NORTHERN ILLINOIS UNIVERSITY

PHYSICS DEPARTMENT

Physics 580 – Solid State Physics Fall 2024

Problem Set #8

Problem Set Due: Thurs, Dec 5, 2024
Read Kittel: Chapters 6 & 7

Problem 1:

Density of States for 1-Dimensional and 3-Dimensional Systems

(a) Derive the density of states for a *1-dimensional system* using the relation between the density of states as a function of energy and wavenumber:

$$D(\varepsilon)d\varepsilon = D(k)dk \quad \text{(}\textit{note}\text{: this is for 1-dimension)}$$

Show that $D(\varepsilon) \sim \frac{1}{\sqrt{\varepsilon}}$ (*give the exact relation*). (hint: google "density of states", and read the Wikipedia section on **Parabolic dispersion**).

(b) What modification must be made to the relation, $D(\varepsilon)d\varepsilon = D(k)dk$, for a *3-dimensional system* to get the result in Kittel? (*hint*: k -space is now 3-dimensional).

Derive Kittel's result using your modified relationship for 3-dimensions (**Eq. 20, Chapter 6**):

$$D(\varepsilon) \equiv \frac{dN}{d\varepsilon} = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \varepsilon^{1/2}$$

Problems #2 & #3 on the next page

Problem 2:

Kinetic energy of electron gas:

Show that the kinetic energy of a three-dimensional gas of N free electrons at $0°K$ is

$$U_0 = \frac{3}{5} N \varepsilon_F$$

where ε_F is the Fermi energy.

Problem 3:

Free electron energies in reduced zone:

Consider the free electron energy bands of an fcc crystal lattice in the approximation of an empty lattice, but in the reduced zone scheme in which all $\vec{k}$'s are transformed to lie in the first Brillouin zone. Plot roughly in the [111] direction the energies of all bands up to six times the lowest band energy at the zone boundary at

$$\vec{k} = \frac{2\pi}{a}\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$$

Let this be the unit of energy. This problem shows why band edges need not necessarily be at the zone center. Several of the degeneracies (band crossings) will be removed when account is taken of the crystal potential.