Physics 210 - Quiz # 3 Solutions

(1)

(1.1) $W = Fx\cos\theta \Rightarrow$

$(c) \rightarrow W = F\Delta x$

$(a) \rightarrow W = F\Delta x \cos 90^\circ = 0$

$(d) \rightarrow W = F\Delta x \cos(90^\circ + 45^\circ) = -Fx\sin 45^\circ$

$(b) \rightarrow W = F\Delta x \cos 180^\circ = -F\Delta x$

Answer: (c): c, a, d, b

(1.2) Apply conservation of energy

$$mgh + \frac{1}{2}mV_i^2 = \frac{1}{2}mV_f^2$$

all balls have same initial speed

all balls have same initial height

$\Rightarrow V_f$ is the same for all balls

Answer: (d) All balls strike the ground at the same time

(1.3) $U_s = \frac{1}{2}Kx^2$

(a) double $x \rightarrow U_s = \frac{1}{2}K(2x)^2 = 4\left[\frac{1}{2}Kx^2\right]$

$\rightarrow$ potential energy increases by factor of 4

(b) triple $x \rightarrow U_s = \frac{1}{2}K(3x)^2 = 9\left[\frac{1}{2}Kx^2\right]$

$\rightarrow$ potential energy increases by factor of 9

(1.4) $\frac{1}{2}m_1 v_1^2 = \frac{1}{2}m_2 v_2^2$

$$m_1 v_1^2 = m_2 v_2^2 \Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{m_2}{m_1}}$$

$$p = mv \Rightarrow \frac{p_1}{p_2} = \frac{m_1 v_1}{m_2 v_2} = \frac{m_1}{m_2}\sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{m_1^2}{m_2^2}\cdot\frac{m_2}{m_1}}$$

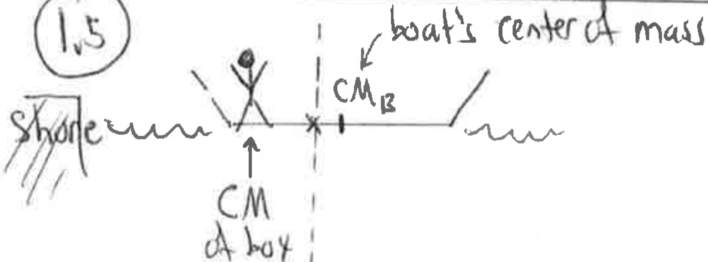
$$\frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}}$$

If $m_1 < m_2$ then $\sqrt{\frac{m_1}{m_2}} < 1$

$$\Rightarrow \frac{p_1}{p_2} < 1 \Rightarrow p_1 < p_2$$

Answer: (b) $p_1 < p_2$

(1.5)

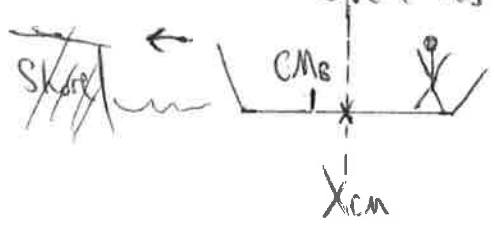


Answer: (c) boat moves toward the shore

center of mass of (Boat + Boy) = X_{CM}

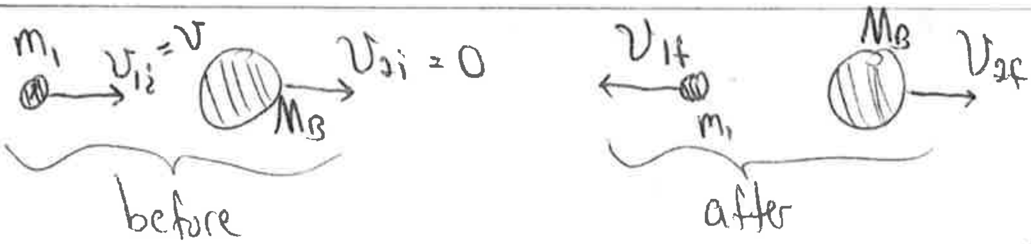
initially, $V_{CM} = 0$ (everything is at rest)

Since there are no external forces, $V_{CM} = 0$ always even as boy walks across boat.



