# Newtonian Mechanics

## Unit #4



So far in class we have studied the equations of motion for an object undergoing constant acceleration:

⇒ This is the study of Galilean Mechanics

Now it is time to fully study Newtonian Mechanics:

⇒ The study of how forces influence the motion of objects

We know that it is forces that cause an object to accelerate. Forces cause a change in velocity of an object, and a change in velocity is an acceleration:

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

In this coure we will be dealing with forces that are constant in time: $F(t) = F_0 = \text{constant}$

For forces that are constant in time and position, we can apply the equations of motion of Galilean Mechanics

$$\frac{F(x,t) = F_0 = \text{constant}}{\left(\text{Can use Galilean Mechanics}\right)} \Rightarrow \begin{cases} \text{Eq. M1: } a = \text{constant} \\ \text{Eq. M2: } v = v_0 + at \\ \text{Eq. M3: } x = x_0 + v_0 t + \frac{1}{2}at^2 \end{cases}$$

$\underline{F(x,t) = F(x)}$ = Varies with position, constant with time

$\Rightarrow$ Cannot Use Galilean Mechanics

$a \neq$ constant

$\underline{F(x,t) = F(t)}$ = Varies with time, constant with position

$\Rightarrow$ Cannot use Galilean Mechanics.

The following Newton's Laws are obeyed for any force $F(x,t)$

(A) Newton's 1st Law: (essentially Galileo's Law of Inertia)
A body will stay at rest or in motion at a constant velocity unless acted upon by an outside force.

(B) Newton's 2nd Law: The total force on an object causes an acceleration proportional to its mass:

$$\vec{F}_{Total} = m\vec{a} \quad \left(\text{units of force} = \text{Newtons} \equiv N\right)$$

A body's resistance to motion (inertia) is the mass of an object. A more massive object (like a bowling ball) is harder to accelerate than a less massive one (such as a marble)

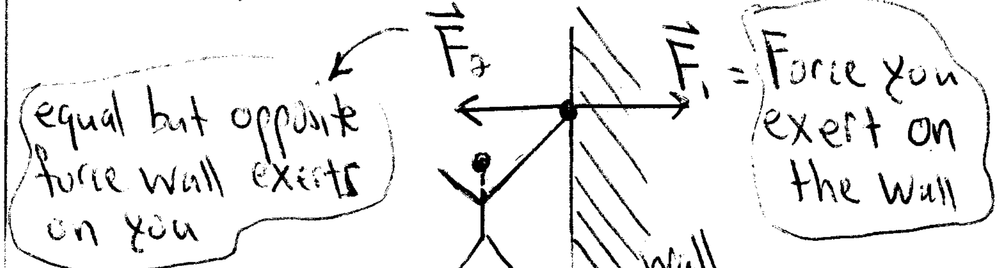
$$\vec{a} = \frac{\vec{F}_{Total}}{m}$$

the more massive an object, the smaller the acceleration.
→ the harder it is to accelerate

⇒ Thus, mass = inertia
= resistance to a change in motion

(C) Newton's 3rd Law: For every force there is an equal but opposite force (that is, force always comes in pairs).

Example: If you are leaning against a wall, you are exerting a force against the wall, and the wall is exerting an equal but opposite force against you.



Note that force is a vector:

$\vec{F}_{Total}$ = Vector sum of all forces acting upon an object.

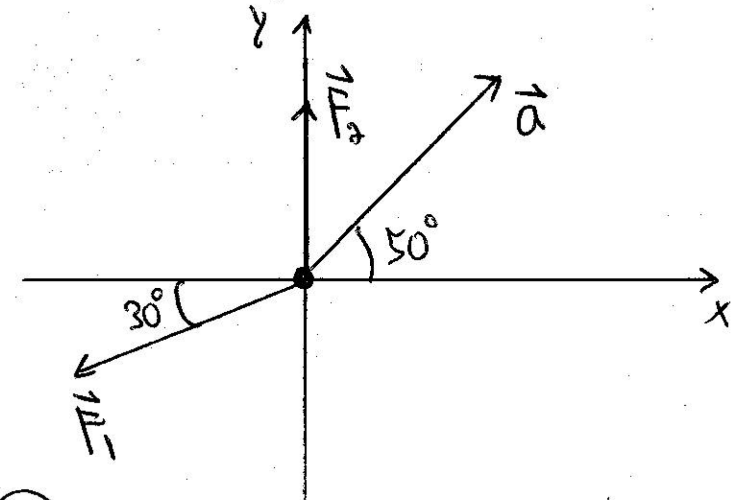
$= F_{Total,x}\,\hat{i} + F_{Total,y}\,\hat{j}$

