# Temperature & Ideal Gas

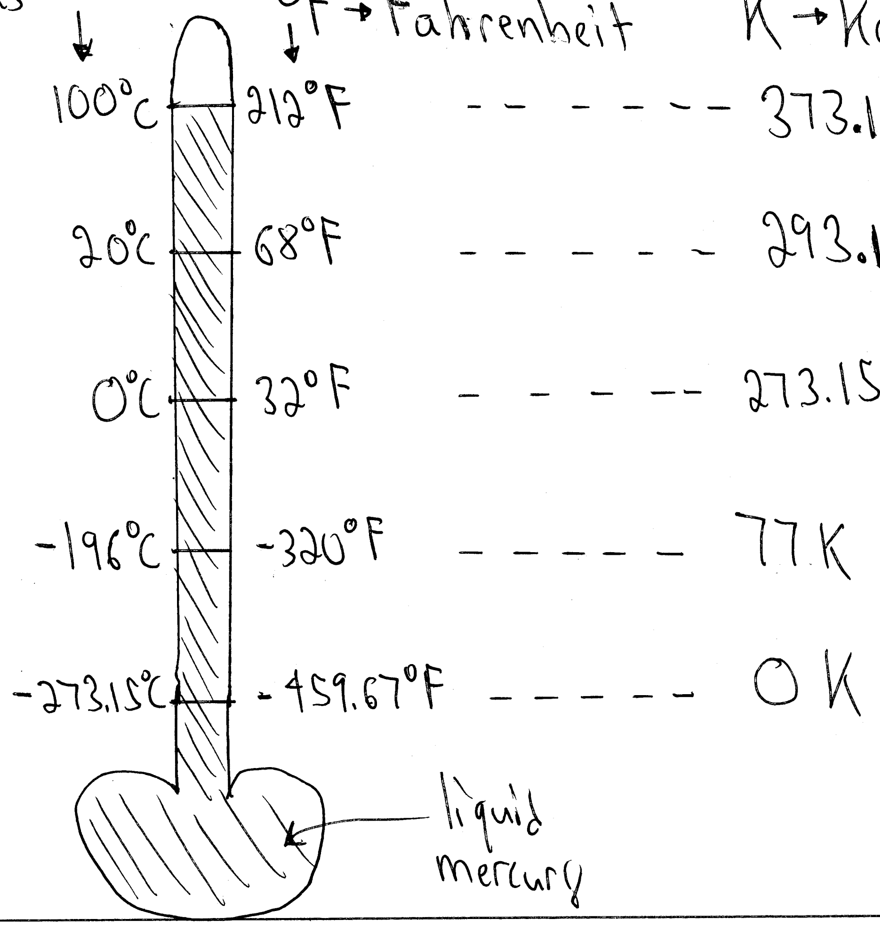


**Temperature**: A quantitative measure of warmth

temperature = comparing whether something is hotter or colder than another object.

One way to measure temperature is with a mercury thermometer: the liquid will expand or contract with temperature ⇒ the height of mercury inside a glass tube is then proportional to temperature

Celcius → °C   °F → Fahrenheit   K → Kelvin




**Conversion Factors:**

$°C = (°F - 32) \cdot 5/9$

$°F = 9/5 \cdot °C + 32$

Approximations: $°C \approx (°F/2 - 15)$

$°F \approx (°C + 15) \cdot 2$

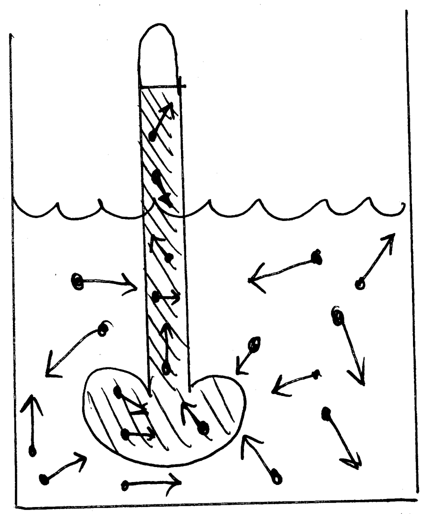
$°K = °C + 273.15$

$80°F \approx 25°C$

$25°C = 77°F$

Note: the thermometer is in contact with an object. When the mercury stops moving (stops expanding) we then measure the temperature.

