

Numerical Summary Measures

- Sample Mean:

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

- Sample Median: The middle number when the observations are arranged in order from smallest to largest.
- Sample Variance:

$$s^2 = \frac{1}{n-1} \left[\sum_{i=1}^n x_i^2 - \frac{1}{n} \left(\sum_{i=1}^n x_i \right)^2 \right]$$

- Chebyshev's Rule: Let $k > 1$. For any set of observations, the proportion of observations within k standard deviations of the mean is at least $1 - \frac{1}{k^2}$.

Probability

- Complement Rule: $P(A') = 1 - P(A)$
- Addition Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- Addition Rule for Two Disjoint Events: $P(A \cup B) = P(A) + P(B)$
- Conditional Probability: If $P(B) > 0$ then

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- Multiplication Rule: $P(A \cap B) = P(A|B)P(B)$
- Multiplication Rule for Independent Events: $P(A \cap B) = P(A)P(B)$

Random Variables and Discrete & Continuous Probability Distributions

- Mean, or expected value, of a discrete random variable X :

$$\mu = E(X) = \sum_{\text{all } x} xp(x)$$

- Variance of a discrete random variable X :

$$\sigma^2 = \text{Var}(X) = \sum_{\text{all } x} x^2p(x) - \mu^2$$

- If X is a binomial random variable based on n trials, each with success probability equal to p , then

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, 3, \dots, n$$

where $\mu = np$, $\sigma^2 = np(1-p)$, and $\sigma = \sqrt{np(1-p)}$.

- If X is a normal random variable with mean μ and standard deviation σ , then a standard normal random variable is given by

$$Z = \frac{X - \mu}{\sigma}$$

Sampling Distributions

- Properties of the sample mean \bar{X} :

$$E(\bar{X}) = \mu_{\bar{X}} = \mu \quad \text{and} \quad V(\bar{X}) = \sigma_{\bar{X}}^2 = \frac{\sigma^2}{n}$$

Confidence Intervals Based on a Single Sample

- Confidence Interval for μ when n is large or the population is normal, and σ is known:

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

- Confidence Interval for μ when the population is normal and σ is unknown:

$$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

- Confidence interval for a population proportion p when n is large enough so that $n\hat{p} \geq 5$ and $n(1 - \hat{p}) \geq 5$:

$$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

- The minimum sample size required for a $100(1 - \alpha)\%$ confidence interval of width w for μ when the population is normal and σ is known:

$$n = \left(\frac{\sigma z_{\alpha/2}}{b} \right)^2$$

where $b = \frac{1}{2}w$.

- The minimum sample size required for a $100(1 - \alpha)\%$ confidence interval of width w for a population proportion p is

$$n = \hat{p}(1 - \hat{p}) \left[\frac{z_{\alpha/2}}{b} \right]^2$$

where \hat{p} is either a reasonable estimate of p based on past experience, or \hat{p} is set to 0.5 to obtain a conservative estimate of n , and $b = \frac{1}{2}w$.

Hypothesis Tests Based on a Single Sample

- Hypothesis test for μ when n is large or the population is normal, and σ is known:

– Null Hypothesis: $H_0 : \mu = \mu_0$

– Test Statistic:

$$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

– Alternative hypotheses, rejection regions, and p -values:

Alternative Hypothesis	Rejection Region	p -value
$H_a : \mu > \mu_0$	$Z \geq z_\alpha$	$P(Z \geq z)$
$H_a : \mu < \mu_0$	$Z \leq -z_\alpha$	$P(Z \leq z)$
$H_a : \mu \neq \mu_0$	$ Z \geq z_{\alpha/2}$	$2P(Z \geq z)$ if $z \geq 0$ $2P(Z \leq z)$ if $z < 0$

- Hypothesis test for μ when the population is normal and σ is unknown:

- Null Hypothesis: $H_0 : \mu = \mu_0$
- Test Statistic:

$$T = \frac{\bar{x} - \mu_0}{S/\sqrt{n}}$$

- Alternative hypotheses, rejection regions, and p -values:

Alternative Hypothesis	Rejection Region
$H_a : \mu > \mu_0$	$T \geq t_\alpha$
$H_a : \mu < \mu_0$	$T \leq -t_\alpha$
$H_a : \mu \neq \mu_0$	$ T \geq t_{\alpha/2}$

- Hypothesis test for a population proportion p :

- Null Hypothesis: $H_0 : p = p_0$
- Test Statistic:

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

- Alternative hypotheses, rejection regions, and p -values:

Alternative Hypothesis	Rejection Region	p -value
$H_a : p > p_0$	$Z \geq z_\alpha$	$P(Z \geq z)$
$H_a : p < p_0$	$Z \leq -z_\alpha$	$P(Z \leq z)$
$H_a : p \neq p_0$	$ Z \geq z_{\alpha/2}$	$2P(Z \geq z)$ if $z \geq 0$ $2P(Z \leq z)$ if $z < 0$

Hypothesis Tests and Confidence Intervals Based on Two Samples

- Confidence interval for $\mu_1 - \mu_2$ based on two independent samples where σ_1 and σ_2 are known, and the populations are normal or n_1 and n_2 are both large:

$$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

- Confidence interval for $\mu_1 - \mu_2$ based on two independent samples where σ_1 and σ_2 are unknown but equal, and the populations are normal:

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2} \sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}$$

where

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

and the degrees of freedom are $df = n_1 + n_2 - 2$.

- Confidence interval for $\mu_1 - \mu_2$ based on paired observations from a normal population:

$$\bar{d} \pm t_{\alpha/2} \frac{s_D}{\sqrt{n}}$$

where the degrees of freedom are $df = n - 1$.

- Confidence interval for $p_1 - p_2$:

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2} \sqrt{\frac{\hat{p}_1(1 - \hat{p}_1)}{n_1} + \frac{\hat{p}_2(1 - \hat{p}_2)}{n_2}}$$

- Hypothesis test for $\mu_1 - \mu_2$ based on two independent samples where σ_1 and σ_2 are known, and the populations are normal or n_1 and n_2 are both large:

- Null Hypothesis: $H_0 : \mu_1 - \mu_2 = \Delta_0$
- Test Statistic:

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

- Alternative hypotheses, rejection regions, and p -values:

Alternative Hypothesis	Rejection Region	p -value
$H_a : \mu_1 - \mu_2 > \Delta_0$	$Z \geq z_\alpha$	$P(Z \geq z)$
$H_a : \mu_1 - \mu_2 < \Delta_0$	$Z \leq -z_\alpha$	$P(Z \leq z)$
$H_a : \mu_1 - \mu_2 \neq \Delta_0$	$ Z \geq z_{\alpha/2}$	$2P(Z \geq z)$ if $z \geq 0$ $2P(Z \leq z)$ if $z < 0$

- Hypothesis test for $\mu_1 - \mu_2$ based on two independent samples where σ_1 and σ_2 are unknown but equal, and the populations are normal:

- Null Hypothesis: $H_0 : \mu_1 - \mu_2 = \Delta_0$
- Test Statistic:

$$T = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

where

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

and the degrees of freedom are $df = n_1 + n_2 - 2$.

- Alternative hypotheses and rejection regions:

Alternative Hypothesis	Rejection Region
$H_a : \mu_1 - \mu_2 > \Delta_0$	$T \geq t_\alpha$
$H_a : \mu_1 - \mu_2 < \Delta_0$	$T \leq -t_\alpha$
$H_a : \mu_1 - \mu_2 \neq \Delta_0$	$ T \geq t_{\alpha/2}$

- Hypothesis test for $\mu_1 - \mu_2$ based on paired observations from a normal population:

- Null Hypothesis: $H_0 : \mu_D = \Delta_0$
- Test Statistic:

$$T = \frac{\bar{D} - \Delta_0}{S_D / \sqrt{n}}$$

- Alternative hypotheses and rejection regions:

Alternative Hypothesis	Rejection Region
$H_a : \mu_D > \Delta_0$	$T \geq t_\alpha$
$H_a : \mu_D < \Delta_0$	$T \leq -t_\alpha$
$H_a : \mu_D \neq \Delta_0$	$ T \geq t_{\alpha/2}$

• Hypothesis test for $p_1 - p_2$:

- Null Hypothesis: $H_0 : p_1 - p_2 = 0$
- Test Statistic:

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_c(1 - \hat{p}_c) \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

where

$$\hat{p}_c = \frac{X_1 + X_2}{n_1 + n_2}$$

- Alternative hypotheses, rejection regions, and p -values:

Alternative Hypothesis	Rejection Region	p -value
$H_a : p_1 - p_2 > 0$	$Z \geq z_\alpha$	$P(Z \geq z)$
$H_a : p_1 - p_2 < 0$	$Z \leq -z_\alpha$	$P(Z \leq z)$
$H_a : p_1 - p_2 \neq 0$	$ Z \geq z_{\alpha/2}$	$2P(Z \geq z)$ if $z \geq 0$ $2P(Z \leq z)$ if $z < 0$