As boy walks to other end of the boat, the center of mass of the boat must move to the left (towards the shore) for X_{CM} to remain at the same place relative to the shore.

(1.6)



Conservation of momentum:

$$m_1 v + \cancel{M_B v_{2i}}^{\,0} = m_1 v_{1f} + \underbrace{\cancel{M_B v_{2f}}}_{\text{assume the velocity of bowling ball is small since its mass (inertia) is so large}}$$

at rest

≈ 0 for a massive object

$$\cancel{m_1} v = \cancel{m_1} v_{1f}$$

$$v_{1f} = v$$

Answer: (c) v

(1.7) $K_E = \frac{1}{2} I \omega^2 \qquad I_{\text{hollow sphere}} = \frac{2}{3} MR^2, \; I_{\text{solid sphere}} = \frac{2}{5} MR^2$

(Hollow sphere): $K_E = \frac{1}{2}\left(\frac{2}{3} MR^2\right)\omega^2 = \frac{1}{3} MR^2\omega^2$

(Solid sphere): $K_E = \frac{1}{2}\left(\frac{2}{5} MR^2\right)\omega^2 = \frac{1}{5} MR^2\omega^2$

$\frac{1}{3} MR^2\omega^2 > \frac{1}{5} MR^2\omega^2$

Answer: (a) the hollow sphere has greater kinetic energy

(1.8) Conservation of angular momentum

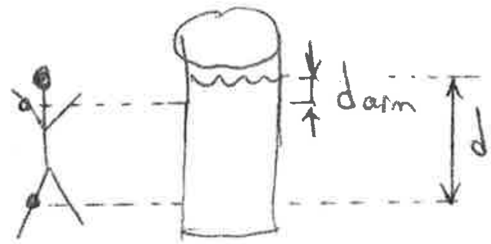
$I_i \omega_i = I_f \omega_f \qquad$ where $I \sim (\text{const}) MR^2$

$$\frac{\omega_f}{\omega_i} \sim \frac{I_i}{I_f} \sim \frac{R_i^2}{R_f^2} \longrightarrow$$

since the radius of the Earth will increase as ice melts, $R_f > R_i \Rightarrow \omega_f < \omega_i$ Earth rotates slower

Answer: (a) The length of a day will increase

(1.9)



$P_{bottom} = P_{top} + \rho g d$

Since $d_{leg} > d_{arm}$ (modeling a person as a cylinder of water)

The pressure in the leg > pressure in the arm

Answer: (b) greater than it is for the arm

(1.10) $F_{bouyant} = B = \underbrace{\rho_{fluid}\, g\, V_{object}}$

the buoyant force does not depend upon the density of the object

Answer: (b) equal to (The buoyant force on lead = the buoyant force on iron if they have the same volume)

(1.11)

$V = \frac{4}{3}\pi r^3$, $\rho = \frac{m}{V} \Rightarrow m = \rho V$

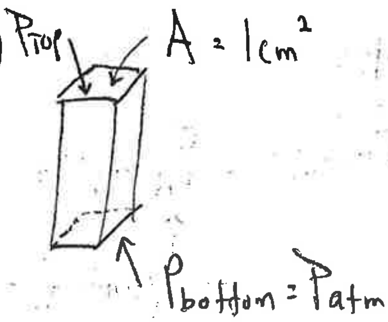
Weight $= mg = \rho V g = \rho g \left(\frac{4}{3}\pi r^3\right)$

(double the radius) $\Rightarrow W = \rho g \left(\frac{4}{3}\pi\right)(2r)^3 = 8\left[\rho g \left(\frac{4}{3}\pi r^3\right)\right]$

$= 8 \times$ (original weight)

$\Rightarrow \boxed{\text{(e) more than 4N}}$

(1.12)



