NORTHERN ILLINOIS UNIVERSITY

PHYSICS DEPARTMENT

Physics 283 – Modern Physics Spring 2023

Problem Set #6

Problem Set Due: Thurs., March 9, 2023
Read Krane: Chapter 5.6, Lecture Notes #1 & #2

1. For the infinite square well potential:

$$U(x)=\begin{cases}0 & \text{for} \quad 0\le x\le L \\ \infty & \text{for} \quad x<0, x>L\end{cases}$$

find the solution to the Schrodinger equation in the three regions:

(a) Region I: $x<0$

(b) Region III: $x>L$

(c) Region II: $0\le x\le L$.
Assume the general solution is $\psi(x)=A\sin kx+B\cos kx$ and apply the condition that $\psi(x)$ is continuous across any boundary.

(d) Find the allowed energies in the infinite square well potential.

(e) Draw the energy level diagram for the 1st 4 allowed energies.

(f) Draw the wavefunctions for the 1st 4 energy states.

(g) Normalize the wavefunction.

2. Krane: **Problem 33** page 175

(a) For Part (a) of this problem: *also* sketch the potential energy barrier and where the energy of the electron is relative to the barrier.
Note: *there is an error in the textbook:* The energy barrier is shown in **Figure 5.26** (not 5.25).

(b) Do Part (b)

(c) Do Part (c)

(d) Sketch the form of a possible solution to the Schrodinger equation in each of the regions of this problem.

3. Krane: **Problem 36** page 175
in addition to each sketch, explain:
   (a) how the wavelength changes in the well (i.e., does it increase or decrease as *x* increases?)—explain why.
   (b) how the amplitude of the wave changes in the well (i.e., does it increase or decrease as *x* increases?)—explain why.
   (c) how does the wavefunction behave outside the box (i.e., is it zero or can the particle exist in the classically forbidden region?)—explain why.

4. Krane: **Problem 37** page 175 (just show calculation)

5. Krane: **Problem 38** page 176 (just show calculation)

6. **Particle in a finite square well potential:** The problem of the particle in the finite potential energy well is similar to that of the infinite well, but the potential energy has the value U_0 for $x<0$ and $x>L$. (see Fig. 5.12 in Krane). Let the electron have and energy $E<U_0$.

(a) Sketch the potential energy well and where the energy of the electron is relative to U_0.

(b) Including six undetermined constants, write down the wave functions for the three regions $x<0$, $0<x<L$, and $x>L$, for the case when $E<U_0$ (see Fig. 5.12)

(c) Two of the coefficients must be set to zero. Which ones? Why?

(d) Keeping in mind the boundary conditions at $x=0$ and $x=L$, sketch the ***three lowest-energy wave functions*** and *probability densities*. Do not attempt to solve for the undetermined constants; just sketch the expected form of the wave function. Let the solutions for the infinite potential well guide you in deciding on the form of the wave functions inside the well.