Bargaining model

(1) $$\Pi_t = \theta_t \left[A_t L_t^{\phi} K_t^{1-\phi} e[W_t / \Omega_t , u_t]^{\phi} \right]^{\frac{\gamma-1}{\gamma}} - W_t L_t - rK_t .$$

Setting the derivative with respect to *K* equal to 0 yields:

$$\frac{d\Pi_t}{dK_t} = 0 = \frac{(1-\phi)(\gamma-1)}{\gamma} \theta_t A_t^{\frac{\gamma-1}{\gamma}} L_t^{\frac{\phi(\gamma-1)}{\gamma}} K_t^{\frac{(1-\phi)(\gamma-1)}{\gamma}-1} e[W_t / \Omega_t , u_t]^{\frac{\phi(\gamma-1)}{\gamma}} - r .$$

The solution for *K* is

(2) $$K_t = r^{\frac{\gamma}{\phi-\phi\gamma-1}} \left(\frac{\gamma}{(1-\phi)(\gamma-1)} \right)^{\frac{\gamma}{\phi-\phi\gamma-1}} \theta_t^{-\frac{\gamma}{\phi-\phi\gamma-1}} A_t^{-\frac{\gamma-1}{\phi-\phi\gamma-1}} L_t^{-\frac{\phi(\gamma-1)}{\phi-\phi\gamma-1}} e[W_t / \Omega_t , u_t]^{-\frac{\phi(\gamma-1)}{\phi-\phi\gamma-1}} .$$

Substituting (2) into (1) yields an expression for profits as a function of *L* and *W*:

$$\begin{aligned} \Pi_t = {} & \theta_t^{-\frac{\gamma}{\phi-\phi\gamma-1}} A_t^{-\frac{\gamma-1}{\phi-\phi\gamma-1}} L_t^{-\frac{\phi(\gamma-1)}{\phi-\phi\gamma-1}} r^{\frac{(1-\phi)(\gamma-1)}{\phi-\phi\gamma-1}} \left(\frac{\gamma}{(1-\phi)(\gamma-1)} \right)^{\frac{(1-\phi)(\gamma-1)}{\phi-\phi\gamma-1}} e[W_t / \Omega_t , u_t]^{-\frac{\phi(\gamma-1)}{\phi-\phi\gamma-1}} \\ & - W_t L_t - \theta_t^{-\frac{\gamma}{\phi-\phi\gamma-1}} A_t^{-\frac{\gamma-1}{\phi-\phi\gamma-1}} L_t^{-\frac{\phi(\gamma-1)}{\phi-\phi\gamma-1}} r^{\frac{(1-\phi)(\gamma-1)}{\phi-\phi\gamma-1}} \left(\frac{\gamma}{(1-\phi)(\gamma-1)} \right)^{\frac{\gamma}{\phi-\phi\gamma-1}} e[W_t / \Omega_t , u_t]^{-\frac{\phi(\gamma-1)}{\phi-\phi\gamma-1}} \end{aligned}$$

which can be expressed as

(3) $$\begin{aligned} \Pi_t = {} & \theta_t^{-\frac{\gamma}{\phi-\phi\gamma-1}} A_t^{-\frac{\gamma-1}{\phi-\phi\gamma-1}} L_t^{-\frac{\phi(\gamma-1)}{\phi-\phi\gamma-1}} r^{\frac{(1-\phi)(\gamma-1)}{\phi-\phi\gamma-1}} e[W_t / \Omega_t , u_t]^{-\frac{\phi(\gamma-1)}{\phi-\phi\gamma-1}} \left(\frac{\gamma}{(1-\phi)(\gamma-1)} \right)^{\frac{\gamma}{\phi-\phi\gamma-1}} \frac{1+\phi\gamma-\phi}{(1-\phi)(\gamma-1)} \\ & - W_t L_t \end{aligned}$$

Setting the derivative of (3) with respect to L equal to 0 yields:

$$\frac{d\Pi_t}{dL_t} = 0 = -\frac{\phi(\gamma-1)}{\phi-\phi\gamma-1}\theta_t^{-\frac{\gamma}{\phi-\phi\gamma-1}} A_t^{-\frac{\gamma-1}{\phi-\phi\gamma-1}} L_t^{-\frac{\phi(\gamma-1)}{\phi-\phi\gamma-1}-1} r^{\frac{(1-\phi)(\gamma-1)}{\phi-\phi\gamma-1}} e[W_t / \Omega_t, u_t]^{-\frac{\phi(\gamma-1)}{\phi-\phi\gamma-1}}$$
$$\times\left(\frac{\gamma}{(1-\phi)(\gamma-1)}\right)^{\frac{\gamma}{\phi-\phi\gamma-1}} \frac{1+\phi\gamma-\phi}{(1-\phi)(\gamma-1)} - W_t$$

