NORTHERN ILLINOIS UNIVERSITY

PHYSICS DEPARTMENT

Physics 283 – Modern Physics Spring 2024

Quiz #2

INSTRUCTIONS:

(1) No graphic calculators are allowed for this quiz.
(2) Do all your work in the examination blue booklet
(3) You may find the following formulas useful:

Chapter Summary

		Section			Section
Time-independent Schrödinger equation	$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U(x)\psi(x) = E\psi(x)$	5.3	Infinite potential energy well	$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$, $E_n = \frac{h^2 n^2}{8mL^2}$ $(n = 1, 2, 3, \ldots)$	5.4
Time-dependent Schrödinger equation	$\Psi(x,t) = \psi(x)e^{-i\omega t}$	5.3	Two-dimensional infinite well	$\psi(x,y) = \frac{2}{L}\sin\frac{n_x\pi x}{L}\sin\frac{n_y\pi y}{L}$, $E = \frac{h^2}{8mL^2}(n_x^2 + n_y^2)$	5.4
Probability density	$P(x) = \lvert\psi(x)\rvert^2$	5.3	Simple harmonic oscillator ground state	$\psi(x) = (m\omega_0/\hbar\pi)^{1/4}\, e^{-(\sqrt{km}/2\hbar)x^2}$	5.5
Normalization condition	$\int_{-\infty}^{+\infty} \lvert\psi(x)\rvert^2\, dx = 1$	5.3	Simple harmonic oscillator energies	$E_n = (n + \frac{1}{2})\hbar\omega_0$ $(n = 0, 1, 2, \ldots)$	5.5
Probability in interval x_1 to x_2	$P(x_1 : x_2) = \int_{x_1}^{x_2} \lvert\psi(x)\rvert^2\, dx$	5.3	Potential energy step, $E > U_0$	$\psi_0(x < 0) = A\sin k_0x + B\cos k_0x$, $\psi_1(x > 0) = C\sin k_1x + D\cos k_1x$	5.6
Average or expectation value of $f(x)$	$\langle f(x)\rangle = \int_{-\infty}^{+\infty} \lvert\psi(x)\rvert^2 f(x)\, dx$	5.3	Potential energy step, $E < U_0$	$\psi_0(x < 0) = A\sin k_0x + B\cos k_0x$, $\psi_1(x > 0) = Ce^{k_1x} + De^{-k_1x}$	5.6
Constant potential energy, $E > U_0$	$\psi(x) = A\sin kx + B\cos kx$, $k = \sqrt{2m(E - U_0)/\hbar^2}$	5.4			
Constant potential energy, $E < U_0$	$\psi(x) = Ae^{k'x} + Be^{-k'x}$, $k' = \sqrt{2m(U_0 - E)/\hbar^2}$	5.4			

Hydrogen quantum numbers	$n = 1, 2, 3, \ldots$ $l = 0, 1, 2, \ldots, n-1$ $m_l = 0, \pm 1, \pm 2, \ldots, \pm l$	7.3
Hydrogen energy levels	$E_n = -\frac{me^4}{32\pi^2\varepsilon_0^2\hbar^2}\frac{1}{n^2}$	7.3
Hydrogen wave functions	$\psi_{n,l,m_l}(r,\theta,\phi) = R_{n,l}(r)\Theta_{l,m_l}(\theta)\Phi_{m_l}(\phi)$	7.3
Radial probability density	$P(r) = r^2\lvert R_{n,l}(r)\rvert^2$	7.4