This can be written as 2 separate equations:

$F_{Total,x} = m a_x \longrightarrow$ component of force in the x-direction

$F_{Total,y} = m a_y \longrightarrow$ component of force in the y-direction

Example: A 2.0 kg object accelerates 3.0 m/sec² by 3 forces, 2 of which are shown below. What is the 3rd force?




$|\vec{F}_1| = F_1 = 10\,N$

$|\vec{F}_2| = F_2 = 20\,N$

$|\vec{a}| = a = 3.0\,m/sec^2$

To find the 3rd force, apply Newton's 2nd Law:

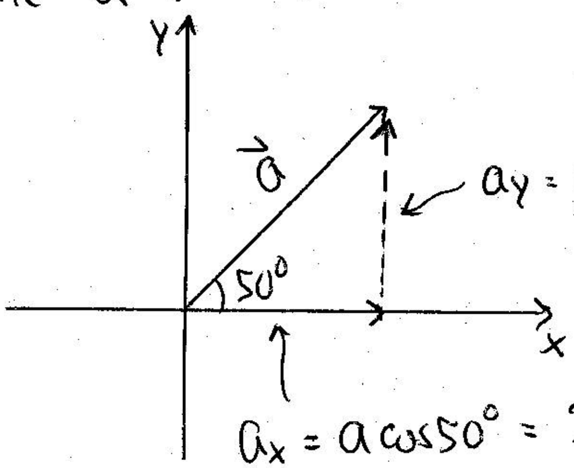
$\vec{F}_{Total} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = m\vec{a}$

Then, $\vec{F}_3 = m\vec{a} - \vec{F}_1 - \vec{F}_2$ = the 3rd force

This can be written as two separate equations for the x and y components of the force:

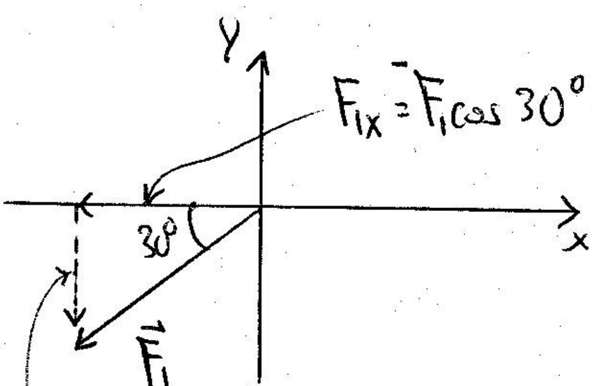
$$\begin{cases} F_{3x} = ma_x - F_{1x} - F_{2x} \\ F_{3y} = ma_y - F_{1y} - F_{2y} \end{cases}$$

Now we just need to determine the components of $\vec{F}_1, \vec{F}_2,$ and $\vec{a}$:



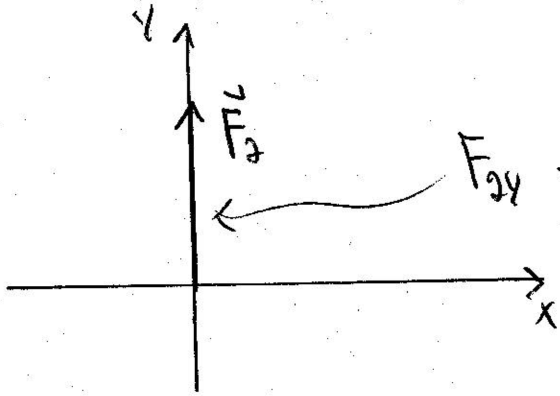

$a_y = a\sin 50° = 3\sin 50°$

$a_x = a\cos 50° = 3\cos 50°$




$F_{1x} = -F_1\cos 30° = -10\cos 30°$ (minus sign needed because it points in the negative x-direction)

