FORCE TABLE Lab #4

INTRODUCTION

All measurable quantities can be classified as either a scalar[1] or a vector[2]. A scalar has only magnitude while a vector has both magnitude and direction. Examples of scalar quantities are the number of students in a class, the mass of an object, or the speed of an object, to name a few. Velocity, force, and acceleration are examples of vector quantities. The statement "a car is traveling at 60 mph" tells us how fast the car is traveling but not the direction in which it is traveling. In this case, we know the speed of the car to be 60 mph. On the other hand, the statement "a car traveling at 60 mph due east" gives us not only the speed of the car but also the direction. In this case the velocity of the car is 60 mph due east and this is a vector quantity.

Unlike scalar quantities that are added arithmetically, addition of vector quantities involves both magnitude and direction. In this lab we will use a force table to determine the resultant of two or more force vectors and learn to add vectors using graphical as well as analytical methods.

DISCUSSION OF PRINCIPLES

Vector Representation

As mentioned above, a vector quantity has both magnitude and direction. A vector is usually represented by an arrow, where the direction of the arrow represents the direction of the vector, and the length of the arrow represents the magnitude of the vector. In three-dimensions, a vector directed out of the page (or along the positive z-axis) is represented by $\odot$ (a circle with a dot inside it) and a vector directed into the page (or along the negative z-axis) is represented by $\otimes$ (a circle with an $\times$ inside it).

In mathematical equations a vector is represented as $\overrightarrow{A}$. In some textbooks a vector is represented by a bold face letter **A**.

The negative of a vector $\overrightarrow{A}$ is a vector of the same length but with a direction opposite to that of $\overrightarrow{A}$. See Fig. 1 below.

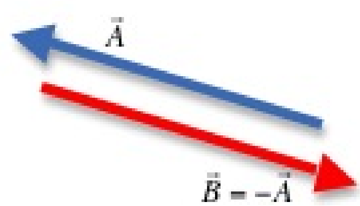


Figure 1: Vectors as arrows

[1]http://en.wikipedia.org/wiki/Scalar_(physics)
[2]http://en.wikipedia.org/wiki/Euclidean_vector

The Cartesian coordinate system is used for graphical representation of vectors. The tail of the vector is placed on the origin and the direction of the vector is defined by an angle, θ (theta), between the positive x-axis and the vector, as shown in Fig. 2.

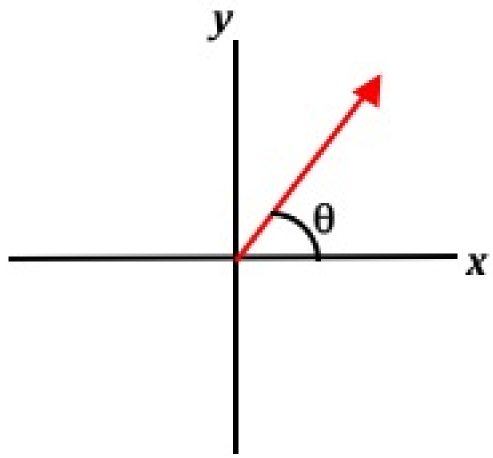


Figure 2: Graphical representation of a vector

Components of Vectors

An important technique when working with vectors mathematically is to break them down into their x and y components. In this example, we will consider the position vector $\overrightarrow{A}$ directed at an angle of 30° from the $+x$-axis and having a magnitude of 8.0 miles. From the head of the vector draw a line perpendicular to the x-axis and a second line perpendicular to the y-axis. We refer to these lines as the projections of the vector on to the x- and y-axes. The projection of the vector on to the y-axis gives the magnitude of the x-component of the vector (green line in Fig. 3 below) and the projection of the vector on the x-axis gives the magnitude of the y-component (red line in Fig. 3).

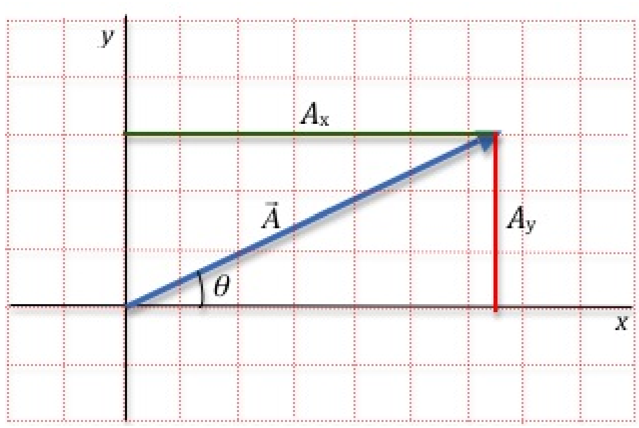


Figure 3: Breaking a vector into x and y components

Note that the green and red lines in the diagram above form two sides of a rectangle with the vector as the diagonal of the rectangle. We can also look at the above situation in two other ways as shown in Fig. 4.