• Hypothesis test for $p_1 - p_2$:

- Null Hypothesis: $H_0 : p_1 - p_2 = \Delta_0$ (where $\Delta_0 \neq 0$)
- Test Statistic:

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - \Delta_0}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}}$$

- Alternative hypotheses, rejection regions, and p -values:

Alternative Hypothesis	Rejection Region	p -value
$H_a : p_1 - p_2 > \Delta_0$	$Z \geq z_\alpha$	$P(Z \geq z)$
$H_a : p_1 - p_2 < \Delta_0$	$Z \leq -z_\alpha$	$P(Z \leq z)$
$H_a : p_1 - p_2 \neq \Delta_0$	$ Z \geq z_{\alpha/2}$	$2P(Z \geq z)$ if $z \geq 0$ $2P(Z \leq z)$ if $z < 0$

The Analysis of Variance

ANOVA Table Summary					
Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p Value
Factor	SSA	$k - 1$	$MSA = \frac{SSA}{k-1}$	$\frac{MSA}{MSE}$	p
Error	SSE	$n - k$	$MSE = \frac{SSE}{n-k}$		
Total	SST	$n - 1$			

Correlation and Linear Regression

- The least squares estimates of β_0 and β_1 are given by

$$\hat{\beta}_1 = \frac{n \sum x_i y_i - (\sum x_i)(\sum y_i)}{n \sum x_i^2 - (\sum x_i)^2}$$

and

$$\hat{\beta}_0 = \frac{\sum y_i - \hat{\beta}_1 \sum x_i}{n} = \bar{y} - \hat{\beta}_1 \bar{x}$$

- The estimated regression line is given by $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$.
- ANOVA Table Summary:

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p -value
Regression	SSR	1	$MSR = \frac{SSR}{1}$	$\frac{MSR}{MSE}$	p
Error	SSE	$n - 2$	$MSE = \frac{SSE}{n-2}$		
Total	SST	$n - 1$			

- Coefficient of Determination:

$$r^2 = \frac{SSR}{SST}$$

- Correlation Coefficient:

$$r = \frac{\sum x_i y_i - \frac{1}{n} (\sum x_i)(\sum y_i)}{\sqrt{\left[\sum x_i^2 - \frac{1}{n} (\sum x_i)^2 \right] \left[\sum y_i^2 - \frac{1}{n} (\sum y_i)^2 \right]}}$$

- Alternate formulas for correlation and regression:

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x \quad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$S_{xy} = \sum xy - \frac{(\sum x)(\sum y)}{n} \quad S_{xx} = \sum x^2 - \frac{(\sum x)^2}{n} \quad S_{yy} = \sum y^2 - \frac{(\sum y)^2}{n}$$

$$r = \frac{S_{xy}}{\sqrt{S_{xx}} \sqrt{S_{yy}}}$$

Table I: Binomial Distribution Cumulative Probabilities (Continued)

$n = 20$																	
x	0.01	0.05	0.10	0.20	0.25	0.30	0.40	p								0.95	0.99
								0.50	0.60	0.70	0.75	0.80	0.90				
0	0.818	0.358	0.122	0.012	0.003	0.001	0.000										
1	0.983	0.736	0.392	0.069	0.024	0.008	0.001										
2	0.999	0.925	0.677	0.206	0.091	0.035	0.004	0.000									
3	1.000	0.984	0.867	0.411	0.225	0.107	0.016	0.001									
4		0.997	0.957	0.630	0.415	0.238	0.051	0.006	0.000								
5		1.000	0.989	0.804	0.617	0.416	0.126	0.021	0.002								
6			0.998	0.913	0.786	0.608	0.250	0.058	0.006	0.000							
7			1.000	0.968	0.898	0.772	0.416	0.132	0.021	0.001	0.000						
8				0.990	0.959	0.887	0.596	0.252	0.057	0.005	0.001	0.000					
9				0.997	0.986	0.952	0.755	0.412	0.128	0.017	0.004	0.001					
10				0.999	0.996	0.983	0.872	0.588	0.245	0.048	0.014	0.003					
11				1.000	0.999	0.995	0.943	0.748	0.404	0.113	0.041	0.01					
12					1.000	0.999	0.979	0.868	0.584	0.228	0.102	0.032	0.000				
13						1.000	0.994	0.942	0.75	0.392	0.214	0.087	0.002				
14							0.998	0.979	0.874	0.584	0.383	0.196	0.011	0.000			
15							1.000	0.994	0.949	0.762	0.585	0.37	0.043	0.003			
16								0.999	0.984	0.893	0.775	0.589	0.133	0.016	0.000		
17								1.000	0.996	0.965	0.909	0.794	0.323	0.075	0.001		
18									0.999	0.992	0.976	0.931	0.608	0.264	0.017		
19										1.000	0.999	0.997	0.988	0.878	0.642	0.182	

