Heat and Calorimetry

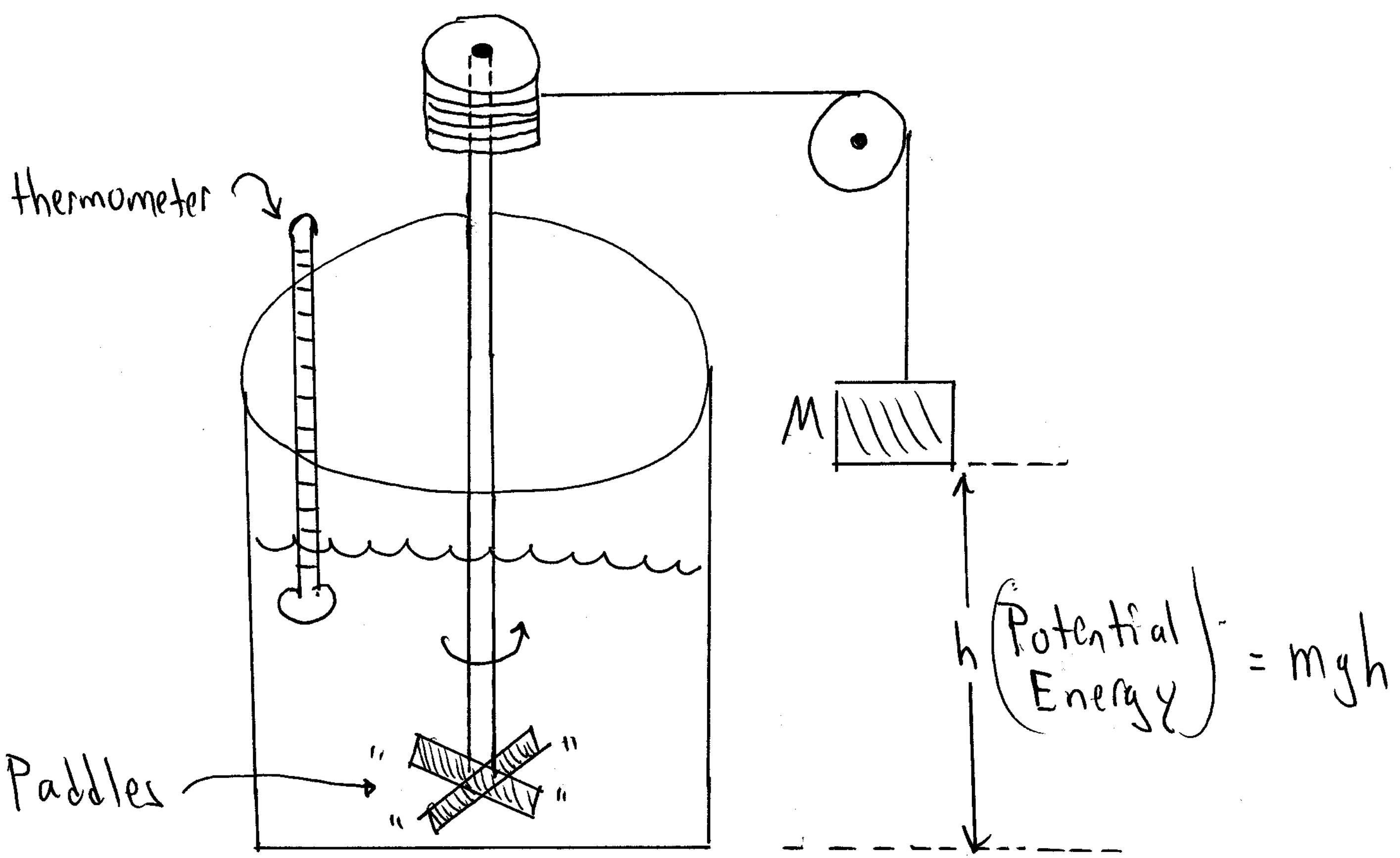
Unit #11

Heat:

What is Heat? James Joules (1840) found that heat is just a form of energy

heat = energy (unit = Joules = J)

Let's examine a "Joules experiment" that demonstrates this:



As the weight falls → the water heats up (due to friction bettween paddles and water).