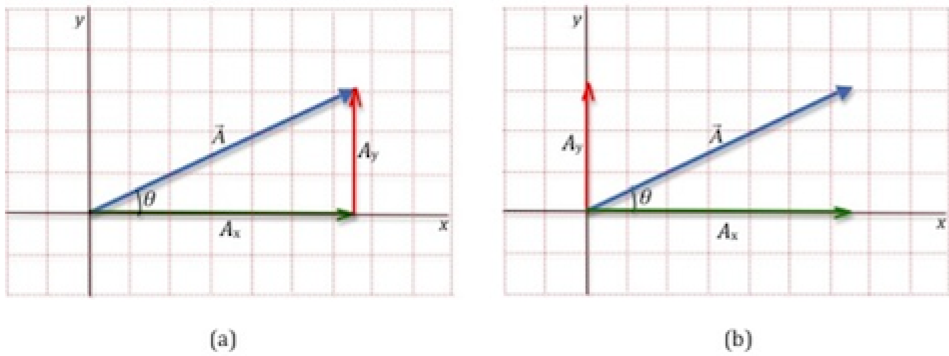


Figure 4: Representing components of a vector

In Fig. 4a we have a right triangle in which the vector is the hypotenuse, the side parallel to the x-axis (green arrow) is the x-component of the vector, and the side parallel to the y-axis (red arrow) is the y-component of the vector. Figure 4b is mathematically equivalent to Fig. 4a, but now A_y is drawn along the y-axis.

Finding components given the magnitude and direction of the vector

We know the direction of the A_x and A_y vectors, but to find their magnitudes we need to use some trigonometric identities. In Fig. 5 the hypotenuse represents the magnitude of the vector $\vec{A}$ and the other two sides of the right triangle represent the x and y components of the vector $\vec{A}$.

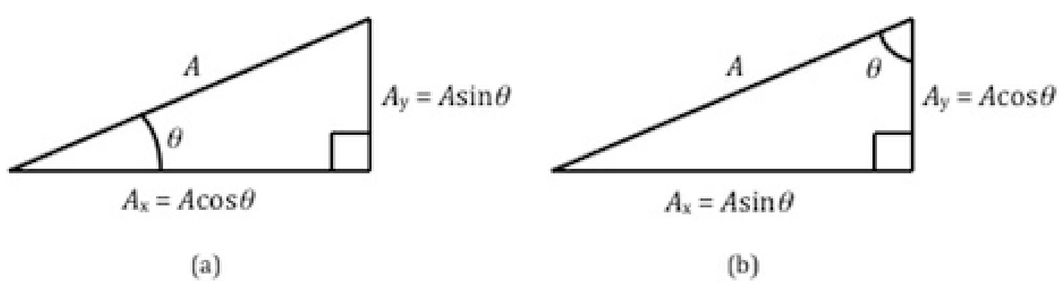


Figure 5: Finding the components of a vector

For any right triangle, we have the following trigonometric identities.

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} \qquad \sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \tag{1}$$

Have the students do this exercise on the graph paper in their notebook - both graphically & analytically

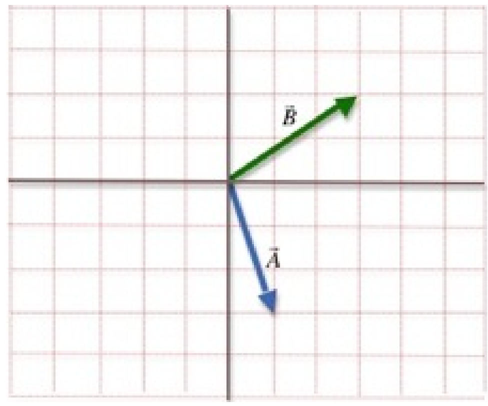


Figure 7: Two vectors

To add two vectors, slide the second vector so that its tail is at the head of the first vector. The sum of the two vectors is a vector drawn from the tail of the first vector to the head of the second vector. In Fig. 8a, $\overrightarrow{B}$ is moved so that its tail is at the head of $\overrightarrow{A}$. Note that the direction of $\overrightarrow{B}$ does not change. The red arrow gives the sum $\overrightarrow{R} = \overrightarrow{A} + \overrightarrow{B}$. Addition is commutative, so you will get the same result by moving $\overrightarrow{A}$ to the head of $\overrightarrow{B}$.