But movement = Kinetic energy



when water molecules collide with the glass tube, some of their Kinetic energy is transferred to the mercury atoms causing the atoms to move and the mercury liquid to expand in the thermometer tube.

Thus, the temperature of matter is proportional to the average Kinetic energy of the constituent particles:

$$\left(K_E\right)_{ave} = \left(\frac{1}{2}mv^2\right)_{ave} \approx K_B T$$

where $K_B$ = Boltzmann's constant $= 1.38 \times 10^{-23}$ J/K

$\boxed{T = \text{temperature in Kelvins}}$

Note: the water is at temperature T.

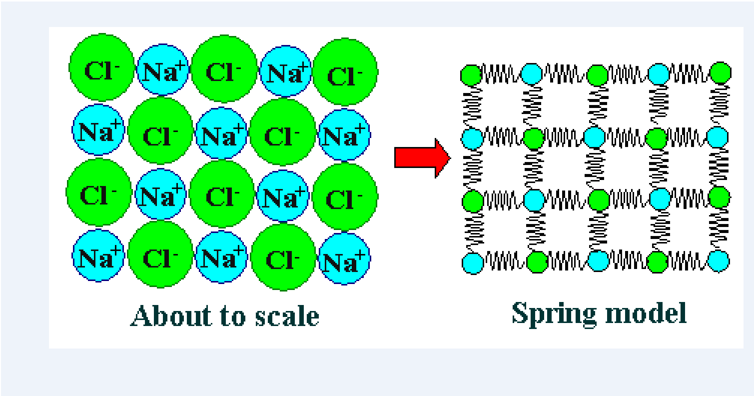
the mercury eventually stops expanding and Stabilizes to a temperature T.

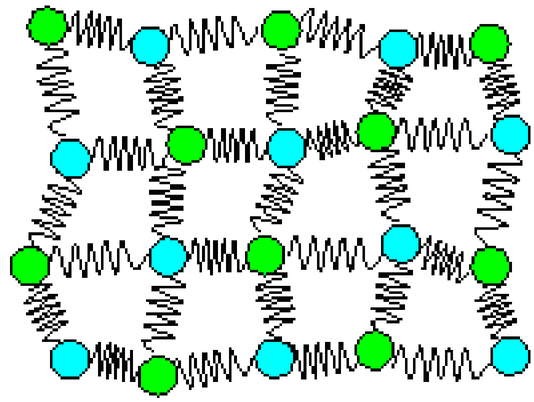
⇒ when the physical properties of a material no longer changes, then the object is in:

⇒ Thermal Equilibrium

This gives rise to: the Zeroth Law of Thermodynamics:

Two or more objects in thermal equilibrium have the same temperature https://www.tf.uni-kiel.de/matwis/amat/iss/kap_4/illustr/i4_1_1.html



Increasing the temperature

Sodium Chloride crystal lattice of atoms

Modeling the lattice as a collection of atoms connected by springs. The springs model the chemical bonds between the atoms