$F_{1y} = -F_1\sin 30° = -10\sin 30°$ (it points in the negative y-direction)




$F_{2y} = F_2 = 20$

(note, $F_{2x} = 0$ since there is no component that points in the x-direction)

Plugging these components in the two equations for $\vec{F}_3$ gives

$$F_{3x} = (2.0\,Kg)(3.0\,m/sec^2)\cos 50^\circ - (-10\,N)\cos 30^\circ - 0$$
$$= 12.517\,N$$

note : Newton = N = Kg m/sec²

$$F_{3y} = (2.0\,Kg)(3.0\,m/sec^2)\sin 50^\circ - (-10\,N)\sin 30^\circ - 20\,N$$
$$= -10.404\,N$$

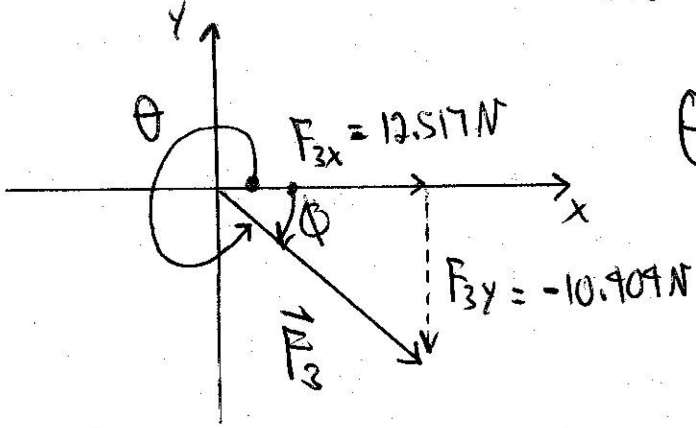
Then,

$$\vec{F}_3 = (12.517\,\hat{i} - 10.404\,\hat{j})\,N$$

Expressing $\vec{F}_3$ in terms of a magnitude and direction:

$$|\vec{F}_3| = F_3 = \sqrt{F_{3x}^2 + F_{3y}^2} = \sqrt{(12.517)^2 + (10.404)^2}\,N$$
$$= 16.27613124\,N$$

$$\phi = \tan^{-1}\left(\frac{F_{3y}}{F_{3x}}\right) = \tan^{-1}\left(\frac{10.404}{12.517}\right) = 39.73232983^\circ$$




$$\theta = 360^\circ - \phi = 320.26767^\circ$$

Then, the answer expressed 2 ways:

$$\boxed{\vec{F}_3 \approx (12.5\hat{i} - 10.4\hat{j})\ N}$$ → unit vector notation

Or

$\vec{F}_3 \approx$ magnitude of 16.3 N at an angle of $320^\circ$ relative to the positive x-axis

# Free Body Diagrams

Free body diagrams are simplified depictions of the interaction of forces with objects. We simplify these interactions by assuming:

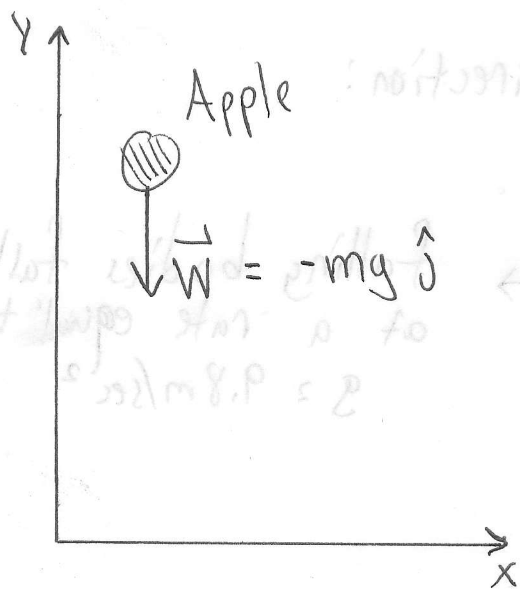
Objects = Point Particles

Forces $\Rightarrow$ Act at the center of mass of the object

Below we will examine different types of forces and their associated free body diagrams.

