Appendix to "Deriving the wage-wage and price-price Phillips curves from a model with efficiency wages and imperfect information"

An expression for $\hat{L}_t$ can be obtained by totally differentiating (14) and dividing by the original equation, yielding

$$\hat{L}_t = \frac{\gamma\hat{W}_t - \hat{Y}_t - \phi(\gamma-1)\hat{A}_t - \phi(\gamma-1)e^{-1}[e_W \frac{W_t}{\hat{\bar{P}}_t^e}\hat{W}_t - e_W \frac{W_t}{\hat{\bar{P}}_t^e}\hat{\bar{P}}_t^e + e_u du_t] - \gamma\hat{\bar{P}}_t}{\phi(\gamma-1)-\gamma}.$$

By making the substitutions $du_t = -s_L\hat{L}_t + s_L\psi\hat{W}_t - s_L\psi\hat{\bar{P}}_t^e$, $ee_w^{-1} = W_t / \bar{P}_t^e$ (from (15)), and $\hat{\bar{P}}_t = \hat{M}_t - \hat{Y}_t$, the solution for $\hat{L}_t$ becomes

$$\hat{L}_t = \frac{[\phi(\gamma-1) - \gamma + \phi(\gamma-1)e^{-1}e_u s_L\psi]\hat{W}_t - \phi(\gamma-1)[1+e^{-1}e_u s_L\psi]\hat{\bar{P}}_t^e - (\gamma-1)\hat{Y}_t + \phi(\gamma-1)\hat{A}_t + \gamma\hat{M}_t}{\phi + \gamma - \phi\gamma + \phi(\gamma-1)e^{-1}e_u s_L}. \qquad \text{(A1)}$$

Since each firm's output (*Q*) equals aggregate demand per firm (*Y*) in a representative firm framework, an equation for $\hat{Y}_t$ can be obtained by differentiating (13) and making the substitutions $du_t = -s_L\hat{L}_t + s_L\psi\hat{W}_t - s_L\psi\hat{\bar{P}}_t^e$ and $ee_W^{-1} = W_t / \bar{P}_t^e$. Accordingly,

$$\hat{Y}_t = \phi\hat{A}_t + \phi\hat{L}_t + \phi\hat{W}_t - \phi\hat{\bar{P}}_t^e - \phi e^{-1}e_u s_L\hat{L}_t + \phi e^{-1}e_u s_L\psi\hat{W}_t - \phi e^{-1}e_u s_L\psi\hat{\bar{P}}_t^e. \qquad \text{(A2)}$$

Substituting (A2) into (A1) yields $\hat{L}_t = \hat{M}_t - \hat{W}_t$. Thus, the unemployment rate can be expressed as

$$du_t = -s_L(\hat{M}_t - \hat{W}_t) + s_L\psi\hat{W}_t - s_L\psi\hat{\bar{P}}_t^e. \qquad \text{(A3)}$$

By substituting (A3) into (16), the following equation is obtained:

$$\left[1-s_L\frac{e_u-e_{Wu}}{e_{WW}}(1+\psi)\right]\hat{W}_t=\left[1-s_L\psi\frac{e_u-e_{Wu}}{e_{WW}}\right]\hat{\bar{P}}_t^{e}-s_L\frac{e_u-e_{Wu}}{e_{WW}}\hat{M}_t\,,$$

which can be expressed as

$$\hat{W}_t=z\hat{\bar{P}}_t^{e}+(1-z)\hat{M}_t\,, \tag{A4}$$

$$\text{where } z=\frac{e_{WW}-s_L\psi(e_u-e_{Wu})}{e_{WW}-s_L(e_u-e_{Wu})(1+\psi)}\,.$$

By substituting $\hat{L}_t=\hat{M}_t-\hat{W}_t$ into (A2) and then substituting the resulting expression for $\hat{Y}_t$ into the equation, $\hat{\bar{P}}_t=\hat{M}_t-\hat{Y}_t$, the following expression for the price level is derived:

$$\hat{\bar{P}}_t=[1-\phi+\phi e^{-1}e_u s_L]\hat{M}_t-\phi\hat{A}_t-\phi e^{-1}e_u s_L(1+\psi)\hat{W}_t+\phi(1+e^{-1}e_u s_L\psi)\hat{\bar{P}}_t^{e}\,. \tag{A5}$$

The variable $\hat{W}_t$ can be eliminated from (A5) by substituting (A4) into (A5), which yields

$$\hat{\bar{P}}_t=[1-\phi+\phi e^{-1}e_u s_L(z+\psi z-\psi)]\hat{M}_t-\phi\hat{A}_t+[\phi-\phi e^{-1}e_u s_L(z+\psi z-\psi)]\hat{\bar{P}}_t^{e}\,. \tag{A6}$$

In addition, the following equation for $\hat{M}_t$ is obtained by substituting (A4) into (A3):

$$\hat{M}_t=\hat{\bar{P}}_t^{e}-\frac{1}{s_L(z+\psi z-\psi)}du_t\,. \tag{A7}$$

Finally, substituting (A7) into (A6) enables the price level to be expressed as

$$\hat{\bar{P}}_t=\hat{\bar{P}}_t^{e}-\frac{1-\phi+\phi e^{-1}e_u s_L(z+\psi z-\psi)}{s_L(z+\psi z-\psi)}du_t-\phi\hat{A}_t$$

$$= \hat{\bar{P}}_t^e - \frac{(1-\phi)[e_{WW} - s_L(e_u - e_{Wu})(1+\psi)] + \phi e^{-1} e_u s_L e_{WW}}{s_L e_{WW}} du_t - \phi \hat{A}_t . \qquad \text{(A8)}$$