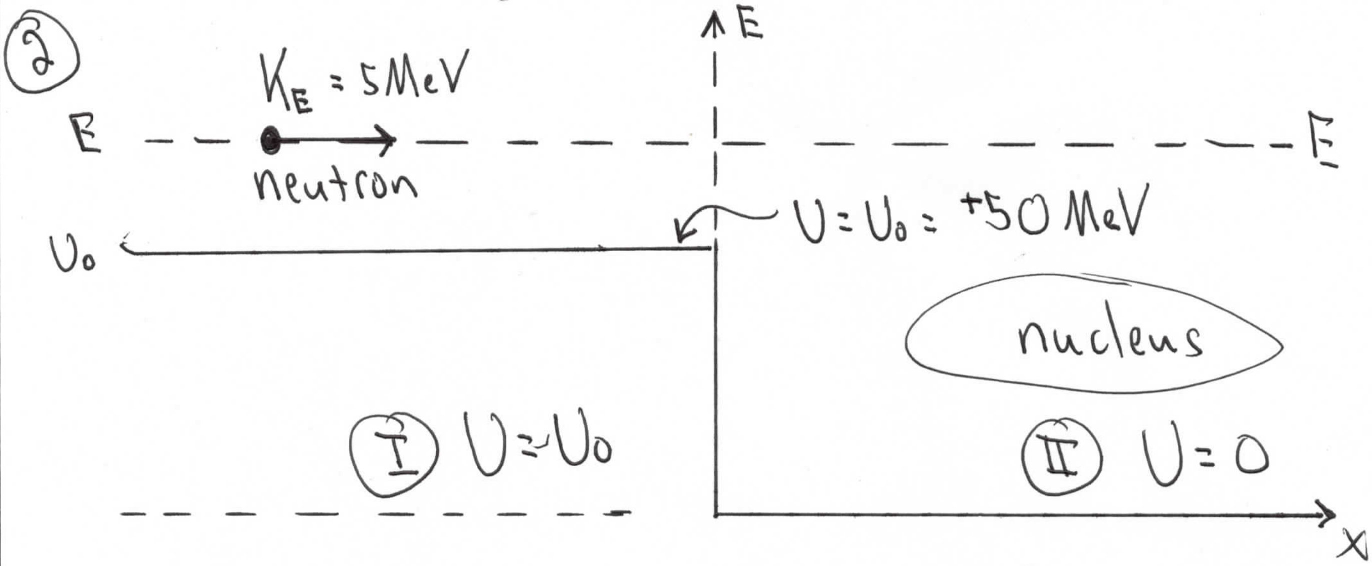
Physics 283

Quiz # 3 Solutions

(2)



(a) $\Psi_I(x) = \underrightarrow{Ae^{iK_I x}} + \underleftarrow{Be^{-iK_I x}}$

wave travelling to the right

reflected wave traveling to the left

(b) $\Psi_{II}(x) = Ce^{iK_{II}x} + De^{-iK_{II}x}$

(c) For Region (I): $-\frac{\hbar^2}{2m}\frac{d^2\Psi_I}{dx^2} + \overset{U_0}{U(x)}\Psi_I = E\,\Psi_I$

$$\frac{d^2\Psi_I}{dx^2} = -\frac{2m}{\hbar^2}(E - U_0)\Psi_I$$

$$\frac{d^2\Psi_I}{dx^2} = -K_I^2\,\Psi_I$$

$$\boxed{K_I = \sqrt{\frac{2m}{\hbar^2}(E - U_0)}}$$

For Region (II): $-\frac{\hbar^2}{2m}\frac{d^2\Psi_{II}}{dx^2} + \cancelto{0}{U(x)}\Psi_{II} = E\,\Psi_{II}$

$U(x) = 0$

$$\frac{d^2\psi_{II}}{dx^2} = \frac{-2mE}{\hbar^2}\psi_{II}$$

$$= -K_{II}^2\psi_{II}$$

$$\boxed{K_{II} = \sqrt{\frac{2mE}{\hbar^2}}}$$

(d) Coefficent D in Region II is zero

$D = 0$ since there is no boundary to reflect from in region II to produce a reflected wave

(e) $\psi_I(x=0) = \psi_{II}(x=0)$

$Ae^0 + Be^0 = Ce^0 \Rightarrow \boxed{A + B = C}$

$$\left.\frac{d\psi_I}{dx}\right]_{x=0} = \left.\frac{d\psi_{II}}{dx}\right]_{x=0}$$

$iK_I Ae^0 - iK_I Be^0 = iK_{II}Ce^0 \Rightarrow \boxed{K_I A - K_I B = K_{II} C}$

(f) Inserting $C = A + B$ into the 2nd relation gives

$$K_I A - K_I B = K_{II}(A + B)$$

$$(K_I - K_{II})A = (K_{II} + K_I)B$$

$$\Rightarrow \frac{B}{A} = \frac{K_I - K_{II}}{K_I + K_{II}}$$

$$R = \frac{K_I |B|^2}{K_I |A|^2} = \left(\frac{K_I - K_{II}}{K_I + K_{II}}\right)^2$$

(g) $E = K_{II} + U = 5\,\text{MeV} + 50\,\text{MeV} = 55\,\text{MeV}$

$$R = \left(\frac{\sqrt{\frac{2m}{\hbar^2}(E-U_0)} - \sqrt{\frac{2mE}{\hbar^2}}}{\sqrt{\frac{2m}{\hbar^2}(E-U_0)} + \sqrt{\frac{2mE}{\hbar^2}}}\right)^2 = \left[\frac{\sqrt{\frac{2m}{\hbar^2}}}{\sqrt{\frac{2m}{\hbar^2}}}\left(\frac{\sqrt{E-U_0} - \sqrt{E}}{\sqrt{E-U_0} + \sqrt{E}}\right)\right]^2$$

$$= \frac{\sqrt{E-U_0} - \sqrt{E}}{\sqrt{E-U_0} + \sqrt{E}} = \left(\frac{\sqrt{(55-50)\,\text{MeV}} - \sqrt{55\,\text{MeV}}}{\sqrt{(55-50)\,\text{MeV}} + \sqrt{55\,\text{MeV}}}\right)^2$$

$= 0.288 \approx 29\%$

Thus, according to this simple theory, a sizeable fraction of the neutrons are reflected.