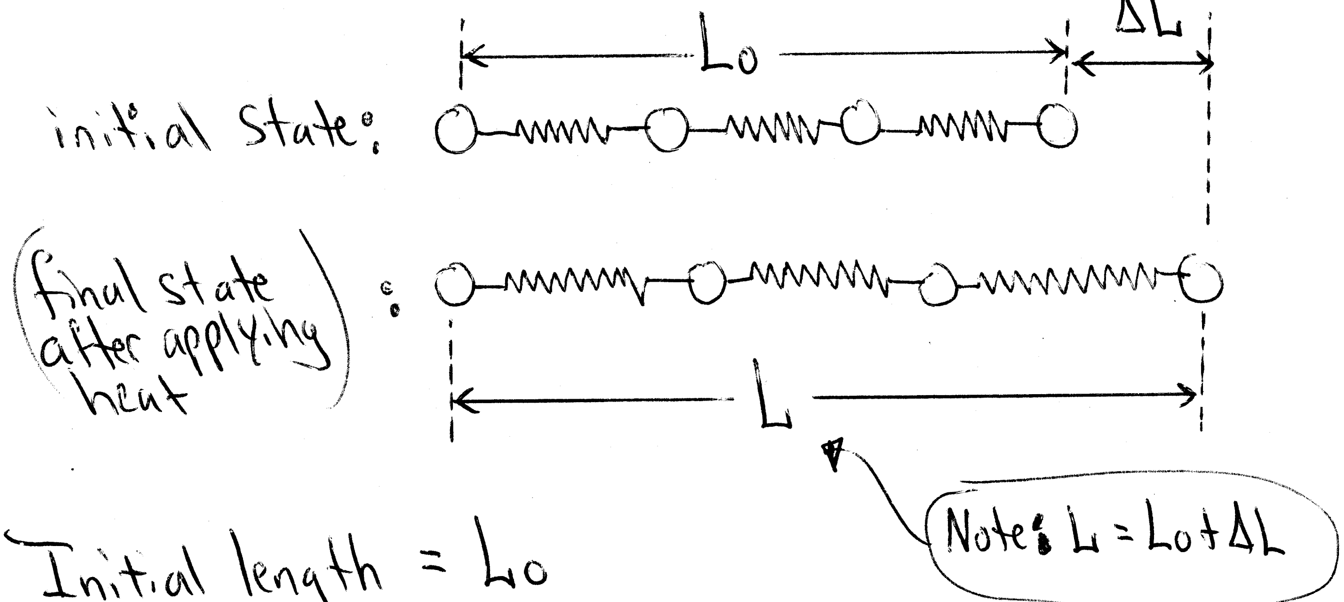
Due to the temperature of the material, the atoms are not stationary -- they are constantly moving about their lattice positions. And when the temperature increases, the entire lattice expands.

Heating causes atoms to move further apart.

→ heating causes thermal expansion

google searches: equipartition theorem (winkepedia)
crystal models matwis (crystal lattice with springs)
Emerald tufts tampl lecture ch1 (heating a lattice)

Example: Heating a line of atoms (linear chain of atoms)




Initial length $= L_o$

Final length (after heating) $= L = L_o + \Delta L$

Change in length $= \Delta L =$ final $-$ initial $= L - L_o$

For small changes in temperature, the linear thermal expansion of a material is directly proportional to the temperature change:

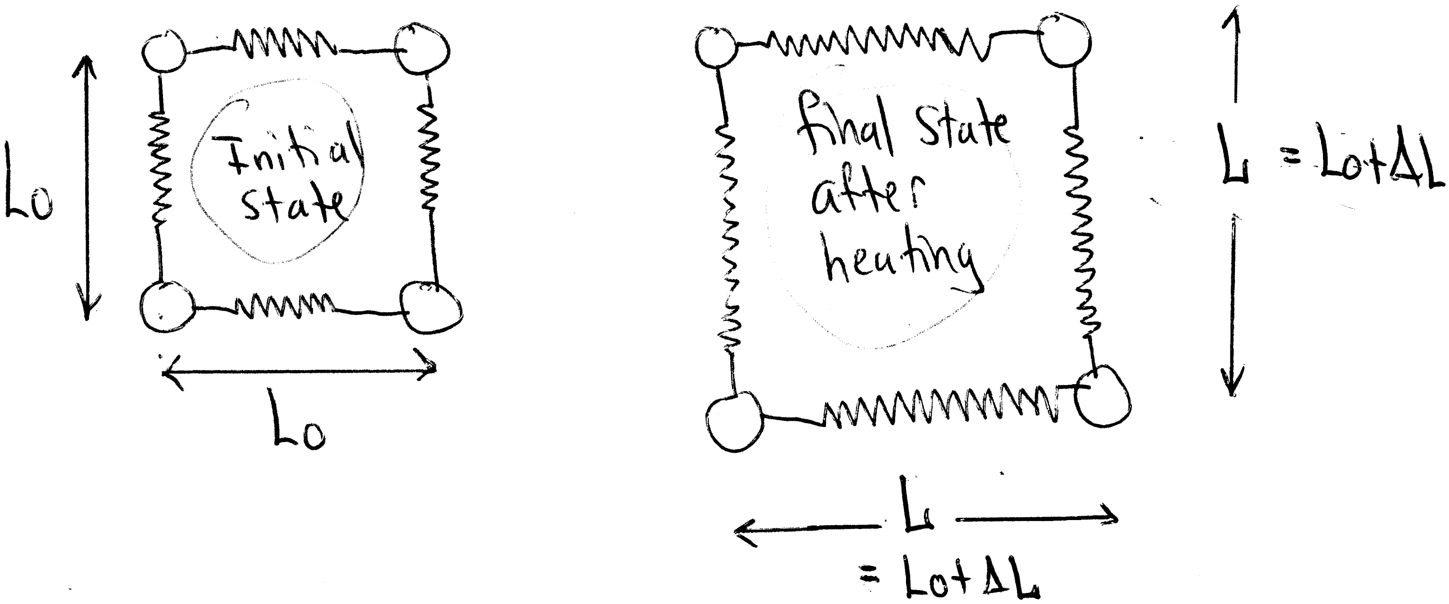
$$\frac{\Delta L}{L_o} = \alpha \Delta T$$

