{1}{\sqrt{2\pi} \cdot \sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad \text{If } -\infty < x < \infty$$

$$Z = \frac{X - \mu}{\sigma}$$

Normal Approximation to Binomial

$$P(X \leq x) \approx P\left(Z \leq \frac{x + 0.5 - np}{\sqrt{np(1-p)}}\right)$$

$$p(x, y) = P(X = x \text{ and } Y = y)$$

$$P[(X, Y) \in A] = \sum_{(x, y) \in A} p(x, y)$$

$$p_X(x) = \sum_y p(x, y) \quad p_Y(y) = \sum_x p(x, y)$$

$$E[h(X, Y)] = \sum_x \sum_y h(x, y) p(x, y)$$

$$\text{Cov}(X, Y) = \begin{cases} E[(X - \mu_X)(Y - \mu_Y)] = \\ \sum_x \sum_y (x - \mu_X)(y - \mu_Y) p(x, y) \\ E(XY) - E(X)E(Y) \end{cases}$$

$$\text{Corr}(X, Y) = \rho_{XY} = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y}$$

$$\text{Cov}(aX + b, cY + d) = ac \text{Cov}(X, Y)$$

If a and c are either both positive or both negative then

$$\text{Corr}(aX + b, cY + d) = \text{Corr}(X, Y)$$

$$\left. \begin{matrix} E(\bar{X}) \\ \mu_{\bar{X}} \end{matrix} \right\} = \mu \quad \left. \begin{matrix} V(\bar{X}) \\ \sigma_{\bar{X}}^2 \end{matrix} \right\} = \frac{\sigma^2}{n} \quad \left. \begin{matrix} SD(\bar{X}) \\ \sigma_{\bar{X}} \end{matrix} \right\} = \frac{\sigma}{\sqrt{n}}$$

$$Y = a_1 X_1 + \dots + a_n X_n = \sum_{i=1}^n a_i X_i$$

$$E(Y) = E(a_1 X_1 + \dots + a_n X_n) = a_1 \mu_1 + \dots + a_n \mu_n$$

If X_1, X_2, \dots, X_n are independent then

$$V(Y) = V(a_1 X_1 + \dots + a_n X_n) = a_1^2 \sigma_1^2 + \dots + a_n^2 \sigma_n^2$$

$$\sigma_Y = \sqrt{a_1^2 \sigma_1^2 + \dots + a_n^2 \sigma_n^2}$$

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \quad n = \left(\frac{2 \cdot z_{\alpha/2} \cdot \sigma}{w} \right)^2$$

$$\bar{x} \pm z_{\alpha/2} \frac{s}{\sqrt{n}}$$

$$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \quad n = \left(\frac{2 \cdot z_{\alpha/2}}{w} \right)^2 \hat{p}(1-\hat{p})$$

$$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} \quad df = n-1$$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$z = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}} \quad df = n-1$$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x \quad \hat{\beta}_1 = \frac{SS_{xy}}{SS_{xx}} \quad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$SS_{xy} = \sum xy - \frac{(\sum x)(\sum y)}{n} \quad SS_{xx} = \sum x^2 - \frac{(\sum x)^2}{n} \quad SS_{yy} = \sum y^2 - \frac{(\sum y)^2}{n}$$

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx}} \sqrt{SS_{yy}}}$$

$$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2} \sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)} \quad df = n_1 + n_2 - 2$$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \quad s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2} \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p}) \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \quad \hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

Table A.1 Cumulative Binomial Probabilities

$$B(x; n, p) = \sum_{y=0}^x b(y; n, p)$$

a. $n = 5$

		<i>p</i>														
		0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
<i>x</i>	0	.951	.774	.590	.328	.237	.168	.078	.031	.010	.002	.001	.000	.000	.000	.000
	1	.999	.977	.919	.737	.633	.528	.337	.188	.087	.031	.016	.007	.000	.000	.000
	2	1.000	.999	.991	.942	.896	.837	.683	.500	.317	.163	.104	.058	.009	.001	.000
	3	1.000	1.000	1.000	.993	.984	.969	.913	.812	.663	.472	.367	.263	.081	.023	.001
	4	1.000	1.000	1.000	1.000	.999	.998	.990	.969	.922	.832	.763	.672	.410	.226	.049

b. $n = 10$

		<i>p</i>														
		0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
<i>x</i>	0	.904	.599	.349	.107	.056	.028	.006	.001	.000	.000	.000	.000	.000	.000	.000
	1	.996	.914	.736	.376	.244	.149	.046	.011	.002	.000	.000	.000	.000	.000	.000
	2	1.000	.988	.930	.678	.526	.383	.167	.055	.012	.002	.000	.000	.000	.000	.000
	3	1.000	.999	.987	.879	.776	.650	.382	.172	.055	.011	.004	.001	.000	.000	.000
	4	1.000	1.000	.998	.967	.922	.850	.633	.377	.166	.047	.020	.006	.000	.000	.000
	5	1.000	1.000	1.000	.994	.980	.953	.834	.623	.367	.150	.078	.033	.002	.000	.000
	6	1.000	1.000	1.000	.999	.996	.989	.945	.828	.618	.350	.224	.121	.013	.001	.000
	7	1.000	1.000	1.000	1.000	1.000	.998	.988	.945	.833	.617	.474	.322	.070	.012	.000
	8	1.000	1.000	1.000	1.000	1.000	1.000	.998	.989	.954	.851	.756	.624	.264	.086	.004
	9	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.994	.972	.944	.893	.651	.401	.096

