NORTHERN ILLINOIS UNIVERSITY

PHYSICS DEPARTMENT

Physics 383 – Modern Physics   Spring 2023

Problem Set #2

Problem Set Due: Thurs., Feb. 2, 2023

Read Krane Chapter 3   (Sections 3.1 & 3.3)

Chapter 3:

1. Krane Problem 1   Draw picture

2. Krane Problem 3   Do steps below

   (a) For Part (a) of this problem: Draw a figure showing a lattice of atoms, the incident and reflected rays, and the Bragg angle (and show what d is in the figure).

   (b) For Part (b) of this problem: Draw the lattice of atoms, the crystals planes that the incident ray is reflecting from, the incident ray, the reflected ray emerging from the crystal surface, and the angle asked for in the problem (see Figures 3.5 and 3.6 in the textbook). Draw this figure BIG (fill at least ¼ of a page) and show (and explain) all your geometric and trigonometric derivations—small figures are usually unreadable and confusing.

   Explain all your mathematical and geometry calculations. Anybody should be able to look at your diagram and explanations and see how you arrived at the angle the reflected ray leaves the surface of the crystal.

3. Krane Problem 15 Do steps below:

(a) Make the following change of variables: $u = \dfrac{hc}{\lambda kT}$. Write the expression for $I(\lambda)$ in terms of the variable u.

(b) Find $\dfrac{dI}{d\lambda}$ by applying the chain rule: $\dfrac{dI}{d\lambda} = \dfrac{dI}{du}\dfrac{du}{d\lambda}$.

(c) Show that when finding the maximum in I by setting $\dfrac{dI}{d\lambda} = 0$, you get the following expression: $e^{-u} + \dfrac{u}{5} - 1 = 0$.

The equation above has no analytical solution. We have to use numerical methods to approximate the solution. We will do this by applying a Taylor Series expansion about the possible solution (ignoring terms of order 2 and higher):

let $f(u) = e^{-u} + \dfrac{u}{5} - 1 = 0$

Then, expanding about the possible solution $u = a$ gives:

$$f(u) = f(a) + f'(a)(u-a) + \ldots.$$

Solving for u gives:

$$u = a - \frac{f(a)}{f'(a)}$$

If one puts in an arbitrary value for a, one gets a first attempt at finding the proper solution (which is the value of u). This value of a is just a first guess at the solution. If one does this iteratively, one can quickly converge on the correct solution:

$$u_{n+1} = u_n - \frac{f(u_n)}{f'(u_n)}$$ (this is called Newton's iterative method).

where u_n is the first guess at the solution, and u_{n+1} is the more accurate solution.

(d) Calculate $f'(u) = \dfrac{df(u)}{du}$.

(e) Using the iterative equation above, find u_1 using the guess: $u_0 = 100$. That is, evaluate: $u_1 = u_0 - \dfrac{f(u_0)}{f'(u_0)}$. You should get $u_1 = 5$. (exactly 5 by surprise).

(f) Find u_2 by using u_1 as your 2nd guess: $u_2 = u_1 - \dfrac{f(u_1)}{f'(u_1)}$ (you should get 4.865..).

(g) Keep doing this until the 6th digit past the decimal place no longer changes—it should then be sufficiently accurate. Be careful not to round off your answers in this iterative approach—you can get very serious truncation errors where the solution does not converge. Show all your values u_{n+1} calculated to at least 6 digits. How many iterations were needed to arrive at the solution?

(h) Derive the Wein's displacement law from your results.

4. Krane Problem 16 Just show derivation
5. Krane Problem 19 Draw Blackbody radiation curve