Joules found that the work, W, done in moving an object a distance h is equal to the heat, Q, produced:

$$W = \begin{pmatrix}\text{Potential}\\ \text{energy}\end{pmatrix} = Mgh = Q$$

$$W = \begin{pmatrix}\text{mechanical}\\ \text{energy}\end{pmatrix} \Rightarrow \text{has been transformed} \Rightarrow \begin{pmatrix}\text{into}\\ \text{heat}\end{pmatrix} = Q$$

Q = Heat = Thermal Energy

Units = Joules (mks Units)

English Units: Calorie (cal)

Conversion factor: 1 cal = 4.186 J

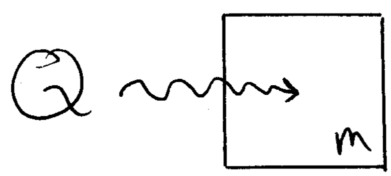
Note: Heat = Energy

Temperature ≠ Energy

temperature is related to energy

$$\Rightarrow \begin{pmatrix}\text{average}\\ \text{kinetic}\\ \text{energy}\end{pmatrix} = \left(\tfrac{1}{2}mv^2\right)_{ave} = \tfrac{3}{2}k_B T$$

Relation between Heat and Temperature:



Let heat energy enter an object of mass m.

Question: How much does its temperature rise?

The greater the amount of heat energy that enters, the greater the change in temperature. Then, heat, Q, is directly proportional to the temperature change

$$\Delta T \sim Q$$

(increases temperature) ← (increase heat) (mass is kept constant)

If we input the same amount of heat, but this time increase the mass of the object, its temperature will not rise as high since now we have to heat a larger object. Then the mass is inversely proportional to the temperature change

$$\Delta T \sim \frac{1}{m}$$

(Q is kept constant)

(temperature does not increase as much) ← (increase mass)

We can then write the relation:

$$\Delta T \sim \frac{Q}{m}$$

$\leftarrow$ as Q increases, ΔT increases

$\leftarrow$ as m increases, ΔT decreases

or

$$Q \sim m\Delta T$$

(putting m on the other side of the relation)

We can now form a phenomenological equation:

$$Q = \left(\begin{matrix}\text{some} \\ \text{constant}\end{matrix}\right) \cdot m\Delta T$$

$\uparrow$

this constant is called

c = specific heat capacity

$$\boxed{Q = mc\Delta T}$$

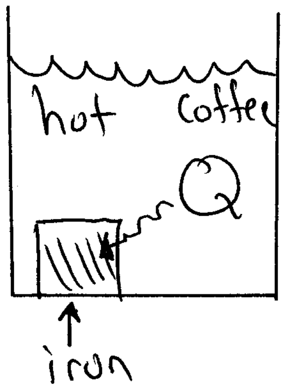
$\leftarrow$ {Relation between heat energy and Temperature}

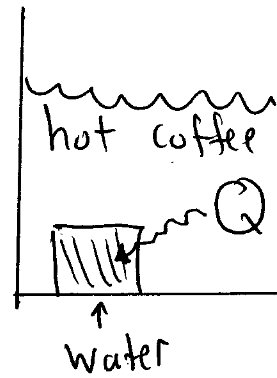
For water: $c = 4186\ J/Kg\,°C$

For iron: $c = 448\ J/Kg\,°C$

notice: It takes a lot more heat to change 1 Kg of water 1°C than to change 1 Kg of iron 1°C.

Example: Let's say we want to cool a cup of coffee. We have 10 grams of iron balls and 10 grams of water. Should we use the iron balls or the water to cool the coffee?