Ⓐ Gravitational Force:

$\vec{W}$ = force directed towards the center of the Earth

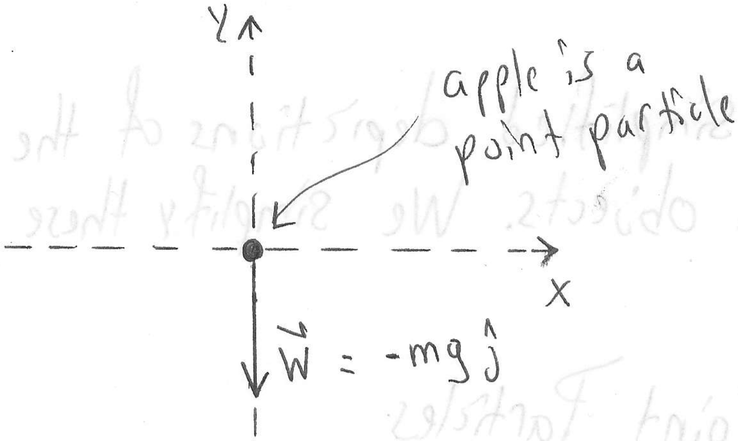



$|\vec{W}| = W = mg$

$g$ = acceleration due to gravity

$= 9.8\ m/sec^2 = 9.8\ N/Kg$

(i) Free Body Diagram:




(ii) Newton's 2nd law $\left(\vec{F} = m\vec{a}\right)$ ← A force causes an object to accelerate

↑ examine x and y-components of this eqn.

$$\begin{cases} F_{Total,y} = -W = ma_y \\ F_{Total,x} = 0 \end{cases}$$

→ note: $\vec{W}$ points in negative y-direction and causes acceleration $a_y$ in the y-direction

→ there are no forces in the x-direction

From the total forces in the y-direction:

$$-mg = ma_y$$

$$\Rightarrow a_y = -g \quad \rightarrow$$ falling bodies fall at a rate equal to $g = 9.8\,m/sec^2$

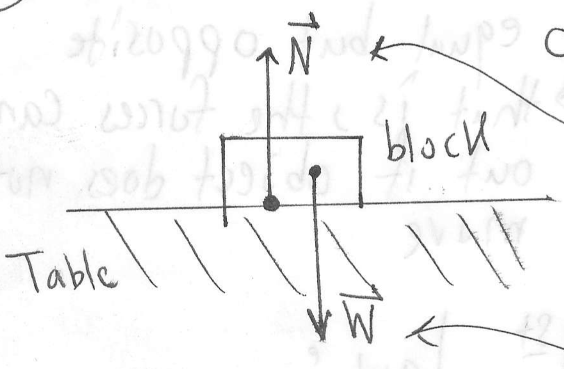
Note: $W$ = weight = magnitude of the gravitational force on a body

$$\boxed{\begin{array}{ccc} \text{Weight} & \neq & \text{mass} \\ \downarrow & & \downarrow \\ \text{Newtons} & \neq & \text{Kg} \end{array}}$$

$$\boxed{\text{Weight} = mg}$$

1 Newton = 0.2248 pounds

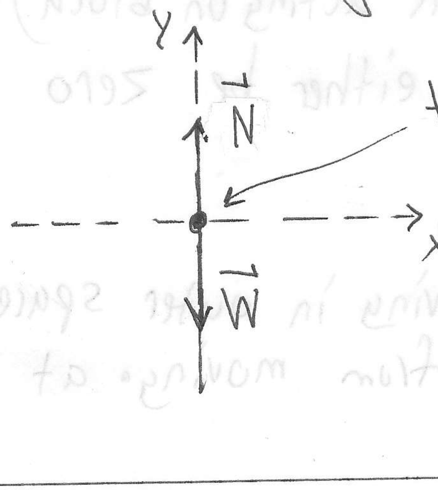
(B) Normal Force: Perpendicular force of a surface on an object. ($\vec{N}$)