c. $n = 15$

		<i>p</i>														
		0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
<i>x</i>	0	.860	.463	.206	.035	.013	.005	.000	.000	.000	.000	.000	.000	.000	.000	.000
	1	.990	.829	.549	.167	.080	.035	.005	.000	.000	.000	.000	.000	.000	.000	.000
	2	1.000	.964	.816	.398	.236	.127	.027	.004	.000	.000	.000	.000	.000	.000	.000
	3	1.000	.995	.944	.648	.461	.297	.091	.018	.002	.000	.000	.000	.000	.000	.000
	4	1.000	.999	.987	.836	.686	.515	.217	.059	.009	.001	.000	.000	.000	.000	.000
	5	1.000	1.000	.998	.939	.852	.722	.403	.151	.034	.004	.001	.000	.000	.000	.000
	6	1.000	1.000	1.000	.982	.943	.869	.610	.304	.095	.015	.004	.001	.000	.000	.000
	7	1.000	1.000	1.000	.996	.983	.950	.787	.500	.213	.050	.017	.004	.000	.000	.000
	8	1.000	1.000	1.000	.999	.996	.985	.905	.696	.390	.131	.057	.018	.000	.000	.000
	9	1.000	1.000	1.000	1.000	.999	.996	.966	.849	.597	.278	.148	.061	.002	.000	.000
	10	1.000	1.000	1.000	1.000	1.000	.999	.991	.941	.783	.485	.314	.164	.013	.001	.000
	11	1.000	1.000	1.000	1.000	1.000	1.000	.998	.982	.909	.703	.539	.352	.056	.005	.000
	12	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.996	.973	.873	.764	.602	.184	.036	.000
	13	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.995	.965	.920	.833	.451	.171	.010
	14	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.995	.987	.965	.794	.537	.140

(continued)

Table A.1 Cumulative Binomial Probabilities (*cont.*)

$$B(x; n, p) = \sum_{y=0}^x b(y; n, p)$$

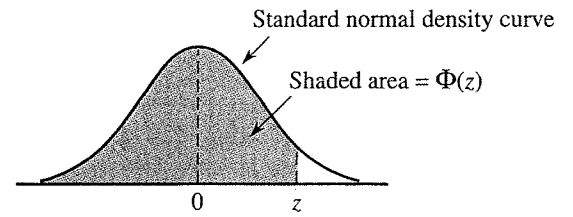
d. $n = 20$

	p														
	0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
0	.818	.358	.122	.012	.003	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000
1	.983	.736	.392	.069	.024	.008	.001	.000	.000	.000	.000	.000	.000	.000	.000
2	.999	.925	.677	.206	.091	.035	.004	.000	.000	.000	.000	.000	.000	.000	.000
3	1.000	.984	.867	.411	.225	.107	.016	.001	.000	.000	.000	.000	.000	.000	.000
4	1.000	.997	.957	.630	.415	.238	.051	.006	.000	.000	.000	.000	.000	.000	.000
5	1.000	1.000	.989	.804	.617	.416	.126	.021	.002	.000	.000	.000	.000	.000	.000
6	1.000	1.000	.998	.913	.786	.608	.250	.058	.006	.000	.000	.000	.000	.000	.000
7	1.000	1.000	1.000	.968	.898	.772	.416	.132	.021	.001	.000	.000	.000	.000	.000
8	1.000	1.000	1.000	.990	.959	.887	.596	.252	.057	.005	.001	.000	.000	.000	.000
9	1.000	1.000	1.000	.997	.986	.952	.755	.412	.128	.017	.004	.001	.000	.000	.000
10	1.000	1.000	1.000	.999	.996	.983	.872	.588	.245	.048	.014	.003	.000	.000	.000
11	1.000	1.000	1.000	1.000	.999	.995	.943	.748	.404	.113	.041	.010	.000	.000	.000
12	1.000	1.000	1.000	1.000	1.000	.999	.979	.868	.584	.228	.102	.032	.000	.000	.000
13	1.000	1.000	1.000	1.000	1.000	1.000	.994	.942	.750	.392	.214	.087	.002	.000	.000
14	1.000	1.000	1.000	1.000	1.000	1.000	.998	.979	.874	.584	.383	.196	.011	.000	.000
15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.994	.949	.762	.585	.370	.043	.003	.000
16	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.984	.893	.775	.589	.133	.016	.000
17	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.996	.965	.909	.794	.323	.075	.001
18	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.992	.976	.931	.608	.264	.017
19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.997	.988	.878	.642	.182

(continued)

Table A.3 Standard Normal Curve Areas

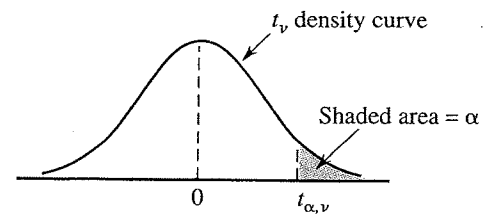
$$\Phi(z) = P(Z \leq z)$$



<i>z</i>	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0038
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3482
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

(continued)

Table A.5 Critical Values for t Distributions



ν	α							
	.10	.05	.025	.01	.005	.001	.0005	
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62	
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598	
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924	
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610	
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869	
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959	
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408	
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041	
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781	
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587	
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437	
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318	
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221	
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140	
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073	
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015	
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965	
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922	
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883	
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850	
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819	
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792	
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767	
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745	
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725	
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707	
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690	
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674	
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659	
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646	
32	1.309	1.694	2.037	2.449	2.738	3.365	3.622	
34	1.307	1.691	2.032	2.441	2.728	3.348	3.601	
36	1.306	1.688	2.028	2.434	2.719	3.333	3.582	
38	1.304	1.686	2.024	2.429	2.712	3.319	3.566	
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551	
50	1.299	1.676	2.009	2.403	2.678	3.262	3.496	
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460	
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373	
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291	