Fluids

Unit #9

Phases of Matter:

There are 5 phases of matter

As temperature increases
↓
atoms tend to get further apart

Low T ↓ High T

(1) Bose-Einstein Condensate
(2) Solid
(3) Liquid
(4) Gas
(5) Plasma (a gas & ions)

Low T ↑ High T

As temperature decreases
↓
atoms tend to get closer together

What we will study in this Chapter will be

Fluids → Liquids, Gases } objects that do not hold their shape

The difference between liquids and gases:

Liquids $\rightarrow$ incompressible fluids $\Rightarrow$ squeezing does not change its volume

Gases $\rightarrow$ compressible fluids $\Rightarrow$ easy to change their volume by squeezing

Mechanics of fluids requires the equations of motion:

(1) position (x)

(2) velocity (v)

(3) acceleration (a)

How do we construct the equations of motion for objects that do not have a definite shape?

Solid Objects	Fluids
Force = F	Pressure = $P = \frac{F}{A} = \frac{\text{force}}{\text{Area}}$
mass = m	Density = $\rho = \frac{m}{V} = \frac{\text{mass}}{\text{Volume}}$

We will describe the properties of fluids using the physical quantities of Pressure and Density instead of Force and mass.

Density of Water: $\rho_{water} = 1000\,Kg/m^3$

Specific Gravity = ratio of density of a material to that of water

$$\text{Specific Gravity} = S.G. = \frac{\rho}{\rho_{water}}$$

$$\text{Pressure} = \frac{F}{A} \Rightarrow \overset{mKs}{\text{units}} = \frac{\text{Newtons}}{m^2} = \text{Pascal} = P_a$$

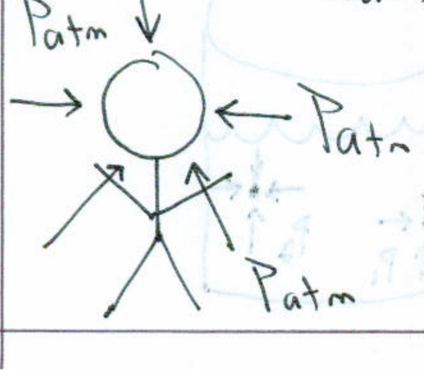
English units = atmosphere = atm

$$= \frac{\text{pounds}}{\text{square inch}} = \frac{lb}{in^2} = psi$$

Conversion factors: $1\,atm = 101300\,P_a = 101.3\,KP_a$

$1\,atm = 14.7\,psi$

Example: What forces do we experience exposed to Earth's environment?



Assume a person's head is a sphere of diameter 10in.

Surface Area of a sphere = $4\pi r^2$

Surface Area $= 4\pi\left(\frac{10\,in}{2}\right)^2 = 4\cdot\pi\cdot 25 = 100\pi\ in^2$

$\approx 300\ in^2$

Force $= F = P\cdot A = 14.7\,\frac{lb}{in^2} \times 300\,in^2 = 4400\ lb > 2$ tons

⇒ Why isn't our head (or body) crushed in Earth's brutal environment?

⇒ through natural evolution, our bodies have evolved to survive in this environment.

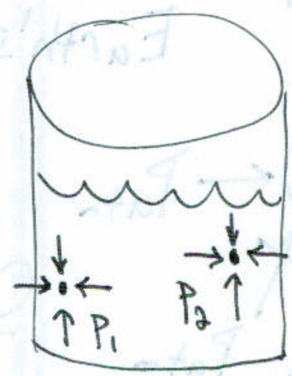
⇒ the blood pressure in our arteries is 20 KPa higher than atmospheric pressure.

Pascal's Principle: A change in pressure at any point in a confined fluid is transmitted everywhere throughout the fluid.