To find the difference of two vectors, we can take the negative of the second vector and add it to the first vector following the steps described above for addition. In other words, $\overrightarrow{A} - \overrightarrow{B} = \overrightarrow{A} + (-\overrightarrow{B})$. This is illustrated in Fig. 8b.

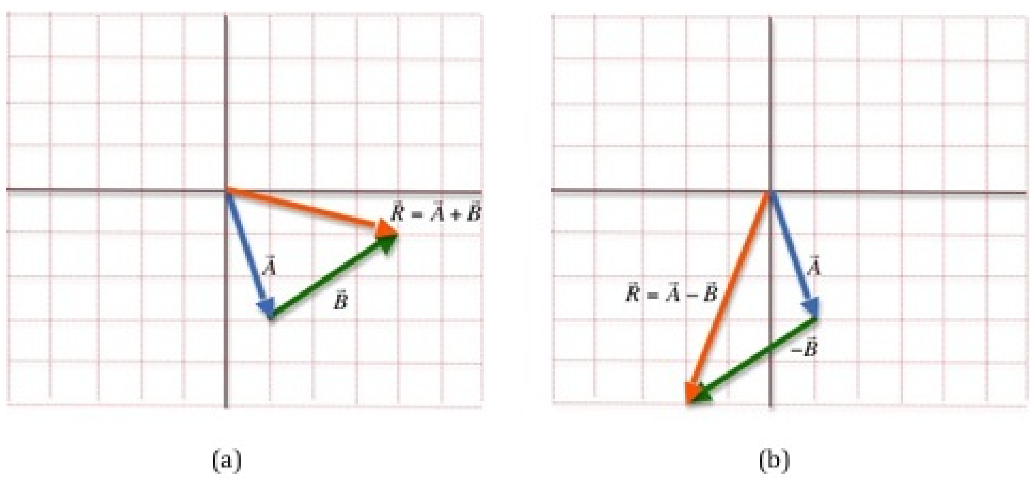


Figure 8: Sum and difference of two vectors

Analytical Method of Adding Vectors

Addition or subtraction of vectors involves breaking up the vectors into its components and then performing the addition or subtraction to the x and y components separately.

$$R_{\text{x}} = A_{\text{x}} + B_{\text{x}} \tag{13}$$

$$R_y = A_y + B_y \tag{14}$$

Now using Eqs. (9) and (12) we can find the magnitude and direction of the resultant vector $\vec{R}$. This process will be the same if you are adding more than two vectors or subtracting vectors.

Example $A_x = 1$ and $B_x = 3$ giving $A_x + B_x = 4$; $A_y = -3$ and $B_y = 2$ giving $A_y + B_y = -1$.

In the case of subtraction $A_x - B_x = -2$; $A_y - B_y = -5$.

Parallelogram Rule for adding Vectors

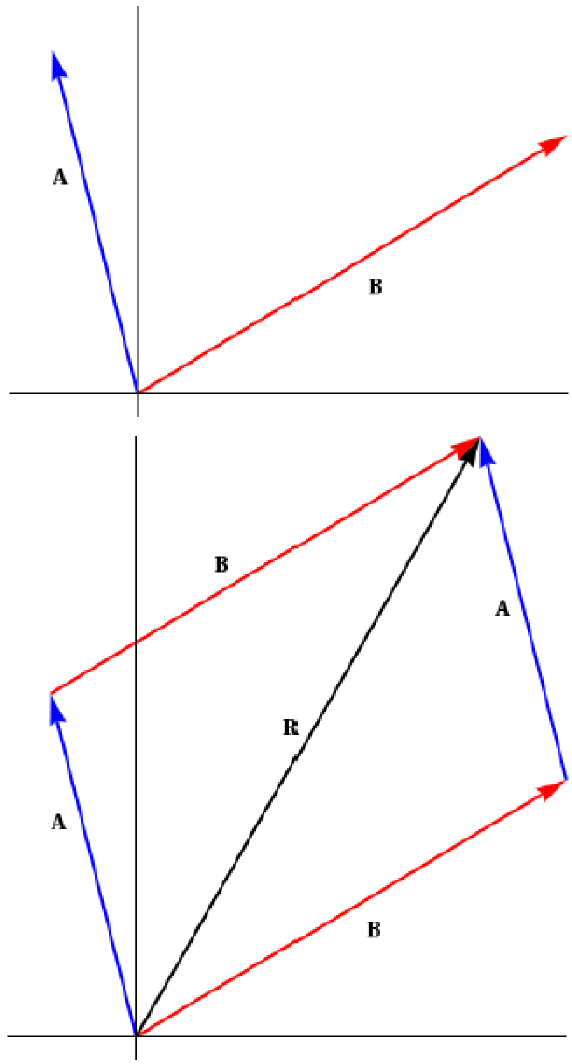


Figure 5: Graphical vector addition of two vectors

Although this graphical method, in which successive vectors are placed head-to-tail, is useful as a visual description, it is limited in precision by that which can be obtained by the drawing instruments. When results more accurate than those provided by graphical analyses are required, analytical methods are applied.

