NORTHERN ILLINOIS UNIVERSITY

PHYSICS DEPARTMENT

Physics 580 – Solid State Physics Fall 2022

Problem Set #5

Problem Set Due: Thurs, Oct. 27, 2022 (have Problem #1 done by Tues)
Read Kittel: Chapter 3

1. Go through Equations (1) through (9) in Kittel Chapter 3 (pages 55-56 in 8th ed; pages 78-79 5th ed). In particular,

   (a) derive Eq. (3)

   (b) make the substitutions in Eq. (5) and derive Eq. (7)

   (c) and derive Eq. (9)

2. Using the Lennard-Jones potential, derive Eq. (14) in Kittel.

3. Kittel: **Problem 3-1**

   1. ***Quantum solid.*** In a quantum solid the dominant repulsive energy is the zero-point energy of the atoms. Consider a crude one-dimensional model of crystalline He^4 with each He atom confined to a line segment of length L. In the ground state the wave function within each segment is taken as a half wavelength of a free particle. (a) Find the zero-point kinetic energy per particle. (b) Find an expression for the force needed to keep the line from expanding. (c) In equilibrium the expansive tendency of the kinetic energy is balanced by the van der Waals (vdW) interaction. The vdW energy between nearest neighbors is given very roughly by $U(L) = -1.6\, L^{-6}\, 10^{-60}$ erg, where L is in cm. Find the equilibrium value of L.

4. Kittel: **Problem 3-2**

   2. ***Cohesive energy of bcc and fcc neon.*** Using the Lennard-Jones potential, calculate the ratio of the cohesive energies of neon in the bcc and fcc structures. The lattice sums for the bcc structures are:

$$\sum_j{}' p_{ij}^{-12} = 9.11418 \ ; \qquad \sum_j{}' p_{ij}^{-6} = 12.2533 \ .$$

5. Kittel: **Problem 3-5**

5. ***Linear ionic crystal.*** Consider a line of $2N$ ions of alternating charge $\pm q$ with a repulsive potential energy A/R^n between nearest neighbors. (a) Show that at the equilibrium separation

$$\text{(CGS)} \qquad U(R_0) = -\frac{2Nq^2 \ln 2}{R_0}\left(1 - \frac{1}{n}\right) .$$

(b) Let the crystal be compressed so that $R_0 \to R_0(1 - \delta)$. Show that the work done in compressing a unit length of the crystal has the leading term $\frac{1}{2}C\delta^2$, where

$$\text{(CGS)} \qquad C = \frac{(n-1)q^2 \ln 2}{R_0} .$$

To obtain the results in SI, replace q^2 by $q^2/4\pi\epsilon_0$. Note: We should not expect to obtain this result from the expression for $U(R_0)$, but we must use the complete expression for $U(R)$.