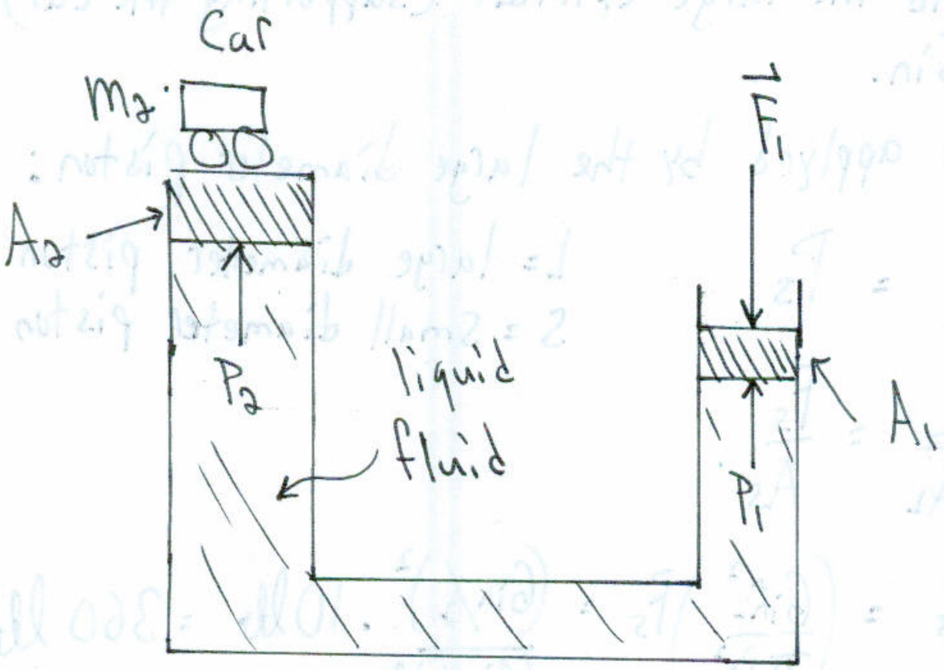
If the weight of the fluid can be ignored (is negligible), the pressure is the same everywhere in the fluid:

$$P_1 = P_2$$

(ignoring gravity)



Example: Hydrolic lift using cylindrical pistons



$$P_1 = P_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Since the fluid is in incompressible $\Rightarrow$ the Volume of fluid displaced by each piston is the same

$$V_1 = V_2$$

$$A_1 d_1 = A_2 d_2$$

(Volume = Area · height)
= A · d

But since $\frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow A_2 = A_1 \cdot \frac{F_2}{F_1}$

Then, $A_1 d_1 = A_1 \cdot \frac{F_2}{F_1} \cdot d_2$

$$\Rightarrow \underbrace{F_1 d_1}_{\text{Work done by Small piston}} = \underbrace{F_2 d_2}_{\text{Work done on large piston}}$$

Example: Let 10 lb be applied to the small 1 in diameter piston, and the large cylinder (supporting the car) has a diameter of 6 in.

(a) Find the force applied by the large diameter piston:

$$P_L = P_s$$

L = large diameter piston
S = small diameter piston

$$\frac{F_L}{A_L} = \frac{F_s}{A_s}$$

$$\Rightarrow F_L = \left(\frac{A_L}{A_s}\right) F_s = \left(\frac{\pi r_L^2}{\pi r_s^2}\right) F_s = \frac{(6\text{in}/2)^2}{(1\text{in}/2)^2} \cdot 10\text{ lb} = 360\text{ lb}$$

(b) If the small piston moved 4 in, how much did the large piston move?