In order to use analytical methods for vector addition, all vectors are described through the use of unit vectors. A unit vector is a vector having a magnitude of one (unaccompanied by any units) with a set orientation. Its only use is as a description of a specific direction in space. In the x-y coordinate system, it is the usual practice to assign the unit vector $\hat{\mathbf{i}}$ in the direction of the positive x-axis and the unit vector $\hat{\mathbf{j}}$ in the direction of the positive y-axis.

Any single vector can be expressed as the sum of two (or more) component vectors. In Figure 4, the vector $\mathbf{A}$ has a magnitude of A and an orientation of θ from the x-axis.

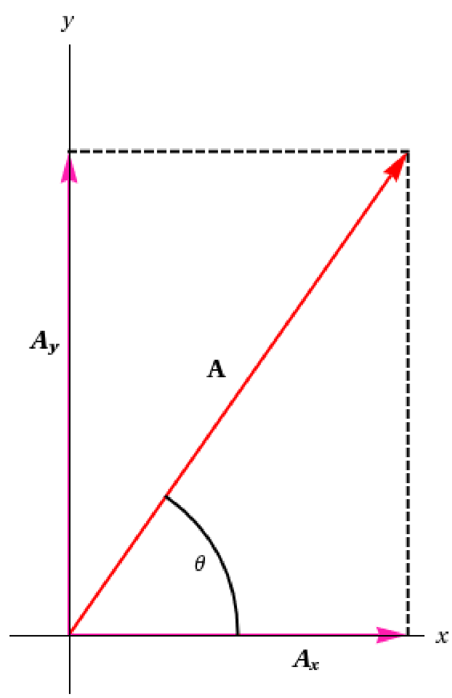


Figure 4: Vector $\mathbf{A}$ graphically resolved into components $\mathbf{A_x}$ and $\mathbf{A_y}$.

This vector can be considered to be the resultant of two component vectors: $\mathbf{A_x} = A_x\hat{\mathbf{i}}$ pointing along the $\hat{\mathbf{i}}$ direction and $\mathbf{A_y} = A_y\hat{\mathbf{j}}$ pointing along the $\hat{\mathbf{j}}$ direction; or symbolically as follows.

$$\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} \tag{3}$$

$$\mathbf{A} = \mathbf{A_x} + \mathbf{A_y} \tag{4}$$

The component $\mathbf{A_x}$ is positive or negative as A_x is in the $+\hat{\mathbf{i}}$ or $-\hat{\mathbf{i}}$ direction, and $\mathbf{A_y}$ is positive or negative as A_y is in the $+\hat{\mathbf{j}}$ or $-\hat{\mathbf{j}}$ direction.

In Figure 6, the vectors **C** and **D** are added analytically. In mathematical terms, the vectors can be written

