**Utility Loss from Imperfect Information in the Effort Model of Campbell (2006)**

This appendix derives a specific functional form for $VL(\overline{w}^e - \overline{w})$, based on the effort model of Campbell (2006). In this model, workers' utility in period $t$ depends on consumption ($c$) and effort ($e$), and can be expressed by the equation, $U[c_t, e_t] = \ln[c_t] + \alpha e_t - \eta e_t^2$, with $\eta>0$.[1] The probability of a worker being dismissed is assumed to equal $m(1-e_t)^2$, where $m$ is the firm's monitoring intensity. Given reasonable simplifying assumptions, the present value of an employed worker's expected utility is demonstrated to equal

$$(1) \qquad V(W, \overline{W}, e) = \frac{(1+r)[\ln W + \alpha e - \eta e^2]}{r+q+m(1-e)^2} + \frac{q+m(1-e)^2}{r+q+m(1-e)^2}\frac{1+r}{r}\left[\ln \overline{W} - A\right],$$

where $q$ is the probability of an exogenous separation (i.e., a separation not related to a worker's effort), and $r$ is the interest rate.[2] In addition, $[(1+r)/r][\ln \overline{W} - A]$ represents the present value of an unemployed individual's expected utility. The variable $A$ represents the difference (in flow terms) between the present value of $\ln \overline{W}$ and the present value of an unemployed individual's expected utility. (This difference arises from workers' utility or disutility from providing effort and from the current and future spells of unemployment experienced by workers who are currently unemployed.) The value of $A$ depends on unemployment benefits, the monitoring

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[1] The value of $\alpha$ can be positive if workers derive positive utility from effort at lower levels of effort. However, the assumption that $\eta>0$ means that the marginal utility of effort becomes negative at higher levels of effort. In addition, it should be noted that the utility function in Campbell (2006) also includes a term incorporating fairness considerations, but this term is ignored in the present study.

[2] The model of Campbell (2006) assumes that the economy is in a steady-state equilibrium, so time subscripts are not used in the derivation in this appendix. Equation (1) in the present study is equivalent to equation (8) in Campbell (2006), although there are some minor differences that are unimportant to the derivation in the present study. The present study uses the notation $V(W, \overline{W}, e)$ instead of $V^{EC}$, expresses the present value of an unemployed individual's utility as $[(1+r)/r][\ln \overline{W} - A]$ instead of $V^{UN}$, assumes that $\rho$ (the ratio between average consumption and average wages) equals 1, and substitutes $\ln W$ for $\ln \overline{W} + (W/\overline{W}) - 1$, since the two expressions are approximately equal if $W$ is close to $\overline{W}$.

intensity at other firms, the probability of hire for unemployed individuals, and the values of $q$, $r$, $\alpha$, and $\eta$. If it is assumed that unemployment benefits are a fixed fraction of the average wage, then $A$ can be treated as a constant.

Another way of expressing equation (1) is

$$(2) \qquad V(W,\overline{W},e)=\frac{(1+r)[\ln W-\ln\overline{W}+A+\alpha e-\eta e^2]}{r+q+m(1-e)^2}+\frac{1+r}{r}\left[\ln\overline{W}-A\right].$$

In (2) the level of effort that maximizes utility is determined by the first-order condition:

$$0=\frac{dV}{de}=(1+r)\frac{[r+q+m(1-e)^2][\alpha-2\eta e]+2[\ln W-\ln\overline{W}+A+\alpha e-\eta e^2]m(1-e)}{[r+q+m(1-e)^2]^2},$$

which implies that

$$(3) \qquad \begin{aligned}0=&(2\eta-\alpha)me^2-2[\eta(r+q+m)+m(\ln W-\ln\overline{W}+A)]e\\&+\alpha(r+q+m)+2m(\ln W-\ln\overline{W}+A).\end{aligned}$$

As demonstrated in Campbell (2006), the utility-maximizing level of effort is

$$(4) \qquad e=\frac{X-\sqrt{X^2-2m(2\eta-\alpha)X+m(r+q+m)(2\eta-\alpha)^2}}{m(2\eta-\alpha)},$$

$$\text{where} \quad X=\eta(r+q+m)+m(\ln W-\ln\overline{W}+A).$$

Let $w=\ln[W]$ and $\overline{w}=\ln[\overline{W}]$, where $\overline{W}$ represents the true value of the average wage. In addition, let $e(\overline{w}^e)$ represent the optimal level of effort for a worker who believes that the average wage (in logs) is $\overline{w}^e$. Then the present value of utility can be expressed as

$$V(w, \overline{w}, e(\overline{w}^e)) = \frac{(1+r)[w-\overline{w}+A+\alpha e(\overline{w}^e)-\eta e(\overline{w}^e)^2]}{r+q+m(1-e(\overline{w}^e))^2} + \frac{1+r}{r}\left[\overline{w}-A\right]. \tag{5}$$

