NORTHERN ILLINOIS UNIVERSITY

PHYSICS DEPARTMENT

Physics 283 – Modern Physics Fall 2023

Quiz #3

INSTRUCTIONS:

(1) No graphic calculators are allowed for this quiz.
(2) Do all your work in the examination blue booklet
(3) You may find the following formulas useful.

Chapter Summary

		Section
Time-independent Schrödinger equation	$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+U(x)\psi(x)=E\psi(x)$	5.3
Time-dependent Schrödinger equation	$\Psi(x,t)=\psi(x)e^{-i\omega t}$	5.3
Probability density	$P(x)=\lvert\psi(x)\rvert^2$	5.3
Normalization condition	$\int_{-\infty}^{+\infty}\lvert\psi(x)\rvert^2\,dx=1$	5.3
Probability in interval x_1 to x_2	$P(x_1{:}x_2)=\int_{x_1}^{x_2}\lvert\psi(x)\rvert^2\,dx$	5.3
Average or expectation value of $f(x)$	$\left\langle f(x)\right\rangle=\int_{-\infty}^{+\infty}\lvert\psi(x)\rvert^2 f(x)\,dx$	5.3
Constant potential energy, $E>U_0$	$\psi(x)=A\sin kx+B\cos kx$, $k=\sqrt{2m(E-U_0)/\hbar^2}$	5.4
Constant potential energy, $E<U_0$	$\psi(x)=Ae^{k'x}+Be^{-k'x}$, $k'=\sqrt{2m(U_0-E)/\hbar^2}$	5.4
Infinite potential energy well	$\psi_n(x)=\sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$, $E_n=\frac{h^2n^2}{8mL^2}\ (n=1,2,3,\ldots)$	5.4
Two-dimensional infinite well	$\psi(x,y)=\frac{2}{L}\sin\frac{n_x\pi x}{L}\sin\frac{n_y\pi y}{L}$, $E=\frac{h^2}{8mL^2}(n_x^2+n_y^2)$	5.4
Simple harmonic oscillator ground state	$\psi(x)=(m\omega_0/\hbar\pi)^{1/4}\,e^{-(\sqrt{km}/2\hbar)x^2}$	5.5
Simple harmonic oscillator energies	$E_n=(n+\frac{1}{2})\hbar\omega_0\ (n=0,1,2,\ldots)$	5.5
Potential energy step, $E>U_0$	$\psi_0(x<0)=A\sin k_0x+B\cos k_0x$, $\psi_1(x>0)=C\sin k_1x+D\cos k_1x$	5.6
Potential energy step, $E<U_0$	$\psi_0(x<0)=A\sin k_0x+B\cos k_0x$, $\psi_1(x>0)=Ce^{k_1x}+De^{-k_1x}$	5.6

Hydrogen quantum numbers	$n=1,2,3,\ldots$; $l=0,1,2,\ldots,n-1$; $m_l=0,\pm1,\pm2,\ldots,\pm l$	7.3
Hydrogen energy levels	$E_n=-\frac{me^4}{32\pi^2\varepsilon_0^2\hbar^2}\frac{1}{n^2}$	7.3
Hydrogen wave functions	$\psi_{n,l,m_l}(r,\theta,\phi)=R_{n,l}(r)\Theta_{l,m_l}(\theta)\Phi_{m_l}(\phi)$	7.3
Radial probability density	$P(r)=r^2\lvert R_{n,l}(r)\rvert^2$	7.4