$$\mathbf{C} = C_x\hat{\mathbf{i}} + C_y\hat{\mathbf{j}} \tag{12}$$

$$\mathbf{D} = D_x\hat{\mathbf{i}} + D_y\hat{\mathbf{j}} \tag{13}$$

$$\mathbf{R} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}}. \tag{14}$$

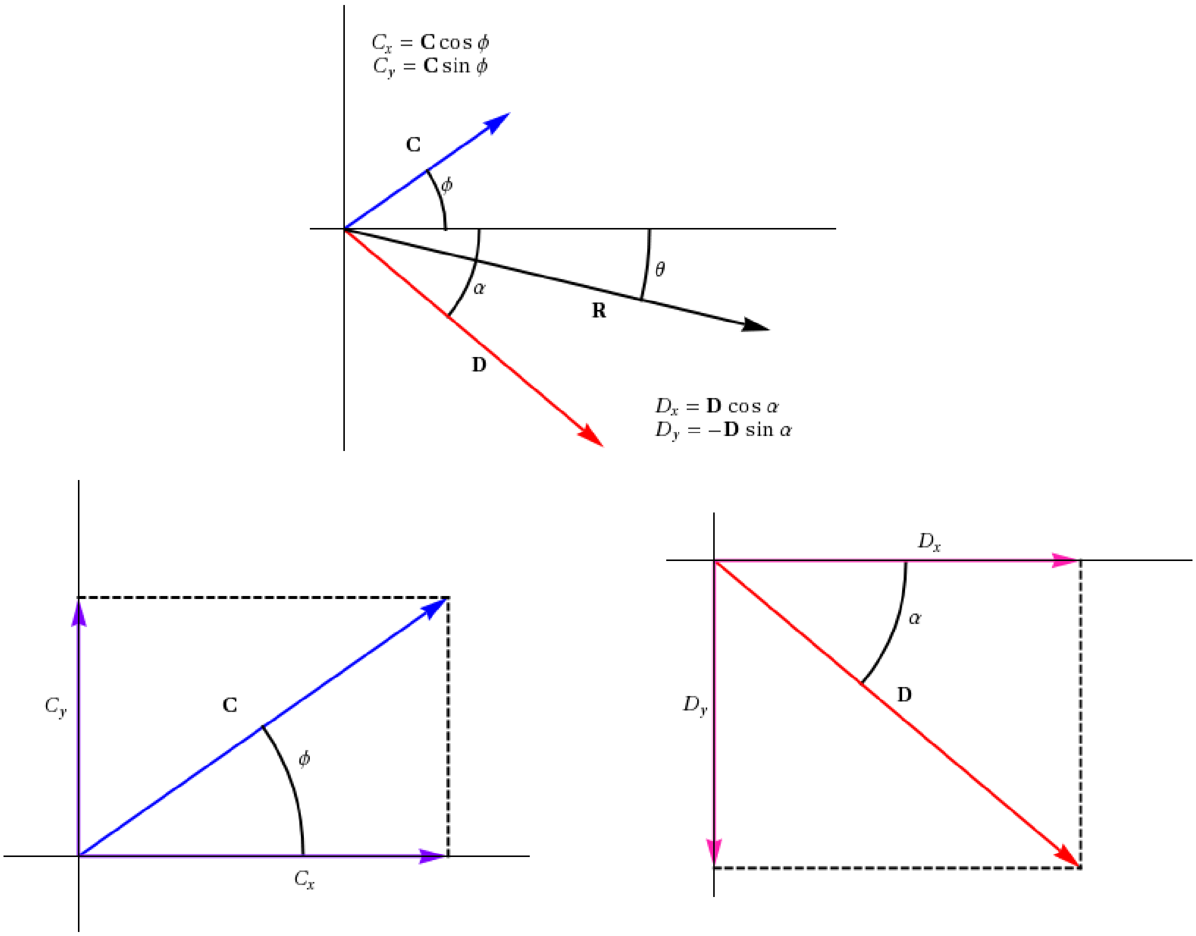


Figure 6: Vectors **C** and **D** resolved into components graphically and analyically, along with their resultant vector **R**.

Since the resultant, **R**, is the sum of the vectors **C** and **D**

$$\mathbf{R} = \mathbf{C} + \mathbf{D} \tag{15}$$

$$\mathbf{R} = C_x\hat{\mathbf{i}} + D_x\hat{\mathbf{i}} + C_y\hat{\mathbf{j}} + D_y\hat{\mathbf{j}} \tag{16}$$

$$\mathbf{R} = (C_x + D_x)\hat{\mathbf{i}} + (C_y + D_y)\hat{\mathbf{j}}. \tag{17}$$

A comparison with equation (14) gives the values for the x-and y-components of the resultant:

$$R_x = C_x + D_x \tag{18}$$

$$R_y = C_y + D_y. \tag{19}$$

From the components of **R**, the magnitude and direction of the resultant are

$$|\mathbf{R}| = \sqrt{R_x^2 + R_y^2} \tag{20}$$

$$\theta = \arctan \tfrac{R_y}{R_x}. \tag{21}$$

If more than two vectors are being added, equations (18) and (19) can be generalized to

$$R_x = A_x + B_x + C_x + D_x + \cdots \tag{22}$$

$$R_y = A_y + B_y + C_y + D_y + \cdots \tag{23}$$

and the relationships of equations (20) and (21) can be applied to obtain the magnitude and direction of the resultant.