$\vec{N}$ = normal force = force of table on block

$\vec{W}$ = force (weight) of block downwards

(i) Free body diagram




the block is a point particle

(ii) Newton's $2^{nd}$ law:

$$\left\{\begin{array}{l} F_{Total,y} = F_N - W = ma_y \\ F_{Total,x} = 0 \end{array}\right.$$

$\rightarrow W = mg$

$\rightarrow$ there is no force in the x-direction

In the y-direction, the block does not move

$\Rightarrow \quad F_N - mg = m\cancel{a_y}^{\,0}$ (no acceleration in the y-direction)

$\Rightarrow F_N = mg$
- $\rightarrow$ this is intuitive
- $\rightarrow$ the forces must be equal but opposite
- $\rightarrow$ that is, the forces cancel out if object does not move

This leads to Newton's $1^{st}$ law:

If the net force acting on an object is zero (like $F_N - W = 0$ = net force acting on block) then the object's velocity will either be zero or it will be constant.

(Example, if the block + table were already moving in outer space, these forces could not stop the block from moving.)

These results also leads to Newton's $3^{rd}$ law:

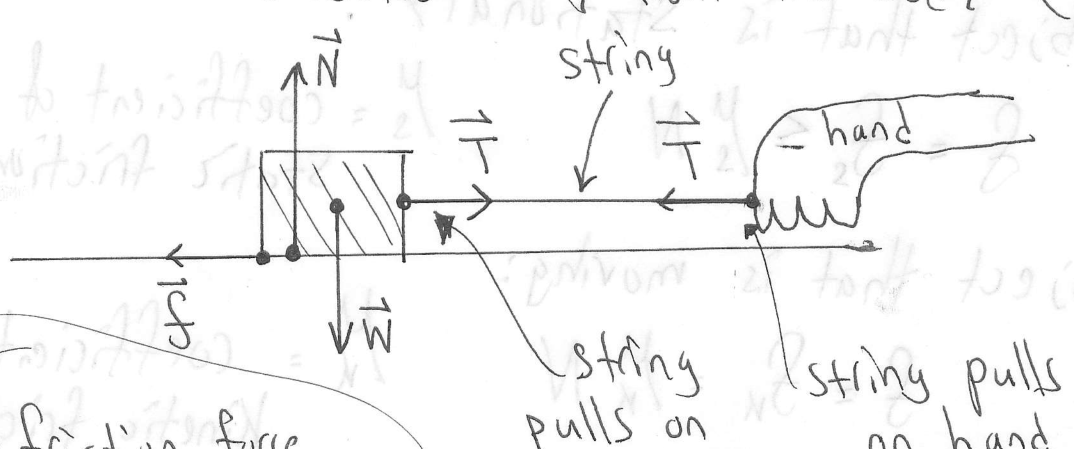
The force of the table on the block (N) is equal to the force of the block on the table (W). These forces are equal but are in opposite directions

Newton's $3^{rd}$ law: Whenever an object exerts a force on a $2^{nd}$ object, the $2^{nd}$ object exerts an equal force in the opposite direction on the $1^{st}$.

