

NORTHERN ILLINOIS UNIVERSITY

PHYSICS DEPARTMENT

Physics 283 – Modern Physics

Spring 2024

Problem Set #2

Problem Set Due: Thurs., Feb. 1, 2024

Read Krane Chapter 3.1, 3.2, 3.3

1. **OpenStax University Physics Vol. 3: Section 3.3:** Problem 35
2. **OpenStax University Physics Vol. 3: Section 3.4:** Problem 39
3. **OpenStax University Physics Vol. 3: Section 3.5:** Problem 49
4. **Krane:** Problem 1 page 100 (draw picture)
5. **Krane:** Problem 7 page 100 (show derivation)
6. **Krane:** Problem 8 page 100 (show derivation)
7. **Krane:** Problem 16 page 101 (show derivation)
8. **Krane:** Problem 15 page 101 (do steps below)

(a) Make the following change of variables: $u = \frac{hc}{\lambda kT}$. Write the expression for $I(\lambda)$ in terms of the variable u .

(b) Find $\frac{dI}{d\lambda}$ by applying the chain rule: $\frac{dI}{d\lambda} = \frac{dI}{du} \frac{du}{d\lambda}$.

(c) Show that when finding the maximum in I by setting $\frac{dI}{d\lambda} = 0$, you get the following expression: $e^{-u} + \frac{u}{5} - 1 = 0$.

The equation above has no analytical solution. We have to use **numerical methods** to approximate the solution. We will do this by applying a Taylor Series expansion about the possible solution (ignoring terms of order 2 and higher):

let $f(u) = e^{-u} + \frac{u}{5} - 1 = 0$

Then, expanding about the possible solution $u = a$ gives:

$$f(u) = f(a) + f'(a)(u - a) + \dots$$

Solving for u gives:

$$u = a - \frac{f(a)}{f'(a)}$$

If one puts in an arbitrary value for a , one gets a first attempt at finding the proper solution (which is the value of u). This value of a is just a first guess at the solution. If one does this iteratively, one can quickly converge on the correct solution:

$$u_{n+1} = u_n - \frac{f(u_n)}{f'(u_n)} \quad (\text{this is called Newton's iterative method}).$$

where u_n is the first guess at the solution, and u_{n+1} is the more accurate solution.

(d) Calculate $f'(u) = \frac{df(u)}{du}$.

(e) Using the iterative equation above, find u_1 using the guess: $u_0 = 100$. That is,

evaluate: $u_1 = u_0 - \frac{f(u_0)}{f'(u_0)}$. You should get $u_1 = 5$. (exactly 5 by surprise).

(f) Find u_2 by using u_1 as your 2nd guess: $u_2 = u_1 - \frac{f(u_1)}{f'(u_1)}$ (you should get 4.865...).

(g) Keep doing this until the 6th digit past the decimal place no longer changes—it should then be sufficiently accurate. Be careful not to round off your answers in this iterative approach—you can get very serious truncation errors where the solution does not converge. **Show all your values** u_{n+1} calculated to at least 6 digits. **How many iterations were needed** to arrive at the solution?

(h) Derive the Wein's displacement law from your results.