The solution for L is

$$(4) \qquad L_t = \left(\frac{1-\phi}{\phi}\right)^{\phi-\phi\gamma-1}\left(\frac{\gamma}{(1-\phi)(\gamma-1)}\right)^{-\gamma} W_t^{\phi-\phi\gamma-1}\theta_t^{\gamma} A_t^{\gamma-1} r^{-(1-\phi)(\gamma-1)} e[W_t / \Omega_t, u_t]^{\phi(\gamma-1)}$$

Substituting (4) into (3) yields

$$\Pi_t = \theta_t^{\gamma} A_t^{\gamma-1}\left(\frac{1-\phi}{\phi}\right)^{-\phi(\gamma-1)}\left(\frac{\gamma}{(1-\phi)(\gamma-1)}\right)^{-\gamma}\frac{1+\phi\gamma-\phi}{(1-\phi)(\gamma-1)} W_t^{-\phi(\gamma-1)} r^{-(1-\phi)(\gamma-1)} e[W_t / \Omega_t, u_t]^{\phi(\gamma-1)}$$
$$-\theta_t^{\gamma} A_t^{\gamma-1}\left(\frac{1-\phi}{\phi}\right)^{\phi-\phi\gamma-1}\left(\frac{\gamma}{(1-\phi)(\gamma-1)}\right)^{-\gamma} W_t^{-\phi(\gamma-1)} r^{-(1-\phi)(\gamma-1)} e[W_t / \Omega_t, u_t]^{\phi(\gamma-1)}$$

which can be expressed as

$$(5) \quad \Pi_t = \theta_t^{\gamma} A_t^{\gamma-1} W_t^{-\phi(\gamma-1)} r^{-(1-\phi)(\gamma-1)} e[W_t / \Omega_t, u_t]^{\phi(\gamma-1)}\left(\frac{1-\phi}{\phi}\right)^{\phi-\phi\gamma-1}\left(\frac{\gamma}{(1-\phi)(\gamma-1)}\right)^{-\gamma}\frac{1}{\phi(\gamma-1)}$$

Suppose workers seek to maximize

$$U = W_t - \overline{U}_t, \qquad \text{where } \overline{U}_t = b\overline{W}_t.$$

Then

$$V = (W_t - b\overline{W_t})^{\beta}$$
$$\times\left(\theta_t^{\gamma} A_t^{\gamma-1} W_t^{-\phi(\gamma-1)} r^{-(1-\phi)(\gamma-1)} e[W_t / \Omega_t, u_t]^{\phi(\gamma-1)} \left(\frac{1-\phi}{\phi}\right)^{\phi-\phi\gamma-1} \left(\frac{\gamma}{(1-\phi)(\gamma-1)}\right)^{-\gamma} \frac{1}{\phi(\gamma-1)}\right)^{1-\beta}$$

Let

$$A = \theta_t^{\gamma} A_t^{\gamma-1} W_t^{-\phi(\gamma-1)} r^{-(1-\phi)(\gamma-1)} e[W_t / \Omega_t, u_t]^{\phi(\gamma-1)} \left(\frac{1-\phi}{\phi}\right)^{\phi-\phi\gamma-1} \left(\frac{\gamma}{(1-\phi)(\gamma-1)}\right)^{-\gamma} \frac{1}{\phi(\gamma-1)}.$$

The bargaining solution is

$$\frac{dV}{dW_t} = 0 = \beta(W_t - b\overline{W_t})^{\beta-1} A^{1-\beta} - (1-\beta)\phi(\gamma-1)(W_t - b\overline{W_t})^{\beta} A^{-\beta} \frac{A}{W_t}$$
$$+ (1-\beta)\phi(\gamma-1)(W_t - b\overline{W_t})^{\beta} A^{-\beta} \frac{A}{e[W_t / \Omega_t, u_t]} \frac{e_W}{\Omega_t}$$

$$0 = \beta - (1-\beta)\phi(\gamma-1)(W_t - b\overline{W_t}) \frac{1}{W_t} + (1-\beta)\phi(\gamma-1)(W_t - b\overline{W_t}) \frac{1}{e[W_t / \Omega_t, u_t]} \frac{e_W[W_t / \Omega_t, u_t]}{\Omega_t}$$

In equilibrium, $W_t = \overline{W_t}$. Therefore,

$$0 = \beta - (1-\beta)\phi(\gamma-1)(1-b) + (1-\beta)\phi(\gamma-1)(1-b)W_t \frac{1}{e[W_t / \Omega_t, u_t]} \frac{e_W[W_t / \Omega_t, u_t]}{\Omega_t}$$

$$\frac{e_W[W_t / \Omega_t, u_t]}{e[W_t / \Omega_t, u_t]} \frac{W_t}{\Omega_t} = \frac{(1-\beta)\phi(\gamma-1)(1-b) - \beta}{(1-\beta)\phi(\gamma-1)(1-b)} = 1 - \frac{\beta}{(1-\beta)\phi(\gamma-1)(1-b)} < 1$$