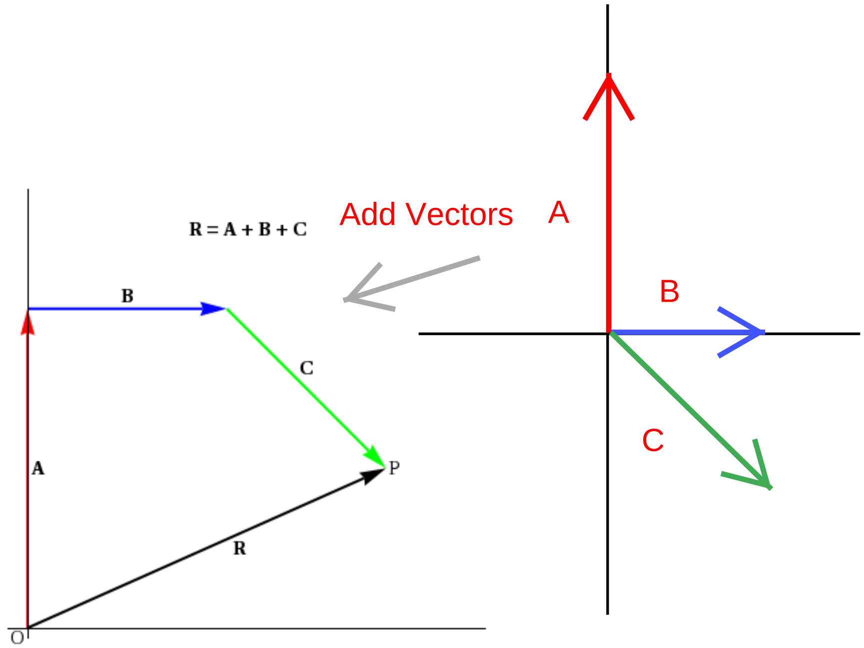


Figure 2: Graphical addition of vectors **A**, **B** and **C**

It should be noted that in this case all paths that represent movement from O to P are equivalent to **R** and to each other. So in Figure 3,

$$\mathbf{R} = \mathbf{A} + \mathbf{B} + \mathbf{C} = \mathbf{C} + \mathbf{A} + \mathbf{B} = \mathbf{C} + \mathbf{D}. \tag{2}$$

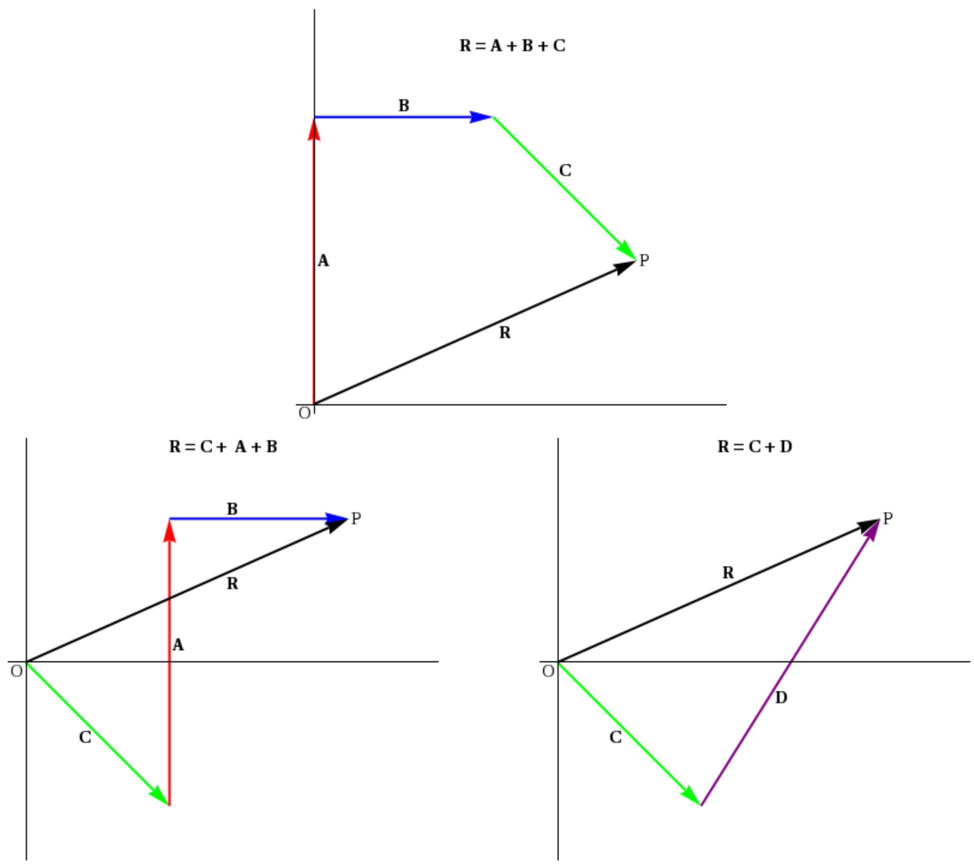


Figure 3: The same resultant **R** is obtained regardless of the order in which the vectors are added.