TABLE 7.1 **Some Hydrogen Atom Wave Functions**

n	l	m_l	$R(r)$	$\Theta(\theta)$	$\Phi(\phi)$
1	0	0	$\frac{2}{a_0^{3/2}} e^{-r/a_0}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2\pi}}$
2	0	0	$\frac{1}{(2a_0)^{3/2}} \left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2\pi}}$
2	1	0	$\frac{1}{\sqrt{3}(2a_0)^{3/2}} \frac{r}{a_0} e^{-r/2a_0}$	$\sqrt{\frac{3}{2}} \cos\theta$	$\frac{1}{\sqrt{2\pi}}$
2	1	±1	$\frac{1}{\sqrt{3}(2a_0)^{3/2}} \frac{r}{a_0} e^{-r/2a_0}$	$\mp\frac{\sqrt{3}}{2} \sin\theta$	$\frac{1}{\sqrt{2\pi}} e^{\pm i\phi}$
3	0	0	$\frac{2}{(3a_0)^{3/2}} \left(1 - \frac{2r}{3a_0} + \frac{2r^2}{27a_0^2}\right) e^{-r/3a_0}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2\pi}}$
3	1	0	$\frac{8}{9\sqrt{2}(3a_0)^{3/2}} \left(\frac{r}{a_0} - \frac{r^2}{6a_0^2}\right) e^{-r/3a_0}$	$\sqrt{\frac{3}{2}} \cos\theta$	$\frac{1}{\sqrt{2\pi}}$
3	1	±1	$\frac{8}{9\sqrt{2}(3a_0)^{3/2}} \left(\frac{r}{a_0} - \frac{r^2}{6a_0^2}\right) e^{-r/3a_0}$	$\mp\frac{\sqrt{3}}{2} \sin\theta$	$\frac{1}{\sqrt{2\pi}} e^{\pm i\phi}$
3	2	0	$\frac{4}{27\sqrt{10}(3a_0)^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0}$	$\sqrt{\frac{5}{8}}(3\cos^2\theta - 1)$	$\frac{1}{\sqrt{2\pi}}$
3	2	±1	$\frac{4}{27\sqrt{10}(3a_0)^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0}$	$\mp\sqrt{\frac{15}{4}} \sin\theta\cos\theta$	$\frac{1}{\sqrt{2\pi}} e^{\pm i\phi}$
3	2	±2	$\frac{4}{27\sqrt{10}(3a_0)^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0}$	$\frac{\sqrt{15}}{4} \sin^2\theta$	$\frac{1}{\sqrt{2\pi}} e^{\pm 2i\phi}$

$$a \sin\theta = m\lambda \text{ for } m = \pm1, \pm2, \pm3, \ldots$$

$$\beta = \frac{\phi}{2} = \frac{\pi a \sin\theta}{\lambda}$$

$$E = N\Delta E_0 \frac{\sin\beta}{\beta}$$

$$I = I_0 \left(\frac{\sin\beta}{\beta}\right)^2$$

$$\theta = 1.22\frac{\lambda}{D}$$

$$m\lambda = 2d\sin\theta,\ m = 1, 2, 3\ldots$$

$$c = 2.99792458 \times 10^8 \text{ m/s} \approx 3.00 \times 10^8 \text{ m/s}$$

$$n = \frac{c}{v}$$

$$\theta_r = \theta_i$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) \text{ for } n_1 > n_2$$

$$I = I_0 \cos^2\theta$$

$$\tan\theta_b = \frac{n_2}{n_1}$$

Euler Identies:

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

$$c = 3 \times 10^{8} \text{ meters/sec}$$

$$e = 1.602 \times 10^{-19}\,\text{C}$$

$$h = 6.626 \times 10^{-34}\,\text{J} \cdot \text{sec}$$

$$hc = 1240\,\text{eV} \cdot \text{nm}$$

$$1 \text{ Å} = 10^{-10} \text{ meters}$$

$$1\,\text{eV} = 1.602 \times 10^{-19}\,\text{J}$$

Show all your work in the solution of each problem.

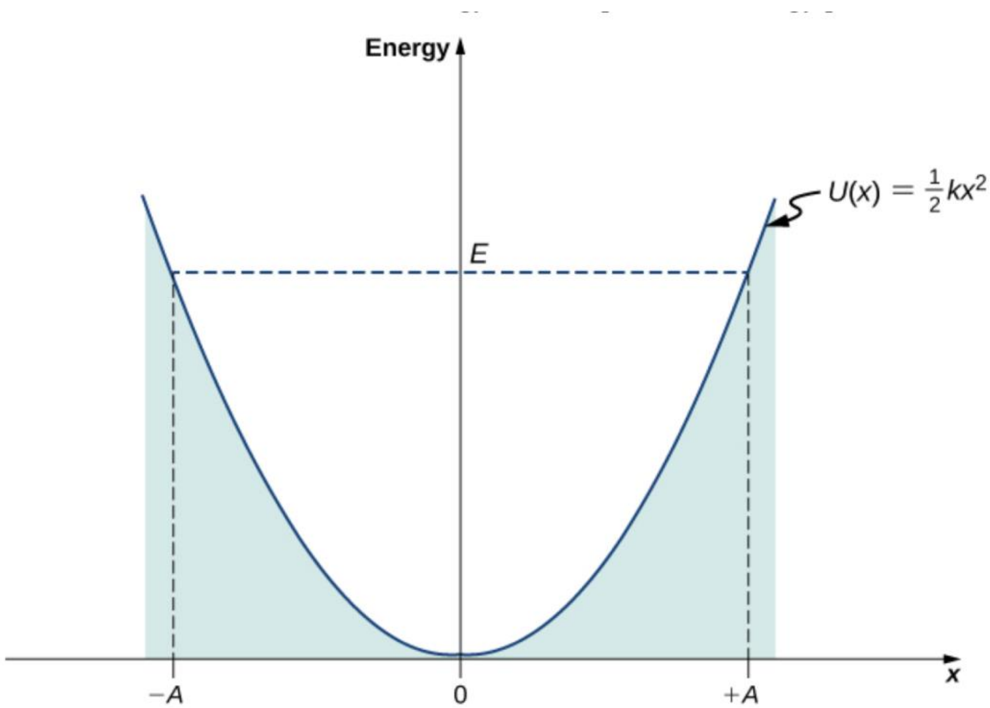
Writing down a number with no work shown gets no credit.