$\frac{\Delta L}{L_o}$: fractional change in length

$\Delta T$: temperature change, $\Delta T = T_{final} - T_{initial}$

$\alpha$ = coefficient of linear thermal expansion

Now, let's examine area expansion of atoms




$$\text{Initial Area} = A_0 = L_0^2$$

$$\text{Final Area} = A = L^2 = (L_0 + \Delta L)^2$$

$$\Delta A = \text{final} - \text{initial} = A - A_0 = (L_0 + \Delta L)^2 - L_0^2$$

$$\Rightarrow \Delta A = (L_0 + \Delta L)(L_0 + \Delta L) - L_0^2$$

$$= L_0^2 + 2L_0 \Delta L + \Delta L^2 - L_0^2$$

$$\Delta A = 2L_0 \Delta L + \Delta L^2$$

Since thermal expansions are usually very small (hard to notice them by eye), then $\Delta L$ is a small number

$$\Delta L \ll 1$$

If $\Delta L$ is is a small number, then, $\Delta L^2$ is an exceptionally small number → so small we can ignore it:

$$\Delta L^2 \approx 0$$

Then,

$$\Delta A = 2L_0 \Delta L + \cancel{\Delta L^2} \approx 0$$

$$\Delta A = 2L_0 \Delta L$$

Let's divide by $A_0$ on both sides of the eqn.

$$\frac{\Delta A}{A_0} = \frac{2L_0 \Delta L}{A_0} = \frac{2L_0 \Delta L}{L_0^2} = 2\frac{\Delta L}{L_0}$$

But, $\frac{\Delta L}{L_0} = \alpha \Delta T$ (a linear thermal expansion)

Then,

$$\boxed{\frac{\Delta A}{A_0} = 2\alpha \Delta T = \gamma \Delta T}$$

Area Thermal Expansion

Greek "gamma"

Then, $\gamma = 2\alpha$ = coefficient of area thermal expansion.

Summary:

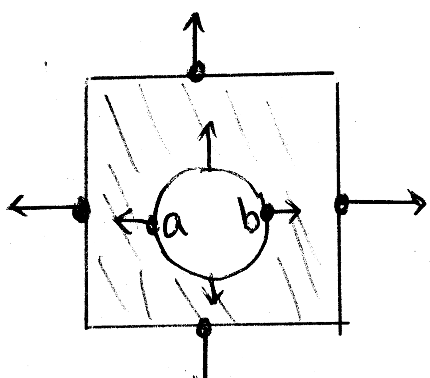
(Linear Thermal Expansion): $\frac{\Delta L}{L_0} = \alpha \Delta T$

(Area Thermal Expansion): $\frac{\Delta A}{A_0} = \gamma \Delta T \quad (\gamma = 2\alpha)$

(Volume Thermal Expansion): $\frac{\Delta V}{V_0} = \beta \Delta T \quad (\beta = 3\alpha)$

this can be deduced from the Area formula

Example: Does a hole in a metal plate get larger or smaller when the plate heats up?




when an object expands, all the atoms try to get a far away from each other as possible. In order for an atom at Point (a) to get as far away as possible from an atom at Point (b), the hole must get larger.

