NORTHERN ILLINOIS UNIVERSITY

PHYSICS DEPARTMENT

Physics 210 – Mechanics & Heat  Fall 2022

Lab #8

Lab Writeup Due: Tue/Wed/Thu, Oct. 25/26/27, 2022
Read OpenStax: Chapter 7.1-7.6, Lecture Notes #7

# Ballistic Pendulum

## Apparatus

A ballistic pendulum is a device consisting of three parts: a spring gun, a ball that can be launched from the gun, and a cup at the end of a pendulum to catch the ball. The spring gun is designed to fire a ball of mass $m_b$ with an initial velocity $v_i$. The pendulum and cup can be moved out of the way. This permits the ball to be fired as a projectile and the initial velocity measured.

When the pendulum is in its lower position, the cup with mass $m_c$ is ready to catch the ball when fired. When the ball is caught in the cup, the energy of the combined cup and ball is used to swing the pendulum up by a final height $h_f$ *above initial position of the center of mass of the combined cup and ball*. A ratchet catches the cup and allows you to read a position measurement by means of a pointer that catches in a groove of the ratchet.

The measurement on the ratchet is marked in units that go from 0 to 40. There are marks at the halfway points (5, 15, 25, 35), and individual ratchet positions represent steps of 1. The ratchet measure can be converted to height by means of the following table:

**Table 1: Conversion of ratchet measurement to height above the base of the ballistic pendulum**

| **Ratchet Measurement** | **Height** (mm) |
|---|---|
| 0 | 65 |
| 10 | 73 |
| 20 | 80 |
| 30 | 88 |
| 40 | 96 |

To find the conversion for other ratchet measurements requires interpolation. Interpolation means estimating the value between two known values. We do this by assuming that the conversion function is a straight line between the two known points. The two points are used to create the equation of a straight line similar to $y = mx + b$. The actual equation will have offsets because the line is only calculated between two points.

For our specific table, we treat the ratchet measurement $x$ as the independent variable, and the height of the ball $h_f$ as the dependent variable. To interpolate, look at your measurement of $x$. If it's not in the table, find the measurement, $x_1$, in the table that is immediately below your measurement and its corresponding height $h_1$. Find the measurement, $x_2$, in the table that is immediately above your measurement and its corresponding height $h_2$. The interpolated height $h_f$ for your measurement is

$$h_f = h_1 + \left(\frac{x - x_1}{x_2 - x_1}\right)\left(h_2 - h_1\right) \tag{1}$$

## Theory

Kinetic energy is the energy of motion. It depends on the mass and velocity of an object. The total kinetic energy of the initial state of the system is

$$K_i = \frac{1}{2} m_b v_i^2 \tag{2}$$

since the cup is stationary when the ball is launched. The total gravitational potential energy of the initial state of the system is

$$U_i = \left(m_b + m_c\right) g h_i = 0 \tag{3}$$

since the ball and cup is defined to have zero gravitational potential energy (that is, $h_i \equiv 0$). Using the principle of *conservation of momentum* (which we will get to later in the course), the velocity $v_{bc}$ instantly after the ball is caught by the cup is given by

$$v_{bc} = v_i \left(\frac{m_b}{m_b + m_c}\right) \tag{4}$$

Using Eq. (5), the total kinetic energy of this intermediate state (instantly after the ball is caught by the cup) is

$$K_{bc} = \frac{1}{2}(m_b + m_c)\left(\frac{m_b}{m_b + m_c}\right)^2 v_i^2 = \frac{1}{2}\frac{m_b^2 v_i^2}{(m_b + m_c)} \quad (5)$$

The total potential energy of this intermediate state, $U_{bc} = U_i = 0$, is still given by Eq. (4) since the ball & cup have not yet started to rise. The total gravitational potential energy of the final state is

$$U_f = (m_b + m_c) g h_f \quad (6)$$

where we have used the total mass of the ball and cup. Note that the total kinetic energy of the final state is zero since the ball & cup come to rest:

$$K_f = 0 \quad (7)$$

Conservation of energy is a fundamental principle of physics—it predicts that the kinetic energy can be fully converted into potential energy.

## Data Collection

***You must show all your calculations to your TA before you leave the lab***.

**<u>Conservation of energy</u>**

(1) Weigh and record in your lab notebook the mass of the ball $m_b$, and record the mass of cup $m_c$ written on the apparatus.

(2) Measure the diameter, $d$, of the ball using the vernier calipers. Record this value in your lab notebook.

(3) Insert the ball into the spring gun, then slide the photogate in front of the ball. Start Logger Pro and fire the gun with the ball into the cup. Record the ratchet measurement, $x$, and the two times measured by photogate.

(4) Calculate the initial velocity, $v_i$, of the ball using its diameter and the two times measured by the photogate.

(5) Use the interpolation formula in Eq. (1) and the data in Table 1 to find the height $h_f$.

(6) Repeat Steps (3) to (5) five times to give you five trial measurements.

(7) Calculate the initial kinetic energy $K_i$, the intermediate kinetic energy $K_{bc}$, and the final potential energy $U_f$ for each trial.

(8) Calculate the total initial energy of the system: $E_{Total}^{initial} = K_i + U_i$, the total intermediate energy of the system: $E_{Total}^{intermediate} = K_{bc} + U_{bc}$, and the total final energy of the system: $E_{Total}^{final} = K_f + U_f$ for each trial.

(9) Compute a percent difference for each trial between $E_{Total}^{initial}$ and $E_{Total}^{final}$ where

$$\%Difference = \frac{|measurement\#1 - measurement\#2|}{½(measurement\#1 + measurement\#2)} \times 100$$

Note that we must use the "*percent difference*" relationship to compare the two values (rather than the "*percent error*" described in Lab#2) because neither measurement is a commonly accepted value of the energy.

(10) Compute a percent difference for each trial between $E_{Total}^{intermediate}$ and $E_{Total}^{final}$.

## Data Analysis

(a) What parts of the determination of the initial velocity in Step (4) were likely to have the greatest source of error? How similar were the velocities for each trial?

(b) What effects might account for the different positions of the ratchet for each trial? Should the conservation of energy still hold even though the ratchet positions are different?

(c) From Steps (13) & (14), explain, using your *%Differences*, which energy conservation relation is more in agreement?:

**(1st)** $E_{Total}^{initial} = E_{Total}^{final}$ or **(2nd)** $E_{Total}^{intermediate} = E_{Total}^{final}$

(d) The **1st** relation above states that kinetic energy is converted into potential energy:

$$\frac{1}{2} m_b v_i^2 = (m_b + m_c) g (h_f - h_i)$$

Why is this statement patently false *for this particular experiment*?

(e) What might be the cause for the discrepancy in energy conservation for the **2nd** relation above?