If the iron and water are initially at room temperature, then heat will flow from the coffee to the iron or water, thus cooling down the coffee.

$$\begin{pmatrix}\text{Heat flow from}\\ \text{coffee to iron}\end{pmatrix} = Q_{iron} = m_{iron} C_{iron} \Delta T_{iron}$$

$$\begin{pmatrix}\text{Heat flow from}\\ \text{coffee to water}\end{pmatrix} = \underbrace{Q_{water} = m_{water} C_{water} \Delta T_{water}}_{}$$

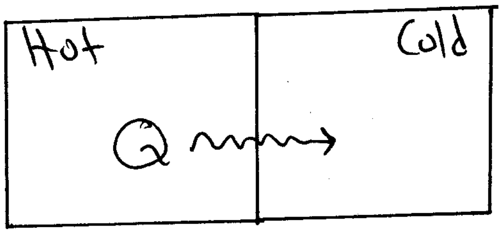
but $C_{water} > C_{iron}$

thus $\underbrace{Q_{water}}_{} > Q_{iron}$

a lot more heat goes into the water

$\Rightarrow$ the water cools down the coffee more since it has the larger specific heat.

Zeroth Law of Thermodynamics:



Heat always flows from the hotter object to the colder one.

If both objects have the same temperature, no heat flows → the objects are in thermal equilibrium.
This is the Zeroth law of Thermodynamics.

Notice: Q = Heat = Thermal energy = (average Kinetic energy)

$= \left(\frac{1}{2}mv^2\right)_{ave}$

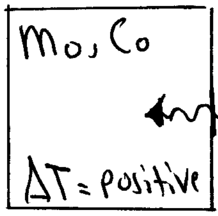
Since Kinetic energy can never be negative, Q can never be a negative number.

Q = positive # always

But if you examine the formula: $Q = mc\Delta T$

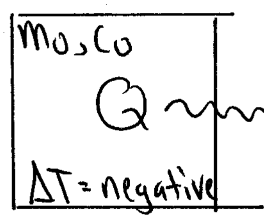
$T_f - T_i = \Delta T$ is negative if $T_f < T_i$

Let's look at this in terms of the heat flow from or into an object of mass m_o:



Q = heat entering object

$= + m_o C_o \Delta T$

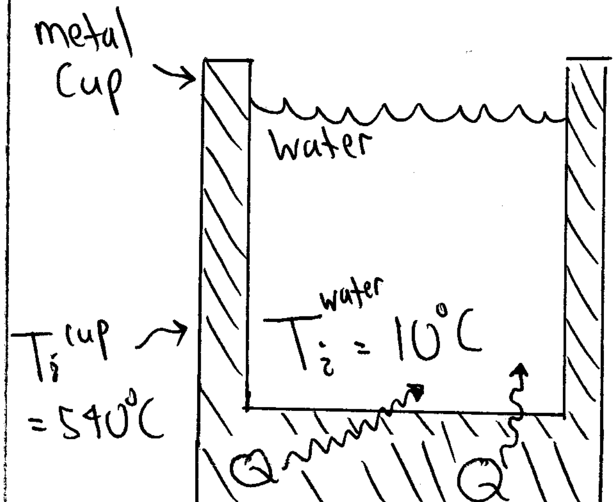


Q = heat leaving object

$= - m_o C_o \Delta T$

When heat leaves an object, use the minus sign to make Q a positive number.

Example: Water in a hot metal cup



$C_{water} = 4186\ J/Kg\,°C$

$C_{cup} = 502\ J/Kg\,°C$

$m_{water} = 400$ grams

$m_{cup} = 150$ grams

$T_i^{water} = 10°C$

$T_i^{cup} = 540°C$

(a) What is the final temperature of the cup and the water when they are in thermal equilibrium?

→ Note: Q is leaving the cup, so we must be careful with signs.

Apply conservation of energy

Heat lost by Cup = Heat gained by water

$$Q_{cup} = Q_{water}$$

$$-m_{cup} C_{cup} \Delta T_{cup} = m_{water} C_{water} \Delta T_{water}$$

minus sign since heat is leaving the cup

$$-m_{cup} C_{cup} \left(T_f^{cup} - T_i^{cup}\right) = m_{water} C_{water} \left(T_f^{water} - T_i^{water}\right)$$

in thermal equilibrium $T_f^{cup} = T_f^{water} \equiv T_f$

Then,

$$-m_{cup} C_{cup} T_f + m_{cup} C_{cup} T_i^{cup} = m_{water} C_{water} T_f - m_{water} C_{water} T_i^{water}$$

