

NORTHERN ILLINOIS UNIVERSITY

PHYSICS DEPARTMENT

Physics 580 – Solid State Physics

Fall 2022

Problem Set #6

Problem Set Due: Thurs, Nov.10, 2022

Read Kittel: Chapter 4

Problem 2

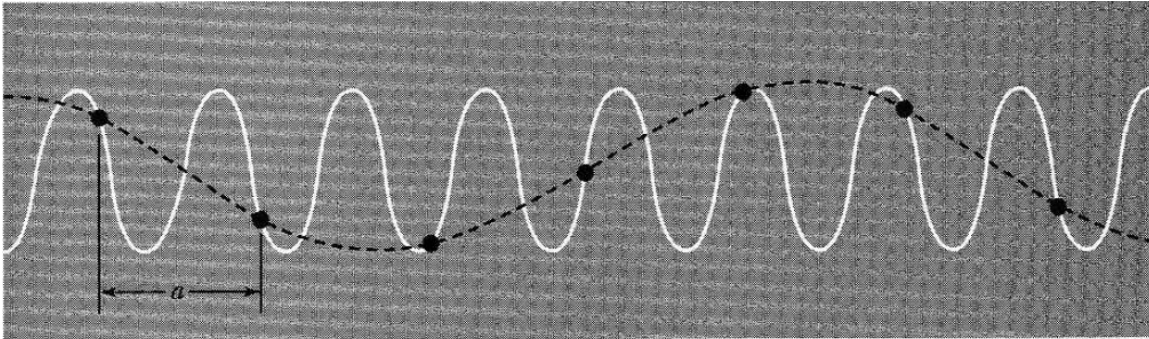
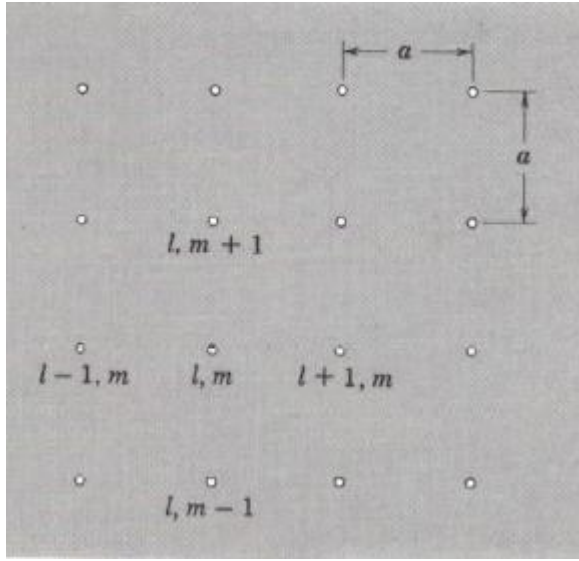


Figure 5 The wave represented by the solid curve conveys no information not given by the dashed curve. Only wavelengths longer than $2a$ are needed to represent the motion.

Explain the significance of this figure from Kittel. That is, why does it mean that for all values of K , we only need to consider only values of K' that lie within the 1st Brillouin zone? Also explain why there are only standing waves at the Brillouin zone boundary.

Problem 2



Square array of lattice constant a .
The displacements considered are normal to the plane of the lattice.

Vibrations of a square lattice. We consider transverse vibrations of a planar square lattice of rows and columns of identical atoms, and let $u_{l,m}$ denote the displacement normal to the plane of the lattice of the atom in the l^{th} column and m^{th} row (see figure above). The mass of each atom is M , and C is the force constant for nearest neighbor atoms.

- (a) Show that the equation of motion is

$$M \frac{d^2 u_{lm}}{dt^2} = C[(u_{l+1,m} + u_{l-1,m} - 2u_{lm}) + (u_{l,m+1} + u_{l,m-1} - 2u_{lm})]$$

- (b) Assume solutions of the form

$$u_{lm} = u(0) \exp[i(lK_x a + mK_y a - \omega t)]$$

where a is the spacing between nearest-neighbor atoms. Show that the equation of motion is satisfied if

$$\omega^2 M = 2C(2 - \cos K_x a - \cos K_y a)$$

This is the dispersion relation for the problem.

- (c) Show that the region of $\vec{K}$ space for which independent solutions exist may be taken as a square of side $2\pi/a$. This is the first Brillouin zone of the square lattice.
- (d) Sketch ω versus K for $K = K_x$ with $K_y = 0$.
- (e) Sketch ω versus K for $K_x = K_y$.
- (f) For $Ka \ll 1$, show that

$$\omega = (Ca^2 / M)^{\frac{1}{2}} (K_x^2 + K_y^2)^{\frac{1}{2}} = (Ca^2 / M)^{\frac{1}{2}} K$$

so that in this limit the velocity is constant.