# Terminology for large numbers of atoms and molecules:

$N$ = # of molecules (or atoms)

$V$ = Volume , $\rho$ = density , $m$ = mass of molecule (atom)

$M_{TOT}$ = total mass of entire collection of molecules (atoms)

$N_A$ = Avagadro's # = $6.022 \times 10^{23}$ molecules/mole

1 mole ≡ # of carbon-12 atoms in 12 grams

1 atomic mass unit ≡ 1u = $1.66 \times 10^{-27}$ Kg

1 carbon-12 atom has a mass exactly = 12u

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$$\text{Number density} = \frac{\#\text{ molecules}}{\text{Volume}} \doteq \frac{N}{V}$$

$$N = \frac{\text{total mass}}{\text{mass of each molecule}} = \frac{M_{TOT}}{m}$$

$$\text{Mass density} = \rho = \frac{M_{TOT}}{V}$$

$$\Rightarrow \text{Number density} = \frac{N}{V} = \frac{1}{V} \cdot \frac{M_{TOT}}{m} = \frac{\rho}{m}$$

$$\# \text{ moles} = \frac{\text{total \# of molecules}}{\text{\# of molecules/mole}} = \frac{N}{N_A}$$

$$\text{but } N = \frac{\text{total mass}}{\text{mass of each molecule}} = \frac{M_{TOT}}{m}$$

$$\Rightarrow \quad \# \text{ moles} = \frac{M_{TOT}}{m} \cdot \frac{1}{N_A}$$

$$\text{Then: mass of each molecule} = m = \frac{M_{TOT}}{\# \text{ moles}} \cdot \frac{1}{N_A}$$

$$\text{molar mass} \equiv \frac{\text{Total mass}}{\# \text{ moles}} = \frac{M_{TOT}}{\# \text{ moles}}$$

$$\text{Then, mass of each molecule} = m = \frac{\text{molar mass}}{N_A}$$

Example: The mass of a helium atom is

$$m_{He} = \frac{\text{molar mass}}{N_A} = \frac{4.00 \text{ g/mole}}{6.022 \times 10^{23} \text{ atoms/mole}} = 6.64 \times 10^{-24} \text{ g/atom}$$

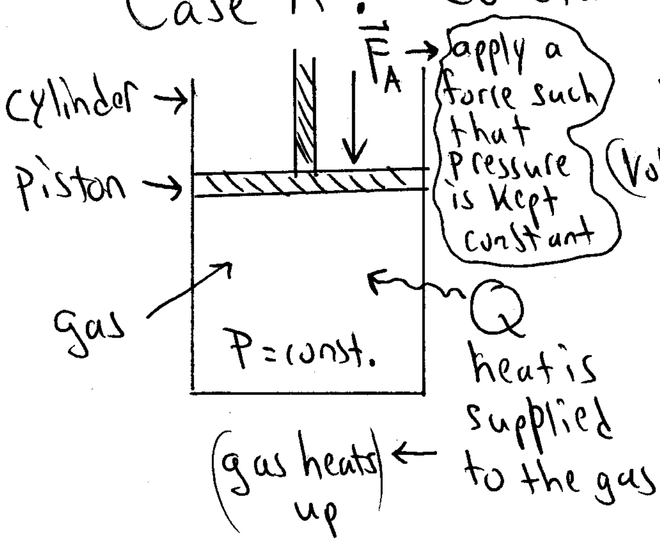
# Ideal Gas Law:

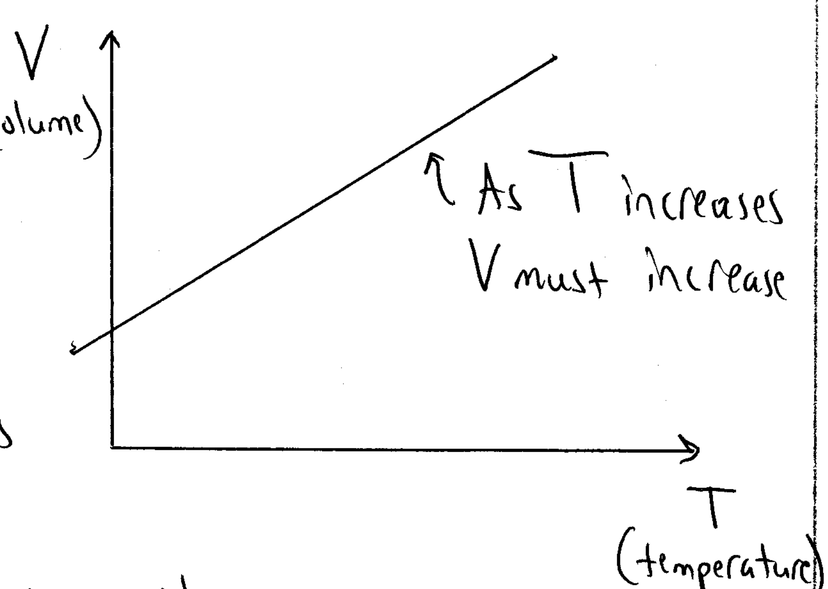
An Ideal Gas: a collection of atoms or molecules where the gravitational and electromagnetic forces between particles are negligible. They only interact with each other through random collisions.