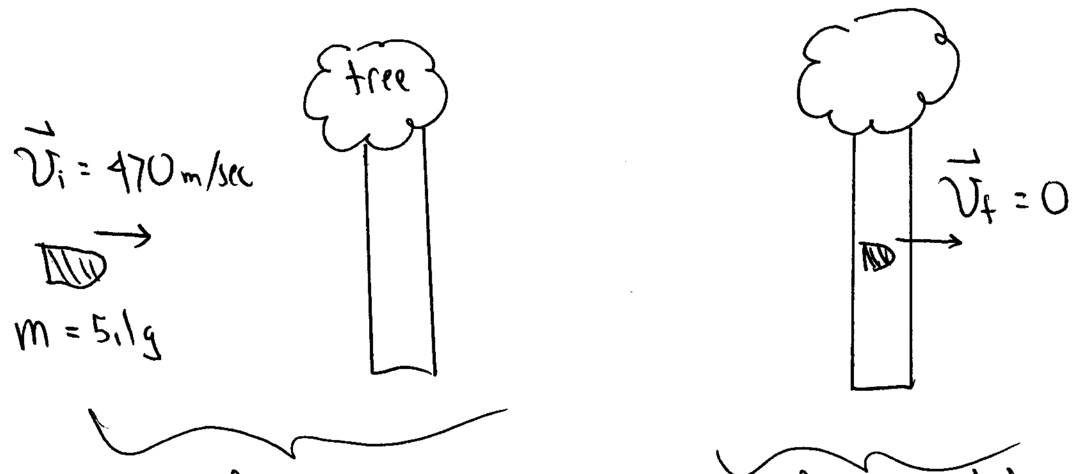
$$T_f = \frac{m_{cup} C_{cup} T_i^{cup} + m_{water} C_{water} T_i^{water}}{m_{water} C_{water} + m_{cup} C_{cup}}$$

$$= \frac{(0.15\,Kg)(502\,J/Kg\,°C)(540°C) + (0.4\,Kg)(4186\,J/Kg\,°C)(10°C)}{(0.4\,Kg)(4186\,J/Kg\,°C) + (0.15\,Kg)(502\,J/Kg\,°C)}$$

$$= 32.81\,°C$$

Example: A 5.1 g lead bullet having a velocity of 470 m/sec hits a tree and gets lodged inside. If half of its kinetic energy goes into heating up the bullet, what is the increase in temperature of the bullet?

Note: $C_{lead} = 128\ J/Kg°C$.



before collision — after collision

$\frac{1}{2}$ (Kinetic energy of bullet) $\longrightarrow$ internal (heat) energy

$\frac{1}{2}$ (Kinetic energy of bullet) $\longrightarrow$ $\left\{ \begin{array}{l} \text{heating tree up} \\ \text{sound energy} \\ \text{vibrational energy in tree} \\ \text{etc.} \end{array} \right.$

$$Q_{bullet} = mc\Delta T = \frac{1}{2} K_E = \frac{1}{2}\left(\frac{1}{2} m v_i^2\right)$$

$$\Delta T = \frac{\frac{1}{4}\cancel{m} v_i^2}{\cancel{m}c} = \frac{v_i^2}{4c} = \frac{(470\ m/sec)^2}{4(128\ J/Kg°C)}$$