Image distance in a plane mirror	$d_o = -d_i$
Focal length for a spherical mirror	$f = \frac{R}{2}$
Mirror equation	$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$
Magnification of a spherical mirror	$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$
Sign convention for mirrors	
Focal length f	+ for concave mirror − for convex mirror
Object distance d_o	+ for real object − for virtual object
Image distance d_i	+ for real image − for virtual image
Magnification m	+ for upright image − for inverted image
Apparent depth equation	$h_i = \left(\frac{n_2}{n_1}\right) h_o$
Spherical interface equation	$\frac{n_1}{d_o} + \frac{n_2}{d_i} = \frac{n_2 - n_1}{R}$
The thin-lens equation	$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$
The magnification m of an object	$m \equiv \frac{h_i}{h_o} = -\frac{d_i}{d_o}$
Optical power	$P = \frac{1}{f}$
Optical power of thin, closely spaced lenses	$P_{total} = P_{lens1} + P_{lens2} + P_{lens3} + \cdots$
Angular magnification M of a simple magnifier	$M = \frac{\theta_{image}}{\theta_{object}}$
Angular magnification of an object a distance L from the eye for a convex lens of focal length f held a distance ℓ from the eye	$M = \left(\frac{25\text{ cm}}{L}\right)\left(1 + \frac{L - \ell}{f}\right)$
Range of angular magnification for a given lens for a person with a near point of 25 cm	$\frac{25\text{ cm}}{f} \le M \le 1 + \frac{25\text{ cm}}{f}$
Net magnification of compound microscope	$M_{net} = m^{obj} M^{eye} = -\frac{d_i^{obj}(f^{eye} + 25\text{ cm})}{f^{obj} f^{eye}}$

Euler Identies: $\cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

$$c = 3\times10^{8}\text{ meters/sec}$$

$$e = 1.602\times10^{-19}\,\text{C}$$

$$h = 6.626\times10^{-34}\,\text{J}\cdot\text{sec}$$

$$\hbar = 6.582\times10^{-16}\,eV\cdot\text{sec}$$

$$hc = 1240\,\text{eV}\cdot\text{nm}$$

$$1\ \text{Å} = 10^{-10}\text{ meters}$$

$$1\,\text{eV} = 1.602\times10^{-19}\,\text{J}$$

Show all your work in the solution of each problem.

Writing down a number with no work shown gets no credit.

Put each problem on a separate page in the blue booklet.

Problem 1 (25 points)

Two lens system: An object is 15 cm to the left of a converging lens having a focal length of 10 cm.

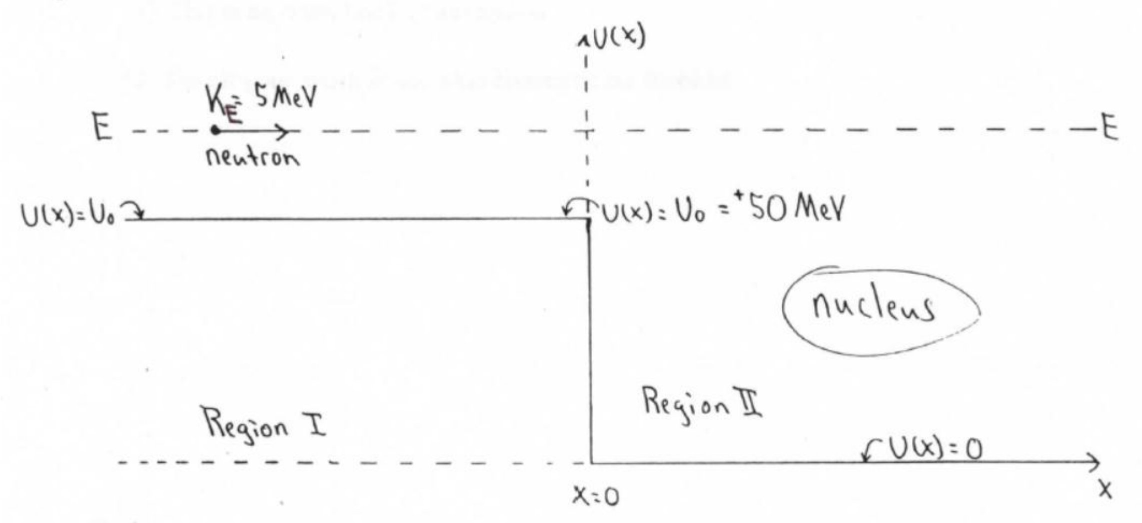
(a) Find the location of the final image.

(b) Find the magnification of the final image.

(c) Verify your conclusions by drawing the ray diagram on the graphing paper provided (put your codename on this sheet and turn it in with your blue booklet).