Real gases having low pressures behave like an ideal gas.

Let's examine the various properties of an ideal gas under certain constraints:

Case A: Constant pressure $\Rightarrow P$ = constant

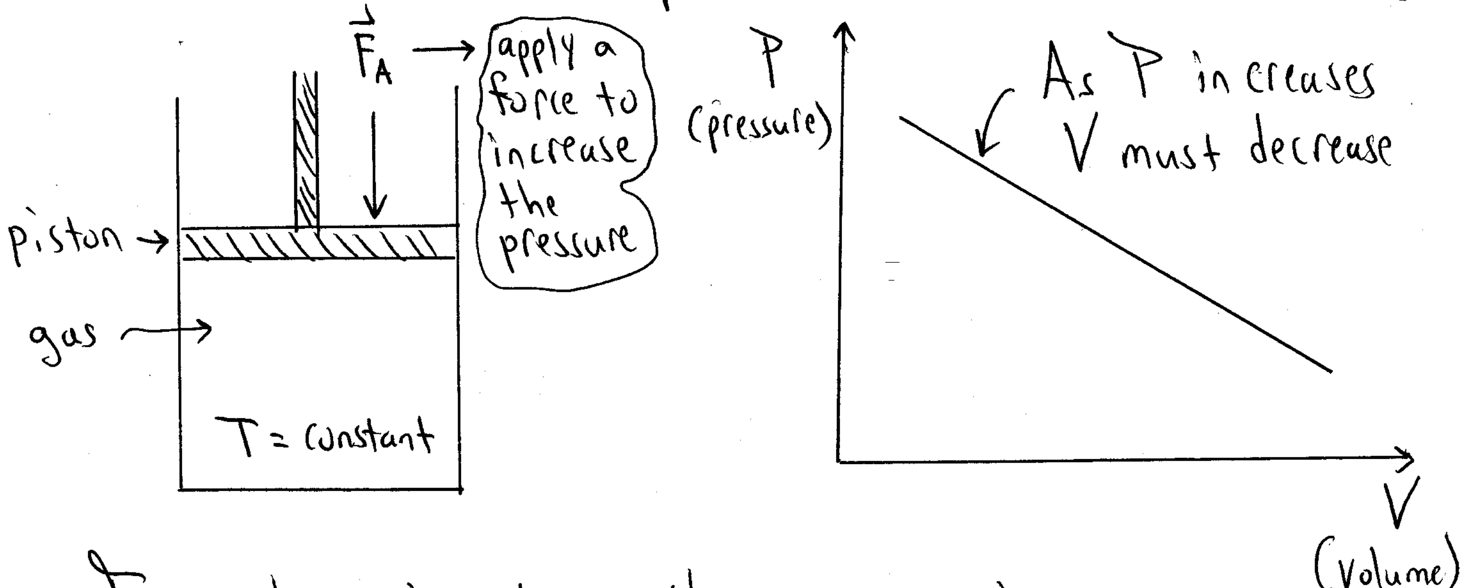



For this situation, the volume is directly proportional to temperature

Case A: $V \sim T$

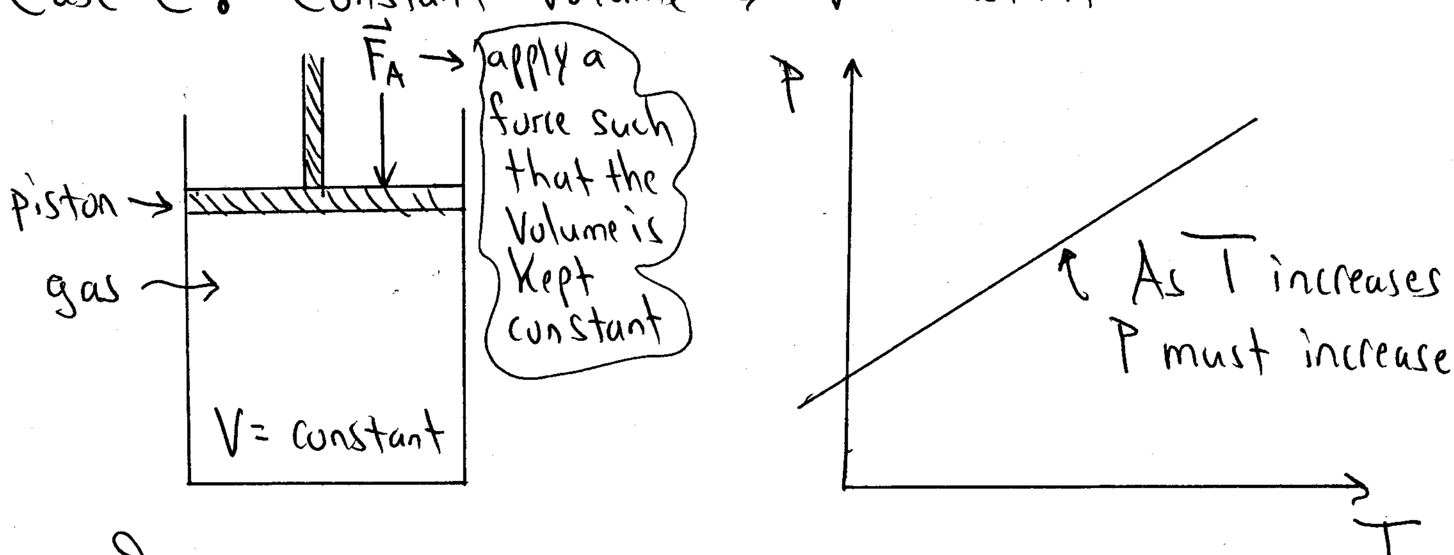