$n = 25$																	
x	0.01	0.05	0.10	0.20	0.25	0.30	0.40	p								0.95	0.99
								0.50	0.60	0.70	0.75	0.80	0.90				
0	0.778	0.277	0.072	0.004	0.001	0.000											
1	0.974	0.642	0.271	0.027	0.007	0.002											
2	0.998	0.873	0.537	0.098	0.032	0.009	0.000										
3	1.000	0.966	0.764	0.234	0.096	0.033	0.002										
4		0.993	0.902	0.421	0.214	0.09	0.009	0.000									
5		0.999	0.967	0.617	0.378	0.193	0.029	0.002									
6		1.000	0.991	0.780	0.561	0.341	0.074	0.007	0.000								
7			0.998	0.891	0.727	0.512	0.154	0.022	0.001								
8			1.000	0.953	0.851	0.677	0.274	0.054	0.004								
9				0.983	0.929	0.811	0.425	0.115	0.013	0.000							
10				0.994	0.970	0.902	0.586	0.212	0.034	0.002	0.000						
11				0.998	0.989	0.956	0.732	0.345	0.078	0.006	0.001						
12				1.000	0.997	0.983	0.846	0.500	0.154	0.017	0.003	0.000					
13					0.999	0.994	0.922	0.655	0.268	0.044	0.011	0.002					
14					1.000	0.998	0.966	0.788	0.414	0.098	0.030	0.006					
15						1.000	0.987	0.885	0.575	0.189	0.071	0.017					
16							0.996	0.946	0.726	0.323	0.149	0.047	0.000				
17							0.999	0.978	0.846	0.488	0.273	0.109	0.002				
18							1.000	0.993	0.926	0.659	0.439	0.220	0.009	0.000			
19								0.998	0.971	0.807	0.622	0.383	0.033	0.001			
20								1.000	0.991	0.910	0.786	0.579	0.098	0.007			
21									0.998	0.967	0.904	0.766	0.236	0.034	0.000		
22									1.000	0.991	0.968	0.902	0.463	0.127	0.002		
23										0.998	0.993	0.973	0.729	0.358	0.026		
24											1.000	0.999	0.996	0.928	0.723	0.222	

Table III Standard Normal Distribution Cumulative Probabilities

Let Z be a standard normal random variable: $\mu = 0$ and $\sigma = 1$.
 This table contains cumulative probabilities: $P(Z \leq z)$.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table III Standard Normal Distribution Cumulative Probabilities (Continued)

Let Z be a standard normal random variable: $\mu = 0$ and $\sigma = 1$.
 This table contains cumulative probabilities: $P(Z \leq z)$.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Special Critical Values: $P(Z \geq z_\alpha) = \alpha$

α	0.10	0.05	0.025	0.01	0.005	0.001	0.0005	0.0001
z_α	1.2816	1.6449	1.9600	2.3263	2.5758	3.0902	3.2905	3.7190
α	0.00009	0.00008	0.00007	0.00006	0.00005	0.00004	0.00003	0.00002
z_α	3.7455	3.7750	3.8082	3.8461	3.8906	3.9444	4.0128	4.1075

Table V Critical Values for the t Distribution

This table contains critical values associated with the t distribution, t_α , defined by the degrees of freedom and α .

