NORTHERN ILLINOIS UNIVERSITY

PHYSICS DEPARTMENT

Physics 580 – Solid State Physics Fall 2022

Final Exam --- Take Home

Problem Set Due: Sun, Dec 11, 2022 @ 8pm
Read Kittel: Chapters 6 & 7

Problem 1: Free electron energies in reduced zone:

Consider the free electron energy bands of an fcc crystal lattice in the approximation of an empty lattice, but in the reduced zone scheme in which all $\vec{k}$'s are transformed to lie in the first Brillouin zone. Plot roughly in the [111] direction the energies of all bands up to six times the lowest band energy at the zone boundary at

$$\vec{k} = \frac{2\pi}{a}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$$

Let this be the unit of energy. This problem shows why band edges need not necessarily be at the zone center. Several of the degeneracies (band crossings) will be removed when account is taken of the crystal potential.

Problem #2 is on the next page

Post to Blackboard scanned images of your solutions (using either your cell phone or the university printers). Make certain that it is a single *.pdf file (do not upload *.jpegs --- you can load jpegs into Word and save as a single pdf file)

Problem 2: Density of States for 1-Dimensional and 3-Dimensional Systems

In class we ran into the problem of deriving the density of states given in Kittel (Eq. 20 in chapter 6): (*note: this is for a 3-dimensional system*)

$$D(\varepsilon) \equiv \frac{dN}{d\varepsilon} = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \varepsilon^{1/2}$$

(a) Using the following relation between the density of states as a function of energy and wavenumber:

$$D(\varepsilon)d\varepsilon = D(k)dk$$

Show that $D(\varepsilon) \sim \dfrac{1}{\sqrt{\varepsilon}}$ (give the exact relation). Note that the relation above is for a *1-dimensional system* (google "density of states", and read the Wikipedia section on **Parabolic dispersion**).

(b) What modification must be made to the relation, $D(\varepsilon)d\varepsilon = D(k)dk$, for a *3-dimensional system* to get the result in Kittel? (*hint*: k-space is now 3-dimensional).

(c) Derive Kittel's equation using your modified relationship for 3-dimensions.

Problem #3 is on the next page

Problem 3: Specific heat of a two-dimensional solid.

In **Problem 2** above, we examined 1-D and 3-D systems. Now consider a two-dimensional system where there is one atom per primitive unit cell. The motion of the atoms is restricted to the direction perpendicular to the plane of the solid.

(a) Find the density of states as a function of k, $D(k)$, for this two-dimensional solid.

(b) Find the density of states as a function of energy, $D(\varepsilon)$, for this model. Draw a labelled picture of this density of states.

(c) Derive expressions for the Debye frequency ω_D and temperature θ_D in terms of the number of atoms per unit area N/A, the sound velocity v_s, and Boltzmann's constant k.

(d) Find an expression for U, the thermal energy of the phonon gas. Leave your answer in terms of a dimensionless but temperature dependent integral: $\int_a^b \frac{x^2 dx}{e^x - 1}$ and give the limits of the integral.

(e) What are the limiting values of the specific heat C for low T and high T?

Problem 4:

Consider a gas of free non-interacting electrons in three dimensions. Find, in terms of the Fermi momentum k_F:

(a) The mean velocity $<\vec{\boldsymbol{v}}>$ of the electrons at $T = 0$. (*note:* velocity is a vector)

(b) The mean speed $<|\vec{\boldsymbol{v}}|>$ of the electrons at $T = 0$.

(c) The root-mean-square speed $\sqrt{<|\vec{\boldsymbol{v}}|^2>}$ at $T = 0$

Hint: you did a very similar problem to this in class on the board where you solved for the kinetic energy in Problem 2 of Problem Set #7. For this problem express all your answers in terms of the Fermi momentum k_F.