Case B: Constant temperature $\Rightarrow T = \text{constant}$




For this situation, the pressure is inversely proportional to the volume:

$$\text{Case B: } P \sim \frac{1}{V}$$

Case C: Constant Volume $\Rightarrow V = \text{constant}$

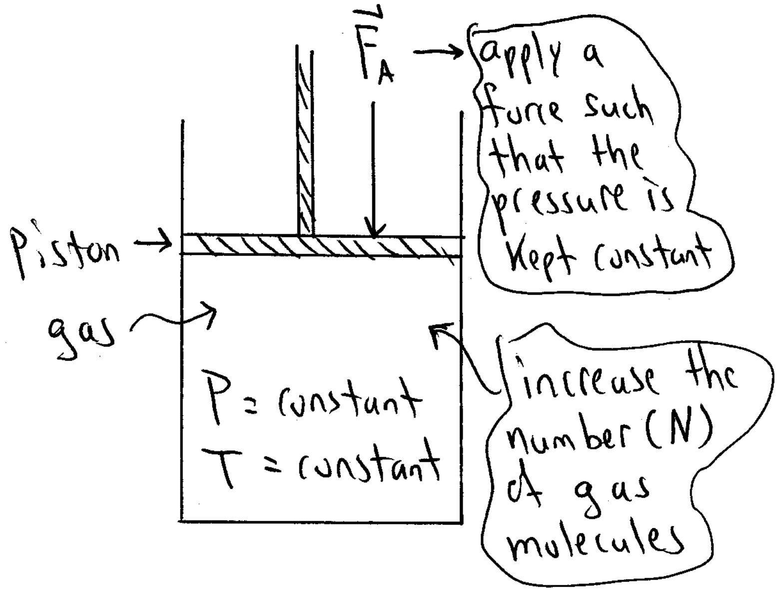


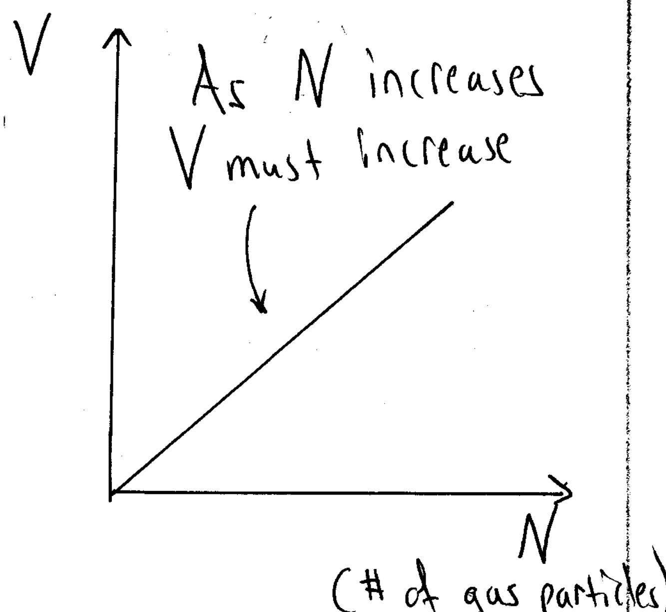

For this situation, the pressure is directly proportional to the temperature:

$$\text{Case C: } P \sim T$$

# Case D: Both P and T are Kept constant






For this situation, as the number of gas particles increase, the volume must increase if P & T are Kept constant:

$$\text{Case D:} \quad V \sim N$$

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We could do all these experiments in a lab. If we did, we could fit the line, find its slope and intercept, and then write an equation of the form

$$y = mx + b \quad \Big\} \text{ equation of a line}$$

For instance, for Case A: $\underbrace{V = mT + b}$

this equation is called a phenomenological equation because it comes from an experiment (not from theory).

Let's derive a general phenomenological equation for the various variables, $V, T, P, N$

$$\left.\begin{array}{l} \text{Case A: } V \sim T \\ \text{Case C: } P \sim T \end{array}\right\} \text{ then the product } \underbrace{PV \sim T}_{\text{must hold}}$$

$PV \sim T$ also satisfies case B where $P \sim \frac{1}{V}$

since $PV \sim T \rightarrow P \sim \frac{T}{V}$

For Case D where $\left.\begin{array}{l} V \sim N \\ PV \sim T \end{array}\right\}$ then $PV \sim NT$

(Then we can write a phenomenological equation): $PV = (\text{const}) \cdot NT$

↓ call this constant something important, like the Boltzmann constant, $K_B$

$$\boxed{PV = K_B N T}$$ → The Ideal Gas Law

where $K_B = 1.38 \times 10^{-23}\ J/{}^{\circ}K$

We can write this equation in another form recalling that

$N$ = # of gas molecules

$n$ = # of moles of gas

$N_A$ = Avagadro's # $= \dfrac{\text{\# of molecules}}{\text{mole}} = 6.02 \times 10^{23} \dfrac{\text{molecules}}{\text{mole}}$

$N_A = \dfrac{N}{n} \Rightarrow N = nN_A$

Then, $PV = K_B N T = K_B (nN_A) T = n(K_B N_A) T$

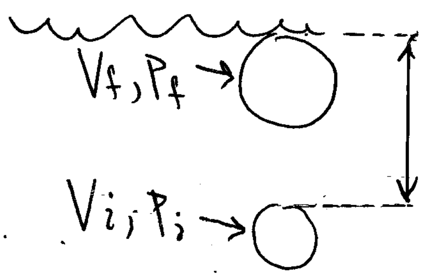
$= nRT$

Thus, $\boxed{PV = nRT}$ where $R = K_B N_A$

another expression for the ideal gas law

$= 8.31\, J/^\circ K\,\text{mole}$

Example: An air bubble has a volume $1.65\ cm^3$ when it is released by a submarine 120m below the surface of a lake. What is the volume of the bubble when it reaches the surface? Assume $T$ and $N$ remain constant.




initially, bubble is at a depth h below the surface $\Big\}\ h = 120m$

$P_f = P_{atm} = 1.01 \times 10^5\ Pa$, $V_i = 1.65\,cm^3$

Applying the Ideal Gas Law gives

$$\begin{Bmatrix} P_i V_i = nRT_i \\ P_f V_f = nRT_f \end{Bmatrix} \quad \text{Since } T_i = T_f$$

then: $P_i V_i = P_f V_f$

$$\Rightarrow \quad V_f = \frac{P_i V_i}{P_f}$$

We need to find the initial pressure, $P_i$. Use the fluid pressure relation:

$$P_{bottom} = P_{Top} + \rho g h$$

$$P_{bottom} = P_i = 1.01 \times 10^5 \, Pa + (1000 \, Kg/m^3)(9.8 \, m/sec^2)(120 \, m)$$
$$= 1.277 \times 10^6 \, Pa$$

Then, $V_f = \dfrac{(1.277 \times 10^6 \, Pa)(1.65 \, cm^3)}{1.01 \times 10^5 \, Pa} = 208 \, cm^3$