Have the students do these two problems
in lab both graphically and analytically

Problem 1 **3-Put – An example in Vector Addition (or poor golf skills)**

A golfer, putting on a green requires three strokes to "hole the ball." During the first putt, the ball rolls 5.0 m due east. For the second putt, the ball travels 2.1 m at an angle of 20° north of east. The third putt is 0.50 m due north. What displacement (magnitude and direction relative to due east) would have been needed to "hole the ball" on the very first putt? Use components to solve this problem.

Solution Identify the three vectors.
Sketch the vectors and show the vector sum. Include a coordinate system.

Identify the components of the three vectors (labeled *a*, *b*, *c*)

$a_x =$ $a_y =$

$b_x =$ $b_y =$

$c_x =$ $c_y =$

Determine the components of the resultant vector (labeled *s*)

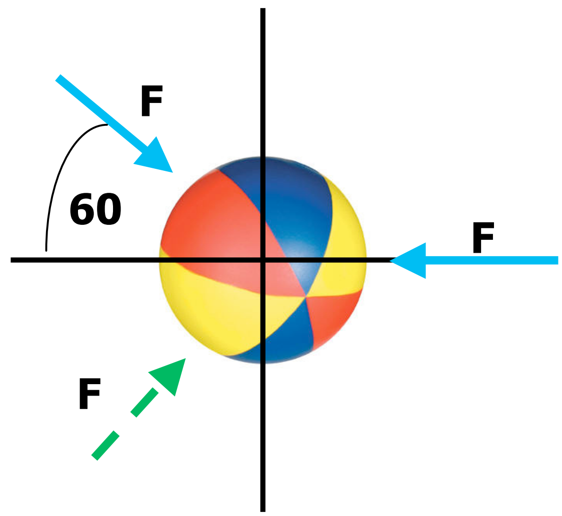
$s_x =$ $s_y =$

Convert this into the magnitude and direction of the resultant vector

$|s| =$

$\theta =$ (measured from the positive *x* axis)

Problem 2 At a picnic, there is a contest in which hoses are used to shoot water at a beach ball from three different directions. As a result, three forces act on the ball, $\mathbf{F_1}$, $\mathbf{F_2}$, and $\mathbf{F_3}$ (see drawing). The magnitudes of $\mathbf{F_1}$ and $\mathbf{F_2}$ are $F_1 = 50.0$ N and $F_2 = 90.0$ N. F_1 acts under an angle of 60° with respect to the x-axis and F_2 is directed along the x-axis. Find the magnitude and direction of $\mathbf{F_3}$ such that the resultant force acting on the ball is zero.



Use 3 holders and arbitrarily find masses and angles that center the ring. Give students two of the masses and angles and let them find the third mass and angle.

OBJECTIVE

The objective of this experiment is to find the equilibrant of one or more known forces using a force table and compare the results to that obtained by analytical method.

EQUIPMENT

Force table
Ruler
Strings
Weight hangers
Assorted weights
Bubble level

PROCEDURE

Given two force vectors you will determine the third force that will produce equilibrium in the system. This third force is known as the equilibrant and it will be equal and opposite to the resultant of the two known forces.

You will use a force table as shown in Fig. 9, and work with force vectors. The force table is a circular platform mounted on a tripod stand. The three legs of the tripod have adjustable screws that can be used to level the circular platform. The circular platform has angle markings, in degrees, on its surface. Two or more pulleys can be clamped at any location along the edge of the platform. In this lab we will use three pulleys. Three strings are attached to a central ring and then each string is passed over a pulley. Masses are added to the other end of the strings.