$$W_{in} = W_{out}$$

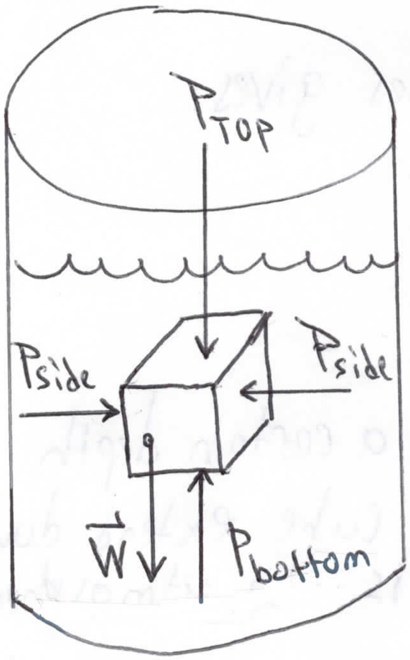
$$F_s d_s = F_L d_L$$

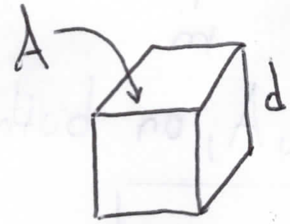
$$\Rightarrow d_L = d_s\left(\frac{F_s}{F_L}\right) = 4\text{in}\left(\frac{10\text{ lb}}{360\text{ lb}}\right) = \frac{1}{9}\text{ inch}$$

To lift a 360 pound object, we need to only push the small piston $\frac{1}{9}$ in = 0.11 inches.

Effect of Gravity on Pressure

Let there be a cylinder filled with a fluid. Let's examine the pressure on the sides of a cube outlined in the fluid.

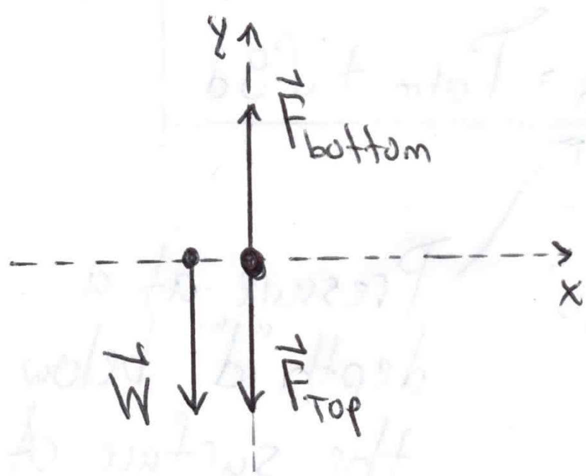




Cube has sides of length d an the area of each side is $A = d^2$

Note, the forces on the sides of the cube cancel out, but the forces on the top and bottom do not because of gravity.

(i) Free body diagram for the cube:



(ii) Newton's 2nd law

$$F_{Total,y} = F_{bottom} - F_{Top} - W = m\overset{0}{\cancel{a_y}}$$

cube of water is static

(1) Convert forces to pressures

$$P = \frac{F}{A} \Rightarrow F = PA$$

$$\Rightarrow F_{Total,y} = P_{bottom}A - P_{Top}A - mg = 0$$

(2) Convert mass to density: $\rho = m/V \Rightarrow m = \rho V = \rho A \cdot d$

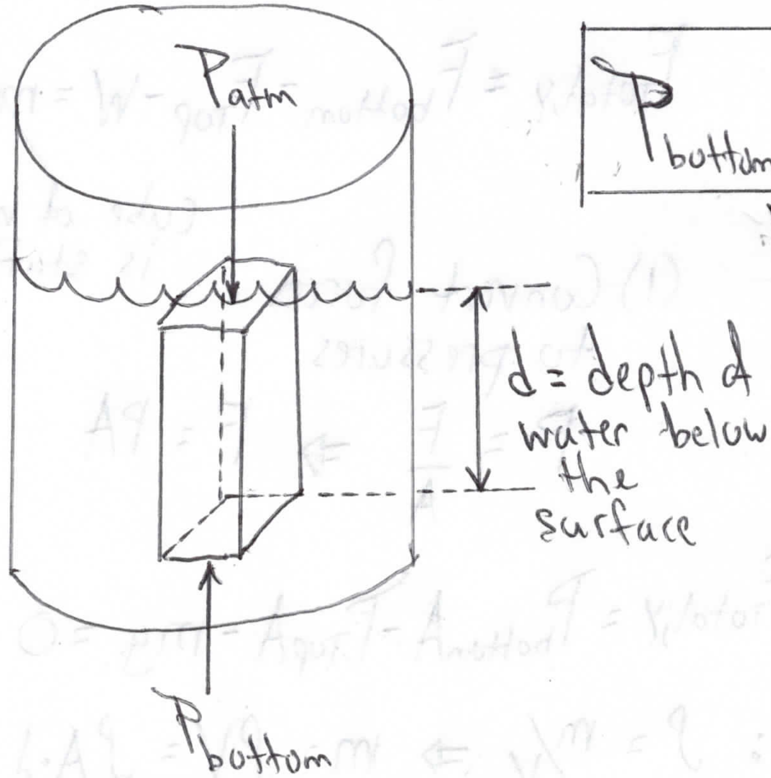
where we have noted that the volume of the cube is $V = A \cdot d$