Note: at the top of the atmosphere there is no air pressure:

$$P_{TOP} = 0$$

$$P_{bottom} = P_{atm} = 1.01 \times 10^5 \; Pa$$

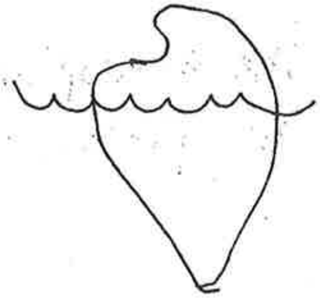
$$P = \frac{F}{A} = \frac{W_{air}}{A} = \frac{m_{air} g}{A} = P_{atm}$$

$$m_{air} = \frac{P_{atm} \cdot A}{g} = \frac{\left(1.01 \times 10^5 \frac{N}{m^2}\right)\left(1 cm^2 \times \left[\frac{1 m}{100 cm}\right]^2\right)}{9.8 \, m/sec^2}$$

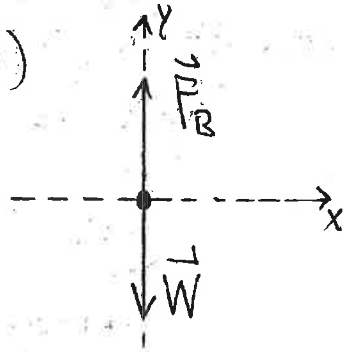
$$= 1.03 \; Kg$$

$$\boxed{\Rightarrow (b)}$$

(2)



(a)



(b)

$$F_{Total,y} = F_B - W = m\cancel{a_y}^{\,0}$$

F_B ↑ buoyant force

(c) From Newton's 2nd Law: $F_B = W \Rightarrow \rho_{\substack{fluid\\displaced}}\, g\, V_{\substack{fluid\\displaced}} = m_{ice}\, g$

but $\rho_{iceberg} = \dfrac{m_{iceberg}}{V_{iceberg}} \Rightarrow m_{iceberg} = \rho_{iceberg} V_{iceberg}$

$$\rho_{\substack{fluid\\displaced}}\, g\, V_{\substack{fluid\\displaced}} = \rho_{iceberg}\, g\, V_{iceberg} \rightarrow \text{from Newton's 2nd Law}$$

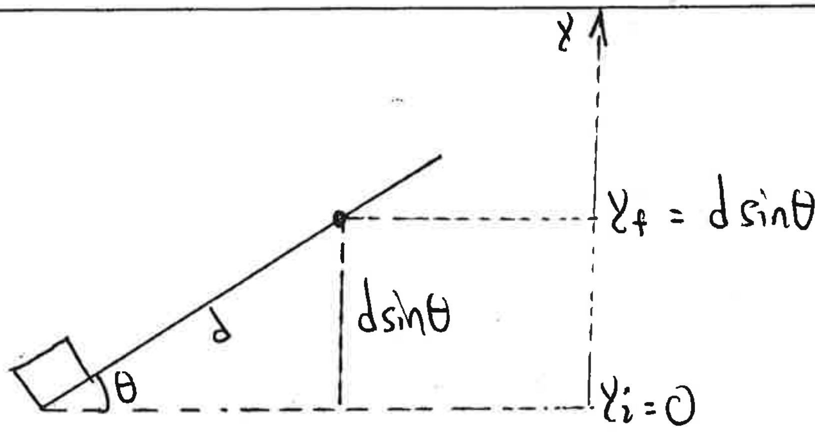
$\rho_{\substack{fluid\\displaced}} \downarrow \rho_{seawater}$; $V_{\substack{fluid\\displaced}} \downarrow V_{\substack{ice\ below\\surface}}$

$$\Rightarrow \rho_{seawater}\, V_{\substack{iceberge\ below\\surface}} = \rho_{iceberg}\, V_{iceberg}$$

$$\frac{V_{\substack{iceberge\ below\\surface}}}{V_{iceberg}} = \frac{\rho_{iceberg}}{\rho_{seawater}} = \frac{0.917}{1.025} = 0.8946$$

$\Rightarrow$ 89.46% of the iceberg's Volume is below the Surface (most of the iceberg is hidden from view from a Ship → very deceiving and dangerous)

3



(a) Conservation of Energy:

$$\underbrace{\tfrac{1}{2} m v_i^2}_{5\,\text{m/sec}} + \underbrace{m g \cancel{y_i}}_{0} = \underbrace{\tfrac{1}{2} m \cancel{v_f^2}}_{0} + m g \underbrace{y_f}_{d\sin\theta} + \underbrace{f_k d}_{\text{frictional energy loss}}$$