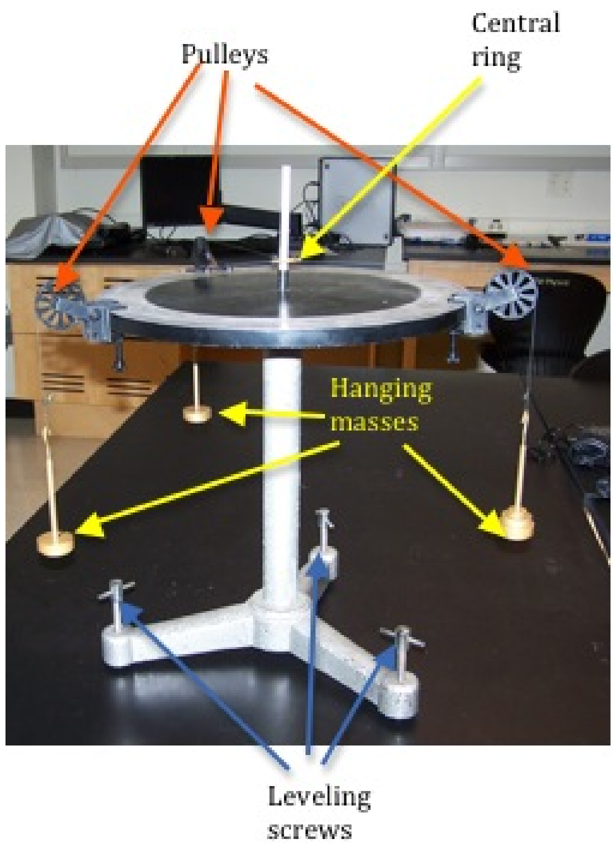


Figure 9: Force table

The hanging masses will produce a tension force in each string. The masses are directly proportional to the gravitational force (which you will learn about later in the course). The tension force in each string is equal to the gravitational force. For example, doubling the mass doubles the force, etc. When the forces are balanced, the ring will be positioned at the exact center of the table. When the forces are not balanced, the ring will rest against one side of the central post.

***Note*:** The force due to each hanging mass will be mg where g is the acceleration due to gravity.

To make it easier to read the angles, assume the x-axis to be from the 180° mark to the 0° mark, with 0° being the positive x direction, and the y-axis to be from the 270° mark to the 90° mark with 90° being the positive y direction. See Fig. 10.

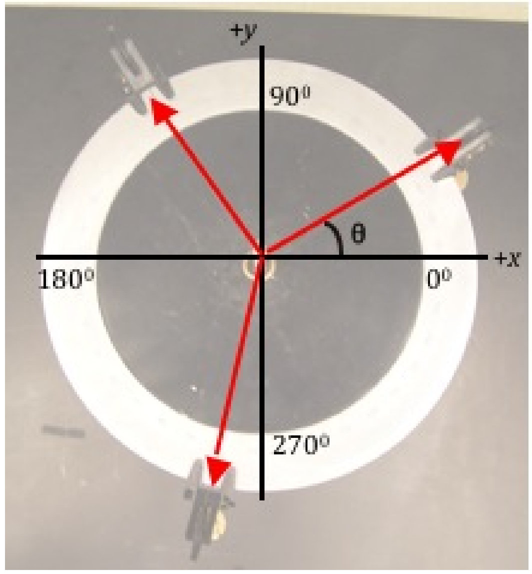


Figure 10: Force table with axes

Procedure A: Finding the Equilibrant of Two Known Forces

1. Use the bubble level to check if the circular platform is horizontal. Use the leveling screws, if necessary, to make the necessary adjustments.

2. You are given two 150 g masses that are to be placed at 60° and 300°.

Remember that the weight hangers have a mass of 50 g each and this needs to be included as part of the hanging mass.

You will determine the magnitude (in newtons) and angle of the third force needed to balance the forces due to these two masses. You will do so by following the instructions below.

3. Draw a Cartesian coordinate system in your lab notebook, and draw the two force vectors on it including the angles.

Each vector in the diagram should be drawn so that the larger the vector the bigger the force it represents.

4. Calculate the x and y components (to the nearest thousandth of a newton) and enter these values in your lab notebook.

5. Find the x and y components of the resultant of the two vectors (the sum of the x and y components) and enter these values in your lab notebook.

This final result is the 3rd force (just like Problem 2 on page 12 above). Find the magnitude and angle of this 3rd force.

8. Add this vector to your Cartesian coordinate system in your lab notebook --> this is your 3rd force that balances out the other two forces and causes the centering ring to be stationary.

9. Position the third string at the angle you determined in step 8 and hang the mass (including the hanger mass) corresponding to the calculated third force to represent the third force.

Make adjustments (if needed) to the mass and the angle until the ring is at the center. Record this value in your lab notebook

10. Compare the calculated and experimental values for the third force by computing the percent difference between the two values.

11. Compare the calculated and experimental values of the angle for the third force by computing the percent difference between the two angle values.

<u>CHECKPOINT 1</u>: Ask your TA to check your diagram, calculations and the set-up on the force table.

For your lab writup: Take cell phone pictures of your lab notebook with all your calculations and upload it to Blackboard. No additional writup is needed.