## (C) Friction & Tension

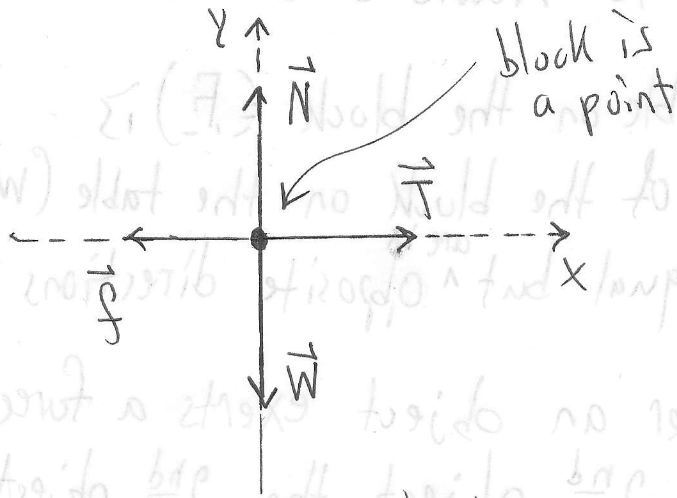
Friction: A force opposite to the direction of motion when one surface slides across another ($\vec{f}$)

Tension: The force of a rope on a body - it is directed away from the body. ($\vec{T}$)




$\vec{f}$ = friction force
$\vec{T}$ = tension force

(i) Free body diagram




(ii) Newton's 2nd law

$$\begin{cases} F_{Total,y} = N - W = \cancel{m a_y}^{\,0} \\ F_{Total,x} = T - f = m a_x \end{cases} \Rightarrow \begin{cases} N - mg = 0 \\ T - f = m a_x \end{cases}$$

Thus we find: $N = mg$ (the normal force is equal to the block's weight)

$$a_x = \frac{T - f}{m}$$

For an object that is stationary:

$$f = f_s \leq \mu_s N$$

$\mu_s$ = coefficient of static friction

For an object that is moving:

$$f = f_k = \mu_k N$$

$\mu_k$ = coefficient of kinetic friction

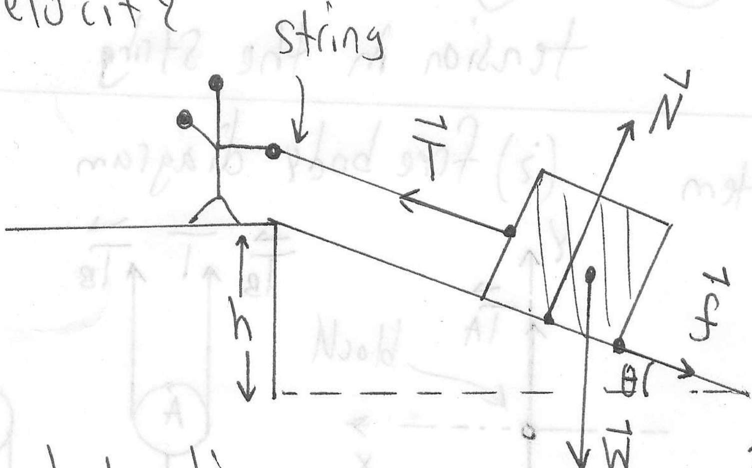
# Equilibrium Examples

Equilibrium: An object does not move or the object moves at a constant velocity: $\vec{v} = 0$ or $\vec{v}$ = constant

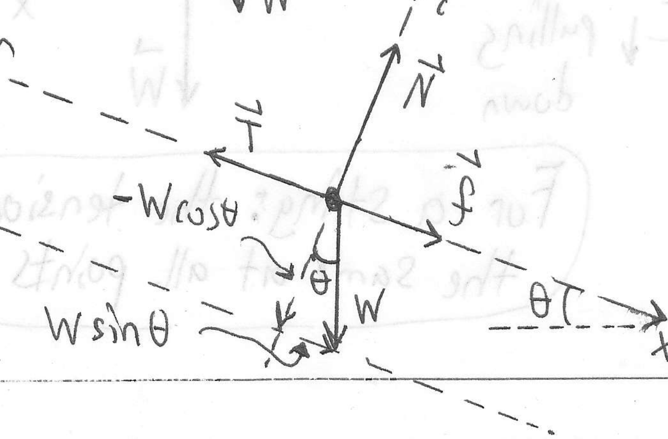
$\Rightarrow$ The net forces on the object are zero : $\vec{F}_{Total} = 0$

