NORTHERN ILLINOIS UNIVERSITY

PHYSICS DEPARTMENT

Physics 580 – Solid State Physics Fall 2020

Problem Set #6

Singularity in density of states.

(a) From the dispersion relation derived in Chapter 4 for a monatomic linear lattice of N atoms with nearest-neighbor interactions, show that the density of modes is

$$D(\omega) = \frac{2N}{\pi} \cdot \frac{1}{(\omega_m^2 - \omega^2)^{\frac{1}{2}}}$$

where ω_m is the maximum frequency.

(b) Make a plot of $D(\omega)$ for Part (a).

(c) Suppose that an optical phonon branch has the form $\omega(K) = \omega_0 - AK^2$, near $K = 0$ in three dimensions. Show that

i. $D(\omega) = (L / 2\pi)^3 (2\pi / A^{\frac{3}{2}})(\omega_0 - \omega)^{\frac{1}{2}}$ for $\omega < \omega_0$

ii. $D(\omega) = 0$ for $\omega > \omega_0$

Here the density of modes is discontinuous.

(d) Make a plot of $D(\omega)$ for Part (b).