Put each problem on a separate page in the blue booklet.

Problem 1 (25 points)

A light beam in air has an angle of incidence of 35° at the surface of a glass plate. What are the angles of reflection and refraction? The index of refraction for the glass plate is 1.50 and assume the index of refraction for air is 1.00.

Problem 2   (25 points)



Sketch the 1st 4 wavefunctions for the harmonic oscillator potential shown above. Explain what is happening to the wavelength and amplitude of the wavefunctions throughout the potential.

Problem 3   (25 points)

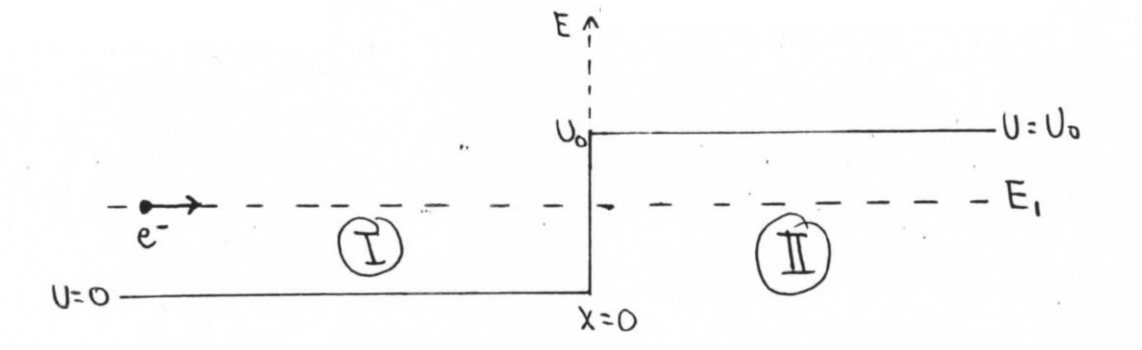
The $n = 2,\ l = 1$ state of hydrogen is: $\psi_{210} = \dfrac{1}{4\sqrt{2\pi}}\left(\dfrac{Z}{a_0}\right)^{3/2} \dfrac{Zr}{a_0} e^{-Zr/2a_0} \cos\theta$.

(a) Calculate the location in r at which the radial probability density is a maximum for this state.

(b) Calculate the expectation value of the radial coordinate, $\langle r \rangle$. You may find the following integral formula useful: $\int_0^\infty x^n e^{-bx} dx = \dfrac{n!}{b^{n+1}}$.

(c) Plot the radial probability density as a function of r, and explain the physical significance of the difference in the answers to part (a) and (b).

Problem 4 (25 points)



$$U(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ U_0 & \text{for} \quad x > 0 \end{cases}$$

The electron has an energy less than the step height: $E < U_0$.

For this problem: ***Use only complex exponentials for wavefunctions***

(a) Write down the wave function, $\psi_I(x)$, for Region I. Let A be the amplitude of the *incident wave* and B be the amplitude of the *reflected wave*.

(b) Write down the wave function, $\psi_{II}(x)$, for region II.

(c) Which coefficient(s) in Part (a) & (b) should be set to zero? Why?

(d) Use the continuity boundary conditions for $\psi(x)$ and its slope at $x = 0$ to find two relationships among the coefficients.

(e) Evaluate the reflection coefficient:

$$R = \frac{\text{reflected probalility flux}}{\text{incident probability flux}} = \frac{k_I |B|^2}{k_I |A|^2}$$

(f) Show that the particle is totally reflected at the step by showing that $R = 1$.