(Stops before coming back down)

$$\tfrac{1}{2} m v_i^2 = m g d \sin\theta + f_k d$$

$$f_k = \frac{m v_i^2}{2d} - m g \sin\theta = m\left(\frac{v_i^2}{2d} - g\sin\theta\right)$$

$$= (12\,\text{kg})\left(\frac{[5.0]^2}{2(1.6\,\text{m})} - (9.8\,\text{m/sec}^2)\sin 30^\circ\right) = 34.95\,\text{N}$$

(b)

$$\underbrace{\tfrac{1}{2} m \cancel{v_{ii}^2}}_{0} + m g \underbrace{y_{ii}}_{d\sin\theta} = \tfrac{1}{2} m v_{ff}^2 + \underbrace{m g \cancel{y_{ff}}}_{0} + \underbrace{f_k d}_{\text{heat energy generated}}$$

(at the top its velocity is zero)

(at the bottom of the slide $y_{ff} = 0$)

$$m g d \sin\theta = \tfrac{1}{2} m v_{ff}^2 + f_k d$$

$$v_{ff} = \sqrt{2d\left(g\sin\theta - f_k/m\right)}$$

$$v_{ff} = \sqrt{2(1.6m)\left[(9.8m/sec^2)\sin 30^\circ - \frac{(34.95N)}{12Kg}\right]}$$

$$= 2.5219\ m/sec$$

(c) $\frac{1}{2}mv_i^2 + \underbrace{\cancel{mgy_i}}_{0} = \frac{1}{2}m\underbrace{\cancel{v_f}^2}_{0} + mg\underbrace{y_f}_{d\sin\theta} + f_k d$

(if just barely gets to the top → it stops)

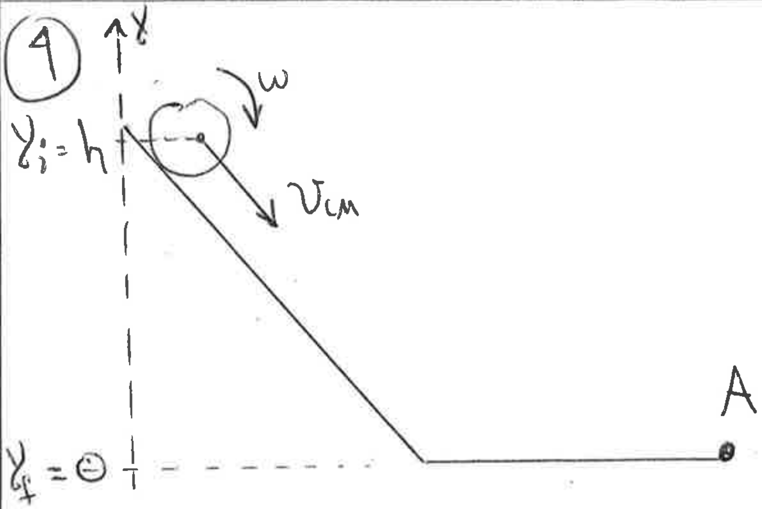
$$\frac{1}{2}mv_i^2 = mgd\sin\theta + f_k d$$

$$v_i = \sqrt{2d(g\sin\theta + f_k/m)}$$

$$= \sqrt{2(2.5m)\left[(9.8m/sec^2)\sin 30^\circ + \frac{34.95N}{12Kg}\right]}$$

$$= 6.25\ m/sec$$

4



(a) Conservation of energy:

$$\underset{\downarrow \atop h}{mgy_i} + \underbrace{\cancelto{0}{\tfrac{1}{2}I\omega_i^2}}_{\text{rotational Kinetic energy}} + \underbrace{\cancelto{0}{\tfrac{1}{2}mv_{cmi}^2}}_{\text{translational Kinetic energy}} = \cancelto{0}{mgy_f} + \tfrac{1}{2}I\omega_f^2 + \tfrac{1}{2}mv_{cmf}^2$$