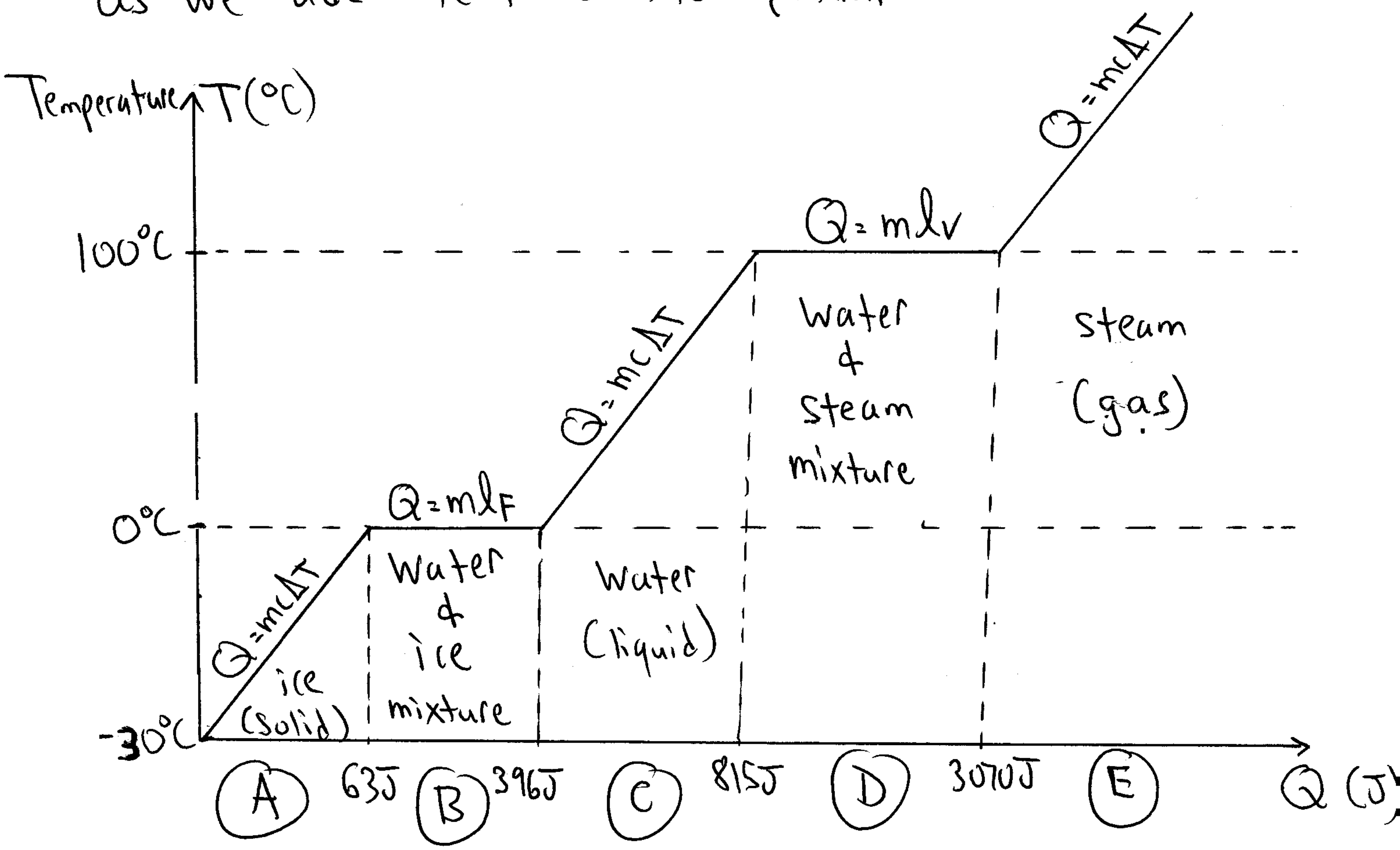
$$= 431°C$$

Phase Changes & Latent Heat:

Let's examine water as it progresses from

ice → water → steam

as we add heat to the system.



(A) → the solid phase of water (ice) gradually rises in temperature as we add heat → the heat gained = $Q = mc\Delta T$

(B) → when the ice begins to melt, its temperature no longer rises → all the heat energy goes into breaking the chemical bonds between the ice molecules
→ the heat gained = $Q = m\ell_F$

where l_F = latent heat of fusion = heat required to change 1 Kg of material from the solid to the liquid state.