$$\Rightarrow P_{bottom}A - P_{Top}A - \underbrace{(\rho A d)}_{m} g = 0$$

Cancelling out the area, A, on both sides gives:

$$\boxed{P_{bottom} = P_{Top} + \rho g d}$$

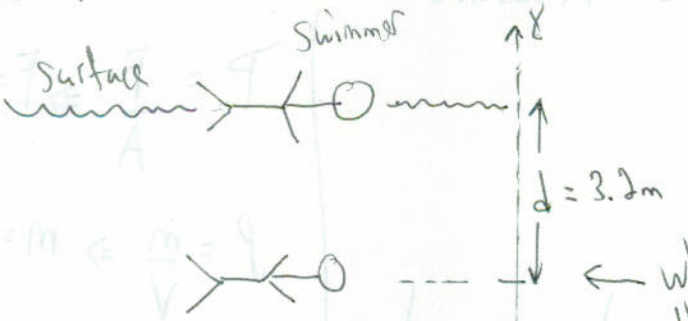
If we wanted to know the pressure at a certain depth below the surface of the fluid, let the cube extend down to that depth. The pressure at the top is the atmospheric pressure: $P_{Top} = P_{atm}$. Then,



$$\boxed{P_{bottom} = P_{atm} + \rho g d}$$

Pressure at a depth "d" below the surface of a fluid.

Example 9.3 :



$A = 0.60 cm^2$ = area of eardrums

← what is the increase in force on the swimmer's eardrum's ?

At surface: Pressure $= P_{atm} = P_i$

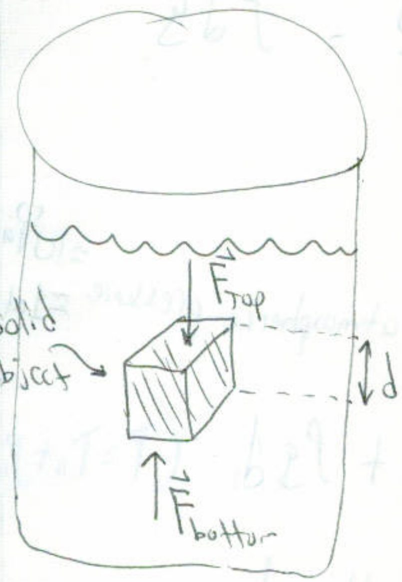
At depth d: Pressure $= P_{atm} + \rho g d = P_f$

Change in pressure $= \Delta P = P_f - P_i = (P_{atm} + \rho g d) - P_{atm} = \rho g d$

$$= \left(1000 \frac{Kg}{m^3}\right) \cdot (9.8 m/sec^2)(3.2 m) = 31.4 KPa$$

Increase in force $= \Delta P \cdot A = (31.4 KPa)\left[0.6 cm^2 \times \left(\frac{1m}{100cm}\right)^2\right] = 1.9 N$

Buoyant Force : An object ~~~~ submerged in a fluid always experiences an upward force.