Note: $v_{cm} = \omega r \Rightarrow \omega = \frac{v_{cm}}{r}$

Then,

$$mgh = \frac{1}{2}I\left(\frac{v_{cmf}}{r}\right)^2 + \frac{1}{2}mv_{cmf}^2$$

let $v_{cmf} \equiv v$ and note that $I = \frac{2}{5}mr^2$

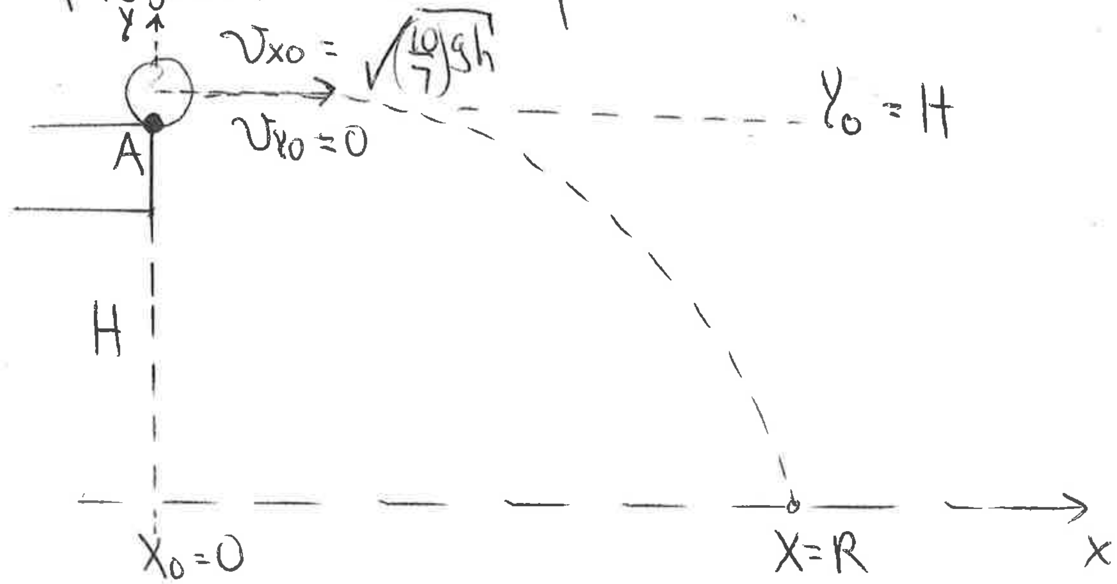
Then,

$$mgh = \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2 + \frac{1}{2}mv^2$$

$$\cancel{m}gh = \frac{1}{5}mv^2 + \frac{1}{2}mv^2 = \frac{2}{10}mv^2 + \frac{5}{10}mv^2 = \frac{7}{10}\cancel{m}v^2$$

$$\Rightarrow v^2 = \left(\frac{10}{7}\right)gh \Rightarrow v = \sqrt{\left(\frac{10}{7}\right)gh}$$

(b) Projectile motion problem



Using Eq. M3x:

$$X = X_0 + v_{xo}t + \tfrac{1}{2}\cancel{a_x t^2}^{\,0}$$

$X \to R$, $X_0 \to 0$, $v_{xo} \to \sqrt{\left(\frac{10}{7}\right)gh}$

$$\Rightarrow R = t\sqrt{\left(\frac{10}{7}\right)gh}$$

To find t, use Eq. M3y:

$$Y = Y_0 + \cancel{v_{yo}t}^{\,0} - \tfrac{1}{2}gt^2$$

$Y \to 0$, $Y_0 \to H$, $v_{yo} \to 0$

$$\Rightarrow 0 = H - \tfrac{1}{2}gt^2 \quad \Rightarrow H = \tfrac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2H}{g}}$$

$$\Rightarrow R = \left(\sqrt{\frac{2H}{g}}\right)\left(\sqrt{\frac{10}{7}gh}\right) = \sqrt{\frac{2H}{\cancel{g}} \cdot \frac{10}{7}\cancel{g}h}$$

$$R = \sqrt{\left(\frac{20}{7}\right)Hh}$$