For water: $l_F = 334{,}000 \ J/Kg$

C → after all the ice has melted, the temperature of the water steadily increases → the heat gained $= Q = mc\Delta T$

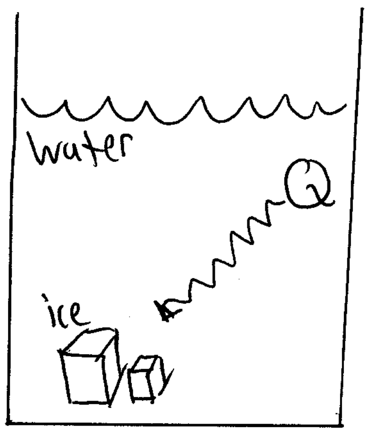
D → when the water begins to vaporize, its temperature no longer rises → all the energy goes into breaking the chemical bonds between the water molecules → the heat gained = $Q = m l_v$

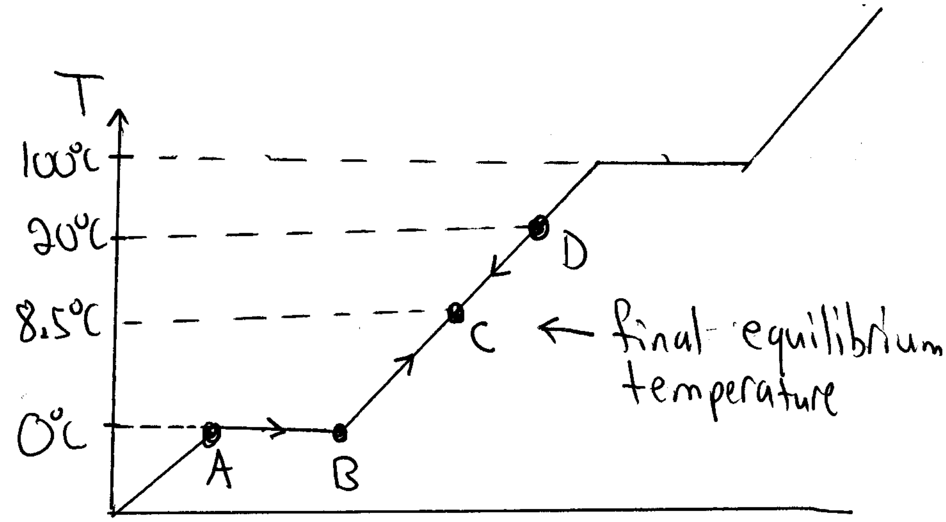
where l_v = latent heat of vaporization = heat required to change 1 Kg of material from the liquid to the vapor state.

For water: $l_v = 2{,}260{,}000 \ J/Kg$

E → after all the water has vaporized, its temperature gradually rises → the heat gained $= Q = mc\Delta T$

Example: Find the latent heat of fusion when ice is added to water in a calorimeter. You are give the following information:





$$\left\{\begin{array}{l} m_{ice} = 30g \\ m_{water} = 200g \\ m_{cup} = 300g \end{array}\right\} \quad \begin{array}{l} T_i^{ice} = 0°C \\ T_i^{water} = 20°C \end{array}$$

Final equilibrium temperature $= T_f = 8.5°C$

$$\left\{\begin{array}{l} C_{water} = 4.186\ J/g°K \\ C_{cup} = 0.38\ J/g°K \end{array}\right\}$$

ice goes from $A \to B \to C$ (it gains heat)

Water goes from $D \to C$ (loses [or releases] heat).

Conservation of Energy:

$$\underbrace{m_i l_F + m_i C_w \Delta T^{w}_{B\to C}}_{\text{ice gains heat energy}} = \underbrace{-m_w C_w \Delta T^{w}_{D\to C} - m_c C_c \Delta T^{c}_{D\to C}}_{\text{water and cup loses heat energy}}$$

$$m_i l_F + m_i C_w (8.5°C - 0°C) = m_w C_w (8.5°C - 20°C) + m_c C_c (8.5°C - 20°C)$$

$$m_i l_F = -m_i C_w (8.5°K) - m_w C_w (-11.5°K) - m_c C_c (-11.5°K)$$

$$l_F = \frac{-30g(4.186 J/g°K)(8.5°K) + (200g)(4.186 J/g°K)(11.5°K) + (300g)(0.38 J/g°K)(11.5°K)}{30g}$$

$$l_F = 329.0 \ J/g$$