$$F_{buoyant} = F_{bottom} - F_{TOP}$$

up

$$= P_{bottom} A - P_{TOP} A$$

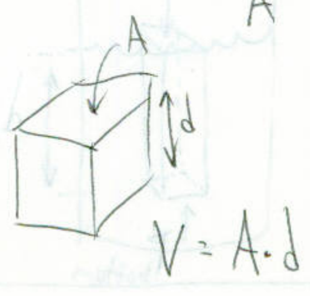
$$= (P_{bottom} - P_{TOP}) A$$

$$= (\rho g d) A$$

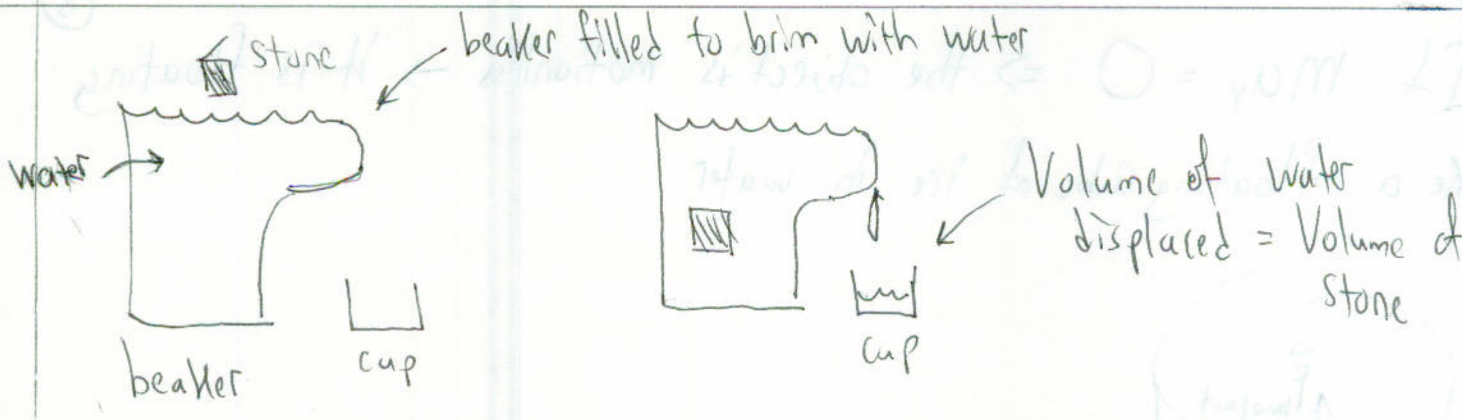
$$= \rho g V$$

(Note $F_{bottom} > F_{TOP}$ since pressure at the bottom is greater than pressure at the top)

$P = \frac{F}{A}$



$V = A \cdot d$



Archimede's Principle #1: A submerged object always displaces a volume of liquid equal to its own volume

$$F_{buoyant} = \rho_{fluid} g V_{object} = \rho_{fluid} g V_{fluid} = \underbrace{m_{fluid} g}_{\text{Weight of the fluid displaced}} =$$

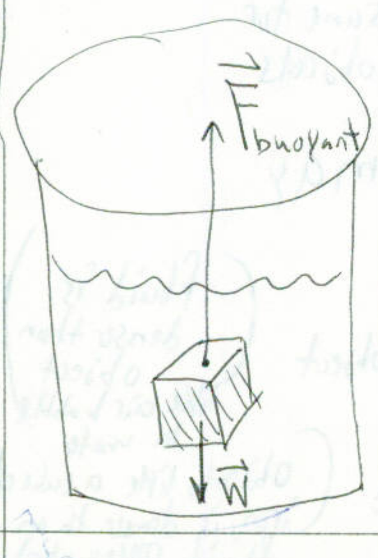
ρ_{fluid}: density of fluid (not the density of object)

V_{object}: Volume of object = Volume of fluid displaced

$\rho_{fluid} = \dfrac{m_{fluid}}{V_{fluid}}$

$\Rightarrow m_{fluid} = \rho_{fluid} V_{fluid}$

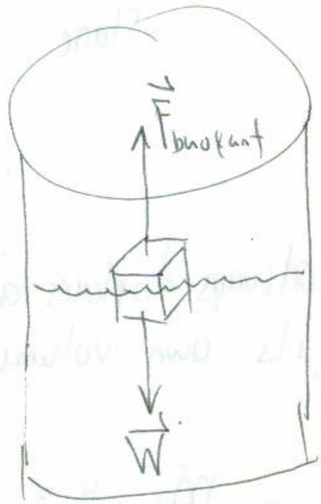
Archimede's Principle #2: An immersed object is buoyed up by a force equal to the weight of the fluid it displaces.