Equation (5) can be approximated by a Taylor series expansion around the point where $\overline{w}^e=\overline{w}$, yielding

$$\begin{aligned} V(w, \overline{w}, e(\overline{w}^e)) \approx V(w, \overline{w}, e(\overline{w})) &+ \left.\frac{dV}{d\overline{w}^e}\right|_{\overline{w}^e=\overline{w}} (\overline{w}^e-\overline{w}) \\ &+ \frac{1}{2}\left.\frac{d^2V}{d(\overline{w}^e)^2}\right|_{\overline{w}^e=\overline{w}} (\overline{w}^e-\overline{w})^2. \end{aligned} \tag{6}$$

The first derivative of $V$ with respect to $\overline{w}^e$ is

$$\frac{dV}{d\overline{w}^e} = \frac{\begin{aligned}&[r+q+m(1-e(\overline{w}^e))^2](1+r)[\alpha e'(\overline{w}^e)-2\eta e(\overline{w}^e)e'(\overline{w}^e)] \\ &+2(1+r)[w-\overline{w}+A+\alpha e(\overline{w}^e)-\eta e(\overline{w}^e)^2]m(1-e(\overline{w}^e))e'(\overline{w}^e)\end{aligned}}{[r+q+m(1-e(\overline{w}^e))^2]^2}.$$

If $N$ is defined as

$$\begin{aligned} N = {} & (2\eta-\alpha)me(\overline{w}^e)^2 - 2[\eta(r+q+m)+m(w-\overline{w}+A)]e(\overline{w}^e) \\ & +\alpha(r+q+m)+2m(w-\overline{w}+A), \end{aligned}$$

then

$$\frac{dV}{d\overline{w}^e} = (1+r)\frac{N}{[r+q+m(1-e(\overline{w}^e))^2]^2}\frac{de}{d\overline{w}^e}.$$

From (3), $N$=0 when $\overline{w}^e = \overline{w}$ . Thus,

(7) $$\left.\frac{dV}{d\overline{w}^e}\right|_{\overline{w}^e=\overline{w}} = 0 .$$

The second derivative of $V$ with respect to $\overline{w}^e$ is

$$\frac{d^2V}{d(\overline{w}^e)^2} = \frac{(1+r)N}{[r+q+m(1-e(\overline{w}^e))^2]^2}\frac{d^2e}{d(\overline{w}^e)^2} + (1+r)\left[\frac{[r+q+m(1-e(\overline{w}^e))^2]^2\dfrac{dN}{d\overline{w}^e}}{[r+q+m(1-e(\overline{w}^e))^2]^4}\right.$$

$$\left. + \frac{4N[r+q+m(1-e(\overline{w}^e))^2]m(1-e(\overline{w}^e))\dfrac{de}{d\overline{w}^e}}{[r+q+m(1-e(\overline{W}^E))^2]^4}\right]\frac{de}{d\overline{w}^e} .$$

Since $N$=0 when $\overline{w}^e = \overline{w}$ , the second derivative at this point is

(8) $$\left.\frac{d^2V}{d(\overline{w}^e)^2}\right|_{\overline{w}^e=\overline{w}} = (1+r)\frac{\dfrac{dN}{d\overline{w}^e}}{[r+q+m(1-e(\overline{w}^e))^2]^2}\frac{de}{d\overline{w}^e}$$

$$= 2(1+r)\frac{\begin{array}{c}(2\eta-\alpha)me(\overline{w})\\ -[\eta(r+q+m)+m(w-\overline{w}+A)]\end{array}}{[r+q+m(1-e(\overline{w}))^2]^2}\left(\frac{de}{d\overline{w}^e}\right)^2 .$$

From equation (4),

$$(2\eta-\alpha)me(\overline{w}) = X - \sqrt{X^2 - 2m(2\eta-\alpha)X + m(r+q+m)(2\eta-\alpha)^2} ,$$

where $X = \eta(r+q+m) + m(w-\overline{w}+A)$ .

Thus, (8) can be expressed as

$$(9)\quad \left.\frac{d^2V}{d(\overline{w}^e)^2}\right|_{\overline{w}^e=\overline{w}} = -2(1+r)\frac{\sqrt{X^2-2m(2\eta-\alpha)X+m(r+q+m)(2\eta-\alpha)^2}}{[r+q+m(1-e(\overline{w}))^2]^2}\left(\frac{de}{d\overline{w}^e}\right)^2 < 0.$$[3]

An approximate solution for a worker's utility can be obtained by substituting (7) and (9) into (6). The utility loss from incorrect expectations about the average wage equals the difference between $V(w, \overline{w}, e(\overline{w}))$ and the expression for $V(w, \overline{w}, e(\overline{w}^e))$ that is obtained from (6). Thus, the utility loss is given by the equation,

$$VL(\overline{w}^e-\overline{w})$$

$$= (1+r)\frac{\sqrt{X^2-2m(2\eta-\alpha)X+m(r+q+m)(2\eta-\alpha)^2}}{[r+q+m(1-e(\overline{w}))^2]^2}\left(\frac{de}{d\overline{w}^e}\right)^2(\overline{w}^e-\overline{w})^2.$$

[3] It is obvious that (9) must be negative if there is a real value solution for *e*. In addition, it can be demonstrated that the expression inside the square root is unambiguously positive, although the derivation is fairly complex and is not reported here.