NORTHERN ILLINOIS UNIVERSITY

PHYSICS DEPARTMENT

Physics 580 – Solid State Physics Fall 2022

Problem Set #7

Problem Set Due: Tues, Nov.22, 2022
Read Kittel: Chapter 5, 6

Problem 1: Singularity in density of states (Chapter 5)

(a) From the dispersion relation derived in Chapter 4 for a monatomic linear lattice of N atoms with nearest-neighbor interactions, show that the density of modes is

$$D(\omega) = \frac{2N}{\pi} \cdot \frac{1}{(\omega_m^2 - \omega^2)^{\frac{1}{2}}}$$

where ω_m is the maximum frequency.

(b) Make a plot of $D(\omega)$ for Part (a).

(c) Suppose that an optical phonon branch has the form $\omega(K) = \omega_0 - AK^2$, near $K = 0$ in three dimensions. Show that

i. $D(\omega) = (L / 2\pi)^3 (2\pi / A^{\frac{3}{2}})(\omega_0 - \omega)^{\frac{1}{2}}$ for $\omega < \omega_0$

ii. $D(\omega) = 0$ for $\omega > \omega_0$

Here the density of modes is discontinuous.

(d) Make a plot of $D(\omega)$ for Part (c).

Problems #2 & #3 on the next page

Problem 2 **(Chapter 6)**

Kinetic energy of electron gas:

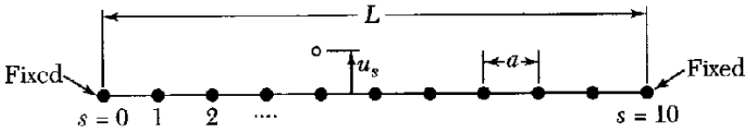
Show that the kinetic energy of a three-dimensional gas of N free electrons at $0°K$ is

$$U_0 = \frac{3}{5} N \varepsilon_F$$

where ε_F is the Fermi energy.

Problem 3 **(Chapter 5)**

For the problem outlined by Kittel on page 109 (8th ed) for the line of atoms fixed on both ends:



explain why the allowed values of K are: $K = \frac{\pi}{L}, \quad \frac{2\pi}{L}, \quad \frac{3\pi}{L}, \ldots,$

Wheras for periodic boundary conditions, the allowed values of K are:

$$K = 0, \quad \pm\frac{2\pi}{L}, \quad \pm\frac{4\pi}{L}, \quad \pm\frac{6\pi}{L}, \ldots,$$

Explain why K = 0 and negative values are not solutions for the fixed boundary condition problem.