df	0.20	0.10	0.05	0.025	0.01	0.005	0.001	0.0005	0.0001
1	1.3764	3.0777	6.3138	12.7062	31.8205	63.6567	318.3088	636.6192	3183.0988
2	1.0607	1.8856	2.9200	4.3027	6.9646	9.9248	22.3271	31.5991	70.7001
3	0.9785	1.6377	2.3534	3.1824	4.5407	5.8409	10.2145	12.9240	22.2037
4	0.9410	1.5332	2.1318	2.7764	3.7469	4.6041	7.1732	8.6103	13.0337
5	0.9195	1.4759	2.0150	2.5706	3.3649	4.0321	5.8934	6.8688	9.6776
6	0.9057	1.4398	1.9432	2.4469	3.1427	3.7074	5.2076	5.9588	8.0248
7	0.8960	1.4149	1.8946	2.3646	2.9980	3.4995	4.7853	5.4079	7.0634
8	0.8889	1.3968	1.8595	2.3060	2.8965	3.3554	4.5008	5.0413	6.4420
9	0.8834	1.3830	1.8331	2.2622	2.8214	3.2498	4.2968	4.7809	6.0101
10	0.8791	1.3722	1.8125	2.2281	2.7638	3.1693	4.1437	4.5869	5.6938
11	0.8755	1.3634	1.7959	2.2010	2.7181	3.1058	4.0247	4.4370	5.4528
12	0.8726	1.3562	1.7823	2.1788	2.6810	3.0545	3.9296	4.3178	5.2633
13	0.8702	1.3502	1.7709	2.1604	2.6503	3.0123	3.8520	4.2208	5.1106
14	0.8681	1.3450	1.7613	2.1448	2.6245	2.9768	3.7874	4.1405	4.9850
15	0.8662	1.3406	1.7531	2.1314	2.6025	2.9467	3.7328	4.0728	4.8800
16	0.8647	1.3368	1.7459	2.1199	2.5835	2.9208	3.6862	4.0150	4.7909
17	0.8633	1.3334	1.7396	2.1098	2.5669	2.8982	3.6458	3.9651	4.7144
18	0.8620	1.3304	1.7341	2.1009	2.5524	2.8784	3.6105	3.9216	4.6480
19	0.8610	1.3277	1.7291	2.0930	2.5395	2.8609	3.5794	3.8834	4.5899
20	0.8600	1.3253	1.7247	2.0860	2.5280	2.8453	3.5518	3.8495	4.5385
21	0.8591	1.3232	1.7207	2.0796	2.5176	2.8314	3.5272	3.8193	4.4929
22	0.8583	1.3212	1.7171	2.0739	2.5083	2.8188	3.5050	3.7921	4.4520
23	0.8575	1.3195	1.7139	2.0687	2.4999	2.8073	3.4850	3.7676	4.4152
24	0.8569	1.3178	1.7109	2.0639	2.4922	2.7969	3.4668	3.7454	4.3819
25	0.8562	1.3163	1.7081	2.0595	2.4851	2.7874	3.4502	3.7251	4.3517
26	0.8557	1.3150	1.7056	2.0555	2.4786	2.7787	3.4350	3.7066	4.3240
27	0.8551	1.3137	1.7033	2.0518	2.4727	2.7707	3.4210	3.6896	4.2987
28	0.8546	1.3125	1.7011	2.0484	2.4671	2.7633	3.4082	3.6739	4.2754
29	0.8542	1.3114	1.6991	2.0452	2.4620	2.7564	3.3962	3.6594	4.2539
30	0.8538	1.3104	1.6973	2.0423	2.4573	2.7500	3.3852	3.6460	4.2340
40	0.8507	1.3031	1.6839	2.0211	2.4233	2.7045	3.3069	3.5510	4.0942
50	0.8489	1.2987	1.6759	2.0086	2.4033	2.6778	3.2614	3.4960	4.0140
60	0.8477	1.2958	1.6706	2.0003	2.3901	2.6603	3.2317	3.4602	3.9621
70	0.8468	1.2938	1.6669	1.9944	2.3808	2.6479	3.2108	3.4350	3.9257
80	0.8461	1.2922	1.6641	1.9901	2.3739	2.6387	3.1953	3.4163	3.8988
90	0.8456	1.2910	1.6620	1.9867	2.3685	2.6316	3.1833	3.4019	3.8780
100	0.8452	1.2901	1.6602	1.9840	2.3642	2.6259	3.1737	3.3905	3.8616
200	0.8434	1.2858	1.6525	1.9719	2.3451	2.6006	3.1315	3.3398	3.7891
∞	0.8416	1.2816	1.6449	1.9600	2.3263	2.5758	3.0902	3.2905	3.7190

Table VII Critical Values for the F Distribution

This table contains critical values associated with the F distribution, F_α , defined by α and degrees of freedom ν_1 and ν_2 .