Net force on a submerged object is:

$$F_{Total,y} = F_{buoyant} - W = ma_y$$

If $ma_y = 0 \Rightarrow$ the object is motionless $\rightarrow$ it is floating like a floating cube of ice in water

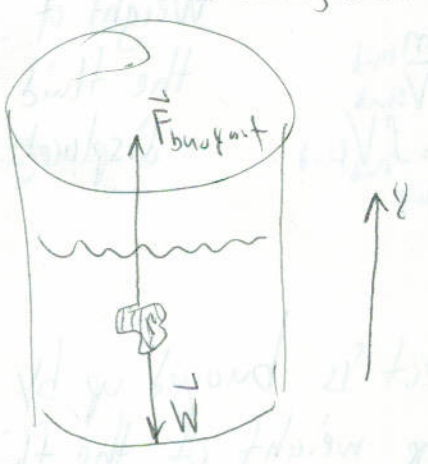


$$\Rightarrow F_{buoyant} - W = 0$$

$$F_{buoyant} = W = \text{weight of floating object}$$

$$\rho_{fluid}\, g\, V_{\substack{fluid \\ displaced}} = m_{object}\, g = \rho_{object}\, g\, V_{object}$$

Submerged object



$$F_{buoyant} - W = ma_y$$

$$\rho_{fluid}\, g\, V_{fluid} - m_{object}\, g = ma_y$$

$$\rho_{fluid}\, g\, V_{fluid} - \rho_{object}\, g\, V_{object} = ma_y$$

Volumes are the same for submerged objects

$$\left(\rho_{fluid} - \rho_{object}\right) g\, V_{fluid} = ma_y$$

For rising object, $a_y > 0$ and $\rho_{fluid} > \rho_{object}$ (fluid is denser than object like air bubble in water)

For sinking object, $a_y < 0$ and $\rho_{object} > \rho_{fluid}$ (object, like a cube of iron, is denser than fluid (like water))

Specific Gravity:

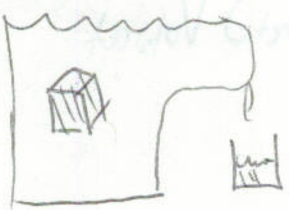
$\underbrace{\text{Density of water at } 3.98°C}_{\text{a standard reference material}} = 1.000\, g/cm^3 = 1000\, \frac{Kg}{m^3}$

$$\text{Specific gravity} = S.G. = \frac{\rho}{\rho_{water}} = \frac{\rho}{1000\, Kg/m^3}$$

Example 9.6 : The golden Falcon → is it made of gold?

Falcon, $\underbrace{W = mg = 24.1N}_{\text{this is its weight}} \rightarrow m = \left(\frac{24.1N}{g}\right)$

Yes!



weight of water = 1.25N

this is the density of gold → answer is Yes!

$$F_{buoyant} = \rho_{fluid}\, g\, V_{fluid} = m_{fluid}\, g = \text{Weight of fluid displaced}$$

$$\rho_{object} = \frac{m_{object}}{V_{object}} = \frac{(24.1N/g)}{V_{object}}$$

$$V_{object} = V_{fluid\ displaced} \Rightarrow V_{fluid} = \frac{m_{fluid}\cancel{g}}{\rho_{fluid}\, g} = \frac{1.25N}{\rho_{fluid}\cdot g} = \frac{1.25N}{\left(1000\frac{Kg}{m^3}\right)\cdot g}$$

$$\rho_{object} = \left(\frac{24.1N}{g}\right)\cdot\frac{(1000Kg/m^3)\cdot \cancel{g}}{1.25N} = 19280\, Kg/m^3 = 1.93\times10^4\, Kg/m^3$$

$\rho_{gold} = 19.3\times10^3\, Kg/m^3$