Problem 2 (25 points)

When a neutron enters a nucleus, it experiences a potential energy which drops at the nuclear surface very rapidly from a constant external value $U = 0$ to a constant internal value of about $U = -50$ MeV. The decrease in the potential is what makes it possible for a neutron to be bound in a nucleus. Consider a neutron incident upon a nucleus with an external kinetic energy $K_E = 5$ MeV, which is typical for a neutron that has just been emitted from a nuclear fission. In this problem we will estimate the probability that the neutron will be reflected at the nuclear surface, thereby failing to enter and have its chance at inducing another nuclear fission.
The model for the neutron-nucleus interaction is shown below as a one-dimensional step potential.



For this problem: ***Use only complex exponentials for wavefunctions***

(a) Write down the wave function, $\psi_I(x)$, for region I. Let A be the amplitude of the *incident wave* and B be the amplitude of the *reflected wave*.
(b) Write down the wave function, $\psi_{II}(x)$, for region II.
(c) What are the expressions for k_I and k_{II} in terms of E and U_0?
(d) Which coefficient(s) in Part (a) & (b) should be set to zero? Why?
(e) Use the continuity boundary conditions for $\psi(x)$ and its slope at $x = 0$ to find two relationships among the coefficients.
(f) Evaluate the reflection coefficient:

$$R = \frac{\text{reflected probalility flux}}{\text{incident probability flux}} = \frac{k_I |B|^2}{k_I |A|^2}$$

(g) For the values of K_E and U_0 given above, numerically evaluate the reflection coefficient for these low energy neutrons, thereby showing the percentage of neutrons that do not contribute to nuclear fission.

$$(\hbar = 6.582 \times 10^{-16} \text{ eV sec}, \quad m_{neutron} = 939.6 \text{ MeV}/c^2)$$

Chapter Summary

		Section
Pauli exclusion principle	*No two electrons in a single atom can have the same set of quantum numbers* (n, l, m_l, m_s).	8.1
Filling order of atomic subshells	$1s, 2s, 2p, 3s, 3d, 4s, 3d, 4p, 5s, 4d, 5p, 6s, 4f, 5d, 6p, 7s, 5f, 6d$	8.2
Capacity of subshell nl	$2(2l+1)$	8.2

		Section
Energy of screened electron	$E_n = (-13.6\,\text{eV})\dfrac{Z_{\text{eff}}^2}{n^2}$	8.3
Moseley's law for K_α X rays	$\Delta E = (10.2\,\text{eV})(Z-1)^2$	8.5
Adding angular momenta l_1, m_{l1} and l_2, m_{l2}	$L_{\max} = l_1 + l_2$, $L_{\min} = \lvert l_1 - l_2 \rvert$, $M_L = m_{l1} + m_{l2}$	8.6
Hund's rules for ground state	First $S = M_{S,\max}$, then $L = M_{L,\max}$; $J = \lvert L - S \rvert$ for $\le$ half filled subshell; $J = \lvert L + S \rvert$ for $>$ half filled subshell	8.6

Problem 3 (25 points)

The electronic configuration of carbon is: $[\mathrm{He}]2s^2 2p^2$. Thus, carbon has two $2p$ subshell electrons outside the closed $2s^2$ subshell.

(a) For the 2 valence electrons, what are their individual quantum numbers ℓ_1, ℓ_2 and s_1, s_2?

(b) Find all the allowed values of the total orbital and spin quantum numbers, L, S, and J for carbon. (remember, for filled subshells, $L = S = 0$, and ignore the Pauli Exclusion Principle for now)

(c) List all the possible quantum states for atomic carbon by specifying each state with the quantum numbers (S, L, J). *How many possible quantum states are there*? (ignore the Pauli Exclusion Principle).

(d) Use Hund's rules to find which (S, L, J) state is the ground state of this system

(e) Write this ground state in spectroscopic notation.

(f) For the (S, L, J) state with the largest J quantum number, show why it violates the Pauli Exclusion Principle (and is thus not allowed).