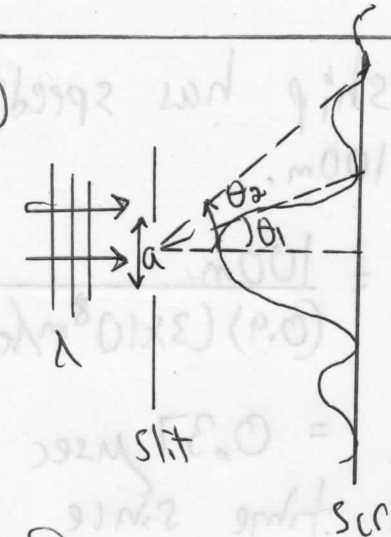
Physics 283

Quiz # 1 Solutions

(1) 

Diffraction condition for a minimum (destructive interference):

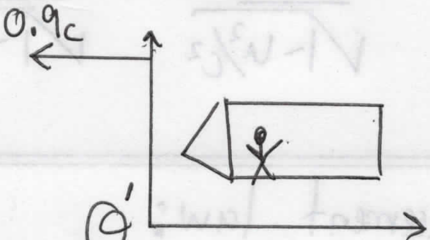
$$a \sin\theta = m\lambda \quad , \; m = \pm 1, \pm 2, \pm 3, \ldots$$

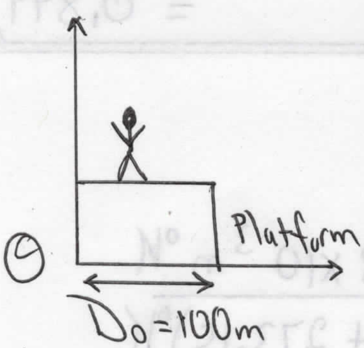
$$a = \frac{m\lambda}{\sin\theta} = \frac{2(600\,nm)}{\sin 45^\circ} = 1697\,nm$$

$$= 1.697\,\mu m$$

$$= 1.697 \times 10^{-6}\,m$$

For 2^{nd} diffraction minimum: $m = 2$

(2) 



(a) Observer O sees spaceship & platform having length $D_0 = 100m$. The spaceship will be seen by O to have a contracted length since it is moving: $L = L_0\sqrt{1 - u^2/c^2}$

The proper length of the spaceship (as seen by O') will then be

$$L_0 = \frac{L}{\sqrt{1 - u^2/c^2}} = \frac{100m}{\sqrt{1 - (0.9c)^2/c^2}} = 229.4m$$

(b) According to O', the platform is moving and will have a contracted length

$$L = L_0\sqrt{1 - u^2/c^2} = 100m\sqrt{1 - (0.9c)^2/c^2} = 43.6\,m$$

(c) According to observer Θ, the rocket ship has speed $u = 0.9c$ and covers a distance $D_0 = 100m$.

Thus $v = \frac{d}{t} \Rightarrow \Delta t_0 = \frac{D_0}{u} = \frac{100m}{0.9c} = \frac{100m}{(0.9)(3\times10^8 m/sec)}$

$= 3.704\times10^{-7}$ sec $= 0.37 \mu sec$

(d) Observer Θ' will measure a dilated time since observer Θ clock is moving:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}} = \frac{0.37 \mu sec}{\sqrt{1-(0.9c)^2/c^2}} = 8.49\times10^{-7} sec$$

$$= 0.849 \mu sec$$

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(3) Using Wein's displacement law:

(a) $$\lambda_{max} T = 2.8978\times10^{-3} m\,°K$$

$$\lambda = \frac{2.8978\times10^{-3} m\,°K}{T} = \frac{2.8978\times10^{-3} m\,°K}{(25+273.15)°K}$$

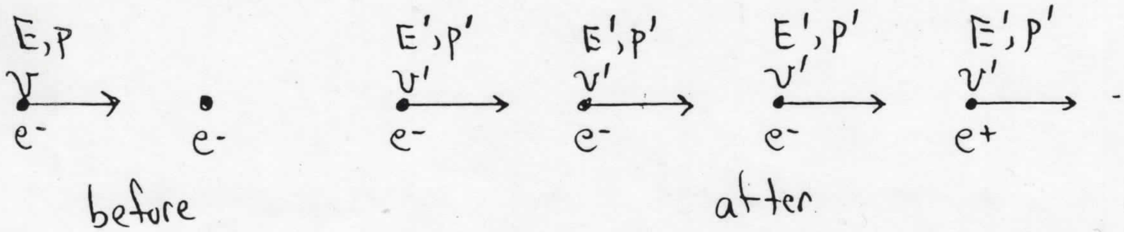
$$= 9.719 \mu m$$

(b) $$T = \frac{2.8978\times10^{-3} m\,°K}{600\times10^{-9} m} = 4830\,°K$$

(c) $$\frac{I_2}{I_1} = \frac{\sigma T_2^4}{\sigma T_1^4} = \frac{T_2^4}{T_1^4} = \frac{(4829.6667)^4}{(25+273.15)^4} = 68854$$

The higher temperature object emits 69,000 times as much thermal radiation compared to the lower temperature object.

(4)



Since all outgoing particles have the same velocity, then they have the same momentum p' and energy E'

Conservation of energy : $E_i = E_f$

$$\underbrace{E}_{\text{energy of moving electron}} + \underbrace{mc^2}_{\text{energy of stationary electron}} = E' + E' + E' + E'$$

Since the problem wants the kinetic energy of the incident electron, we will solve for E first, and then use the relation $E = K + mc^2$ to solve for K. To do this, we also need the momentum equations:

Conservation of momentum : $P_i = P_f$

$$P = p' + p' + p' + p' = 4p'$$

From relation $E^2 = p^2c^2 + m^2c^4$ we have $pc = \sqrt{E^2 - m^2c^4}$

The momentum equations can be expressed in terms of E, E' as follows:

$$P = 4p'$$

$$pc = 4p'c$$

$$p^2c^2 = 16p'^2c^2$$

$$E^2 - m^2c^4 = 16(E'^2 - m^2c^4)$$

Then,

$$\begin{cases} E^2 - m^2c^4 + 16m^2c^4 = 16E'^2 & \leftarrow \text{from momentum equations} \\ (E + mc^2)^2 = 16E'^2 & \end{cases}$$

From energy equation $\rightarrow$ $(E + mc^2)^2 = 16E'^2$

Squaring out energy equations gives:

$$\text{subtract}\quad \left\{\begin{aligned} E^2 + 15m^2c^4 &= 16E'^2 \\ E^2 + 2Emc^2 + m^2c^4 &= 16E'^2 \end{aligned}\right\} \quad \begin{aligned} &\leftarrow \text{squared momentum equations} \\ &\leftarrow \text{squared energy equations} \end{aligned}$$

$$2Emc^2 - 14m^2c^4 = 0$$

$$E = 7mc^2$$

But, $E = K + mc^2$, so

$$K + mc^2 = 7mc^2$$

$$K = 6mc^2 \quad = 6(0.511\,\text{MeV}) \approx 3.07\,\text{MeV}$$