NORTHERN ILLINOIS UNIVERSITY

PHYSICS DEPARTMENT

Physics 210 – Mechanics & Heat Spring 2022

Lab #8

Lab Writeup Due: Tue/Wed/Fri, March 29/30/ April 1, 2022
Read OpenStax: Chapter 7.1-7.6, Lecture Notes #7

Ballistic Pendulum

Apparatus

A ballistic pendulum is a device consisting of three parts: a spring gun, a ball that can be launched from the gun, and a cup at the end of a pendulum to catch the ball. The spring gun is designed to fire a ball of mass m_b with an initial velocity v_i. The pendulum and cup can be moved out of the way. This permits the ball to be fired as a projectile and the initial velocity measured.

When the pendulum is in its lower position, the cup with mass m_c is ready to catch the ball when fired. When the ball is caught in the cup, the energy of the combined cup and ball is used to swing the pendulum up by a final height h_f *above initial position of the center of mass of the combined cup and ball*. A ratchet catches the cup and allows you to read a position measurement by means of a pointer that catches in a groove of the ratchet.

The measurement on the ratchet is marked in units that go from 0 to 40. There are marks at the halfway points (5, 15, 25, 35), and individual ratchet positions represent steps of 1. The ratchet measure can be converted to height by means of the following table:

Table 1: Conversion of ratchet measurement to height above the base of the ballistic pendulum

Ratchet Measurement	**Height** (mm)
0	65
10	73
20	80
30	88
40	96

To find the conversion for other ratchet measurements requires interpolation. Interpolation means estimating the value between two known values. We do this by assuming that the conversion function is a straight line between the two known points. The two points are used to create the equation of a straight line similar to $y = mx + b$. The actual equation will have offsets because the line is only calculated between two points.

For our specific table, we treat the ratchet measurement x as the independent variable, and the height of the ball h_f as the dependent variable. To interpolate, look at your measurement of x. If it's not in the table, find the measurement, x_1, in the table that is immediately below your measurement and its corresponding height h_1. Find the measurement, x_2, in the table that is immediately above your measurement and its corresponding height h_2. The interpolated height h_f for your measurement is

$$h_f = h_1 + \left(\frac{x - x_1}{x_2 - x_1}\right)\left(h_2 - h_1\right) \tag{1}$$

Theory

In an earlier laboratory we studied projectile motion. A ball moving horizontally with initial velocity v_i was allowed to fall freely to the ground a distance d below the launch point. The range R beyond the launch point was derived from the kinematics of an object subject only to gravitational acceleration g. The relation between the velocity, range, and vertical distance was

$$v_i = R\sqrt{\frac{g}{2d}} \tag{2}$$

Kinetic energy is the energy of motion. It depends on the mass and velocity of an object. The total kinetic energy of the initial state of the system is

$$K_i = \frac{1}{2} m_b v_i^2 \tag{3}$$

since the cup is stationary when the ball is launched. The total gravitational potential energy of the initial state of the system is

$$U_i = \left(m_b + m_c\right) g h_i = 0 \tag{4}$$

since the ball and cup is defined to have zero gravitational potential energy (that is, $h_i \equiv 0$). Using the principle of *conservation of momentum* (which we will get to later in the course), the velocity v_{bc} instantly after the ball is caught by the cup is given by

$$v_{bc} = v_i \left(\frac{m_b}{m_b + m_c} \right) \tag{5}$$

Using Eq. (5), the total kinetic energy of this intermediate state (instantly after the ball is caught by the cup) is

$$K_{bc} = \frac{1}{2}\left(m_b + m_c\right)\left(\frac{m_b}{m_b + m_c} \right)^2 v_i^2 = \frac{1}{2}\frac{m_b^2 v_i^2}{\left(m_b + m_c\right)} \tag{6}$$

The total potential energy of this intermediate state, $U_{bc} = U_i = 0$, is still given by Eq. (4) since the ball & cup have not yet started to rise. The total gravitational potential energy of the final state is

$$U_f = \left(m_b + m_c\right) g h_f \tag{7}$$

where we have used the total mass of the ball and cup. Note that the total kinetic energy of the final state is zero since the ball & cup come to rest:

$$K_f = 0 \tag{8}$$

Conservation of energy is a fundamental principle of physics—it predicts that the kinetic energy can be fully converted into potential energy.

Data Collection

You must show all your calculations in Parts A&B to your TA before you leave the lab.

Part A: Muzzle velocity

(1) Make sure the apparatus is clamped to the table, a stop wall is available, and swing the pendulum arm out of the way.

(2) Measure the vertical drop d from the end of the gun to the floor and record this value in your lab notebook.

(3) Fire the gun and note where the ball lands. Tape a sheet of white paper to the floor at the location where the ball lands and place (<u>*do not tape*</u>) sheets of carbon paper over the sheet.

(4) Fire the gun 10 times so that the impacts are recorded on the sheet of white paper.

(5) Measure the horizontal distance from the spot on the floor directly beneath the tip of the gun to the edge of the paper.

(6) Measure and record the distance of each mark on the paper from the edge of the paper and find the mean (or average) distance.

(7) Add the average distance from Step (6) to that in Step (5) to get the range R.

(8) Calculate the initial velocity v_i using Eq. (2).

<u>Part B: Conservation of energy</u>

(9) Weigh and record the mass of the ball m_b, and record the mass of cup m_c written on the apparatus.

(10) Fire the gun with the ball into the cup 10 times, and record the ratchet measurement, x, for each trial. Calculate the mean (or average) ratchet measurement. Use the interpolation formula in Eq. (1) and the data in Table 1 to find the height h_f.

(11) Calculate the initial kinetic energy K_i, the intermediate kinetic energy K_{bc}, and the final potential energy U_f.

(12) Calculate the total initial energy of the system: $E_{Total}^{initial} = K_i + U_i$, the total intermediate energy of the system: $E_{Total}^{intermediate} = K_{bc} + U_{bc}$, and the total final energy of the system: $E_{Total}^{final} = K_f + U_f$.

(13) Compute a percent difference between $E_{Total}^{initial}$ and E_{Total}^{final} where

$$\%Difference = \frac{|measurement\#1 - measurement\#2|}{\frac{1}{2}(measurement\#1 + measurement\#2)} \times 100$$

Note that we must use the "*percent difference*" relationship to compare the two values (rather than the "*percent error*" described in Lab#2) because neither measurement is a commonly accepted value of the energy.

(14) Compute a percent difference between $E_{Total}^{intermediate}$ and E_{Total}^{final}.

Data Analysis

(a) How accurate was the measurement of the range resulting from Step (4)? What effects might account for the different positions of the marks on the floor?

(b) What parts of the determination of the initial velocity in Step (8) were likely to have the greatest error?

(c) How accurate was the measurement in Step (11)? What effects might account for the different positions of the ratchet?

(d) From Steps (14) & (15), explain, using your *%Differences*, which energy conservation relation is more in agreement?:

(1st) $E_{Total}^{initial} = E_{Total}^{final}$ or **(2nd)** $E_{Total}^{intermediate} = E_{Total}^{final}$

(e) The **1st** relation above states that kinetic energy is converted into potential energy:

$$\frac{1}{2} m_b v_i^2 = \left(m_b + m_c\right) g\left(h_f - h_i\right)$$

Why is this statement patently false *for this particular experiment*?

(f) What might be the cause for the discrepancy in energy conservation for the **2nd** relation above?