$\Rightarrow$ 29% of incident neutrons do not contribute to nuclear fission

(3) Carbon: $[He]2s^2 2p^2$

(a) For the two 2p electrons ($l = 1$)

$e^- \# 1$: $l_1 = 1$, $s_1 = 1/2$

$e^- \# 2$: $l_2 = 1$, $s_2 = 1/2$

(b) $\vec{L} = \vec{L}_1 + \vec{L}_2 \Rightarrow L = |l_1 + l_2|$ to $|l_1 - l_2| = |1+1|$ to $|1-1|$

$= 2, 1, 0$

$$\Rightarrow \boxed{L = 2, 1, 0}$$

$\vec{S} = \vec{S}_1 + \vec{S}_2 \Rightarrow S = |S_1 + S_2|$ to $|S_1 - S_2| = |1/2 + 1/2|$ to $|1/2 - 1/2|$

$= 1, 0$

$$\Rightarrow \boxed{S = 1, 0}$$

$\vec{J} = \vec{L} + \vec{S} \Rightarrow$

$J = |L+S|$ to $|L-S|$

$L = 2$: $S = 1 \Rightarrow |2+1|$ to $|2-1| = 3, 2, 1$; $S = 0 \Rightarrow |2+0|$ to $|2-0| = 2$

$L = 1$: $S = 1 \Rightarrow |1+1|$ to $|1-1| = 2, 1, 0$; $S = 0 \Rightarrow |1+0|$ to $|1-0| = 1$

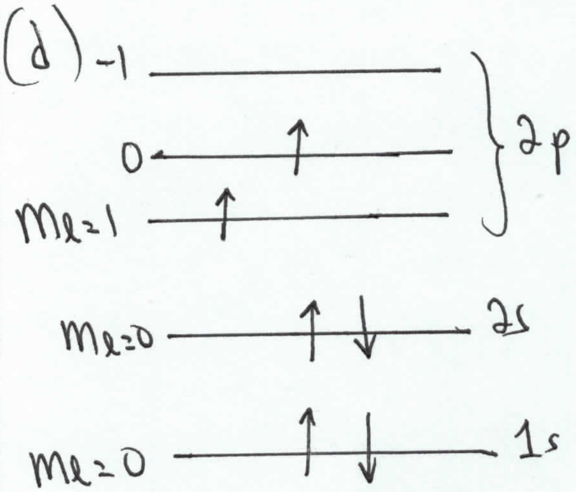
$L = 0$: $S = 1 \Rightarrow |0+1|$ to $|0-1| = 1$; $S = 0 \Rightarrow |0+0|$ to $|0-0| = 0$

$$\Rightarrow \boxed{J = 3, 2, 1, 0}$$

(c) (S, L, J) states: $(1,2,3)$ $(1,2,2)$ $(1,2,1)$
$(0,2,2)$, $(1,1,2)$, $(1,1,1)$, $(1,1,0)$, $(0,1,1)$, $(1,0,1)$
$(0,0,0)$

$\Rightarrow$ 10 possible states

(d)



$M_{L,max} = m_{l1} + m_{l2} = 1 + 0 = 1$
$\Rightarrow L = 1$
$M_{S,max} = m_{s1} + m_{s2} = \frac{1}{2} + \frac{1}{2} = 1$
$\Rightarrow S = 1$

$L = 0, S = 0$ for filled subshells

Since the 2p subshell is less than half filled,

$$J = |L - S| = 1 - 1 = 0$$

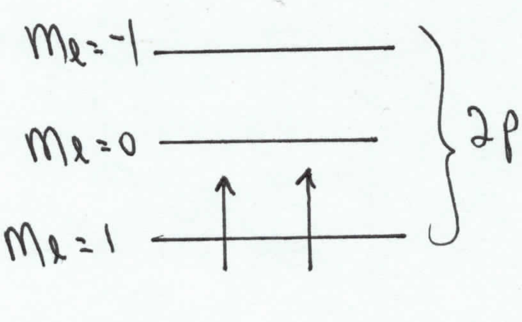
The ground state (S, L, J) is $(1, 1, 0)$

(e) Spectroscopic notation: $L = 1 = P$ state

multiplicity $= 2S + 1 = 2 \cdot 1 + 1 = 3$
$J = 0$

Spectroscopic state: $^3P_0$

(f) State with largest J: $(1, 2, 3) \rightarrow S = 1, L = 2$
$J = 3$



to have $L = 2, S = 1$, we must have $M_L = m_{l1} + m_{l2} = 2$, $M_S = m_{s1} + m_{s2} = 1$
The only way to have $M_S = 1$, $M_L = 2$ is to put 2 spin up electrons in the bottom level

$\Rightarrow$ this violates the Pauli Exclusion principle

Thus, the $(1,2,3)$, $(1,2,2)$, $(1,2,1)$ states are not allowed