NORTHERN ILLINOIS UNIVERSITY

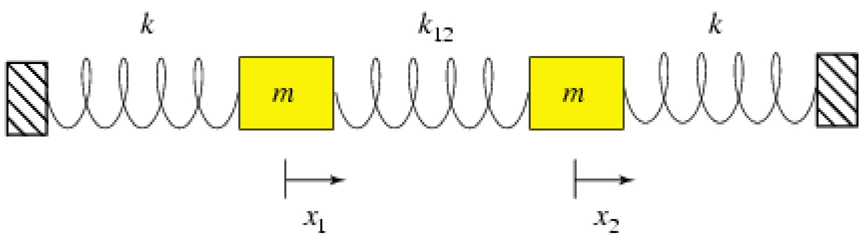
PHYSICS DEPARTMENT

Physics 580 – Solid State Physics Fall 2024

Problem Set #6

Problem Set Due: Thurs, Nov.14, 2024
Read Kittel: Chapter 4

Problem 1 (review of coupled harmonic oscillators)



A pair of objects are connected by springs to immovable walls. The spring constants are k and k_{12}.

(a) Write down the 2nd order differential equations for x_1 and x_2.
Note: the solutions are on the Web:
https://scienceworld.wolfram.com/physics/SpringsThreeSpringsandTwoMasses.html
https://physics.stackexchange.com/questions/689998/equations-of-motion-of-two-bodies-attached-to-three-springs

(b) Assume the objects undergo simple harmonic motion:

$$\ddot{x}_1 = -\omega^2 x_1 \quad \text{and} \quad \ddot{x}_2 = -\omega^2 x_2$$

Insert these expressions into your equations in Part (a) and find the two possible values of the oscillation frequency, ω.

(c) What is the solution for $x_1(t)$ and $x_2(t)$?

(d) What exactly is Kittel solving for with equations (1) to (4) on pages 91 & 92?

Problem 2

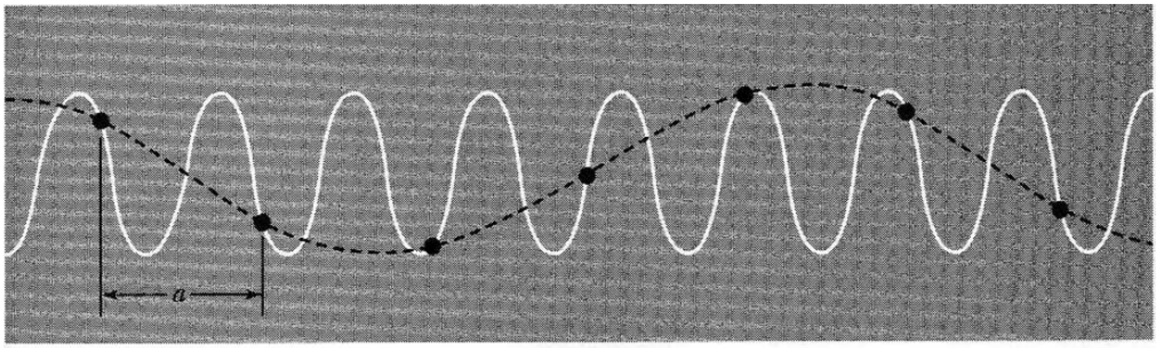
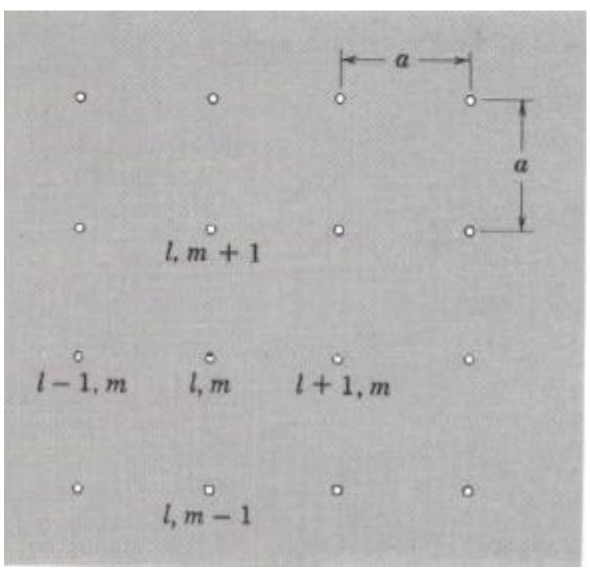


Figure 5 The wave represented by the solid curve conveys no information not given by the dashed curve. Only wavelengths longer than $2a$ are needed to represent the motion.

Explain the significance of this figure from Kittel. That is, why does it mean that for all values of K, we only need to consider only values of K' that lie within the 1st Brillioun zone? Also explain why there are only standing waves at the Brillioun zone boundary.

Problem 3



Square array of lattice constant a. The displacements considered are normal to the plane of the lattice.

Vibrations of a square lattice. We consider transverse vibrations of a planar square lattice of rows and columns of identical atoms, and let $u_{l,m}$ denote the displacement normal to the plane of the lattice of the atom in the l^{th} column and m^{th} row (see figure above). The mass of each atom is M, and C is the force constant for nearest neighbor atoms.

(a) Show that the equation of motion is

$$M\frac{d^2u_{lm}}{dt^2} = C[(u_{l+1,m} + u_{l-1,m} - 2u_{lm}) + (u_{l,m+1} + u_{l,m-1} - 2u_{lm})]$$

(b) Assume solutions of the form

$$u_{lm} = u(0)\exp[i(lK_x a + mK_y a - \omega t)]$$

where a is the spacing between nearest-neighbor atoms. Show that the equation of motion is satisfied if

$$\omega^2 M = 2C(2 - \cos K_x a - \cos K_y a)$$

This is the dispersion relation for the problem.

(c) Show that the region of $\vec{\boldsymbol{K}}$ space for which independent solutions exist may be taken as a square of side $2\pi/a$. This is the first Brillouin zone of the square lattice.

(d) Sketch ω verses K for $K = K_x$ with $K_y = 0$.

(e) Sketch ω verses K for $K_x = K_y$.

(f) For $Ka << 1$, show that

$$\omega = (Ca^2 / M)^{\frac{1}{2}}(K_x^2 + K_y^2)^{\frac{1}{2}} = (Ca^2 / M)^{\frac{1}{2}} K$$

so that in this limit the velocity is constant.