$\alpha = 0.05$

ν_2	ν_1																
	1	2	3	4	5	6	7	8	9	10	15	20	30	40	50	60	100
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.43	19.45	19.46	19.47	19.48	19.48	19.49
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.70	8.66	8.62	8.59	8.58	8.57	8.55
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.86	5.80	5.75	5.72	5.70	5.69	5.66
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.62	4.56	4.50	4.46	4.44	4.43	4.41
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	3.94	3.87	3.81	3.77	3.75	3.74	3.71
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.51	3.44	3.38	3.34	3.32	3.30	3.27
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.22	3.15	3.08	3.04	3.02	3.01	2.97
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.01	2.94	2.86	2.83	2.80	2.79	2.76
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.85	2.77	2.70	2.66	2.64	2.62	2.59
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.72	2.65	2.57	2.53	2.51	2.49	2.46
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.62	2.54	2.47	2.43	2.40	2.38	2.35
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.53	2.46	2.38	2.34	2.31	2.30	2.26
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.46	2.39	2.31	2.27	2.24	2.22	2.19
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.40	2.33	2.25	2.20	2.18	2.16	2.12
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.35	2.28	2.19	2.15	2.12	2.11	2.07
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.31	2.23	2.15	2.10	2.08	2.06	2.02
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.27	2.19	2.11	2.06	2.04	2.02	1.98
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.23	2.16	2.07	2.03	2.00	1.98	1.94
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.20	2.12	2.04	1.99	1.97	1.95	1.91
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.18	2.10	2.01	1.96	1.94	1.92	1.88
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.15	2.07	1.98	1.94	1.91	1.89	1.85
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.13	2.05	1.96	1.91	1.88	1.86	1.82
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.11	2.03	1.94	1.89	1.86	1.84	1.80
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.09	2.01	1.92	1.87	1.84	1.82	1.78
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.01	1.93	1.84	1.79	1.76	1.74	1.70
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	1.92	1.84	1.74	1.69	1.66	1.64	1.59
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.87	1.78	1.69	1.63	1.60	1.58	1.52
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.84	1.75	1.65	1.59	1.56	1.53	1.48
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.77	1.68	1.57	1.52	1.48	1.45	1.39

Table VII Critical Values for the F Distribution

This table contains critical values associated with the F distribution, F_α , defined by α and degrees of freedom ν_1 and ν_2 .

$\alpha = 0.01$

ν_2	ν_1																
	1	2	3	4	5	6	7	8	9	10	15	20	30	40	50	60	100
2	98.50	99.00	99.17	99.25	99.3	99.33	99.36	99.37	99.39	99.40	99.43	99.45	99.47	99.47	99.48	99.48	99.49
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	26.87	26.69	26.50	26.41	26.35	26.32	26.24
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.20	14.02	13.84	13.75	13.69	13.65	13.58
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.72	9.55	9.38	9.29	9.24	9.20	9.13
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.56	7.40	7.23	7.14	7.09	7.06	6.99
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.31	6.16	5.99	5.91	5.86	5.82	5.75
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.52	5.36	5.20	5.12	5.07	5.03	4.96
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	4.96	4.81	4.65	4.57	4.52	4.48	4.41
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.56	4.41	4.25	4.17	4.12	4.08	4.01
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.25	4.10	3.94	3.86	3.81	3.78	3.71
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.01	3.86	3.70	3.62	3.57	3.54	3.47
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.82	3.66	3.51	3.43	3.38	3.34	3.27
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.66	3.51	3.35	3.27	3.22	3.18	3.11
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.52	3.37	3.21	3.13	3.08	3.05	2.98
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.41	3.26	3.10	3.02	2.97	2.93	2.86
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.31	3.16	3.00	2.92	2.87	2.83	2.76
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.23	3.08	2.92	2.84	2.78	2.75	2.68
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.15	3.00	2.84	2.76	2.71	2.67	2.60
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.09	2.94	2.78	2.69	2.64	2.61	2.54
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.03	2.88	2.72	2.64	2.58	2.55	2.48
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	2.98	2.83	2.67	2.58	2.53	2.50	2.42
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	2.93	2.78	2.62	2.54	2.48	2.45	2.37
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	2.89	2.74	2.58	2.49	2.44	2.40	2.33
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.85	2.70	2.54	2.45	2.40	2.36	2.29
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.70	2.55	2.39	2.30	2.25	2.21	2.13
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.52	2.37	2.20	2.11	2.06	2.02	1.94
50	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78	2.70	2.42	2.27	2.10	2.01	1.95	1.91	1.82
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.35	2.20	2.03	1.94	1.88	1.84	1.75
100	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59	2.50	2.22	2.07	1.89	1.80	1.74	1.69	1.60