$\Rightarrow$ The object has zero acceleration $\vec{a} = 0$

(A) A person pulls a crate up a ramp at a constant velocity




(i) free body diagram



(ii) Newton's 2nd law:

$$\begin{cases} F_{Total,y} = N - W\cos\theta = m\cancel{a_y}^{\,0} \\ F_{Total,x} = f - T + W\sin\theta = m\cancel{a_x}^{\,0} \end{cases}$$

$\rightarrow a_y = 0$ since crate does not leave ramp and rise into the air

$\rightarrow a_x = 0$ since velocity is constant

$$\Rightarrow \begin{cases} N - mg\cos\theta = 0 \\ \mu_k N - T + mg\sin\theta = 0 \end{cases} \longrightarrow f = f_k = \mu_k N$$

$\Rightarrow$ Thus we find that $N = mg\cos\theta$ } Normal force

and

$$\mu_k(mg\cos\theta) - T + mg\sin\theta = 0$$

$$\underbrace{T = mg(\mu_k\cos\theta + \sin\theta)}_{\text{tension in the string}}$$

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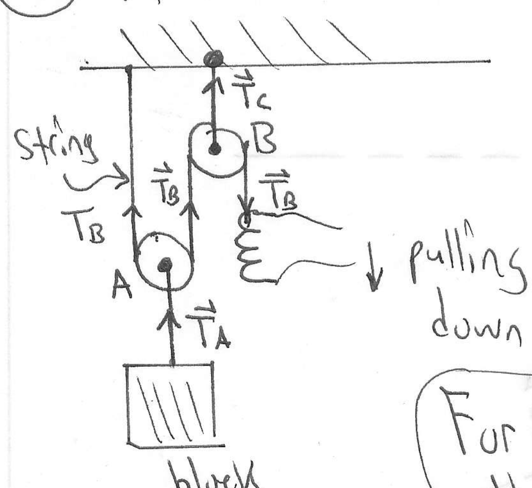
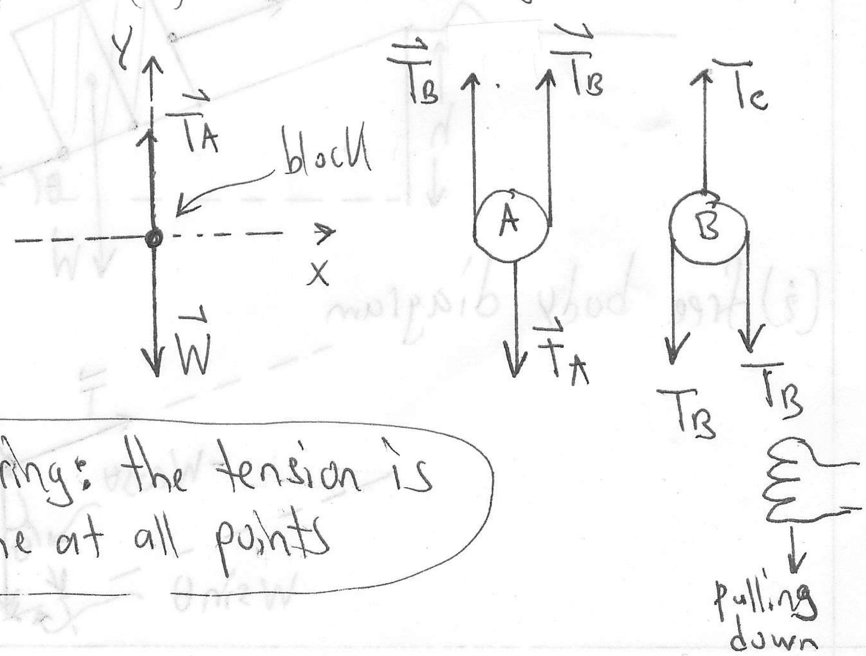
(B) A two Pulley System




(i) free body diagram




For a string: the tension is the same at all points

(ii) Newton's $2^{nd}$ law:

$$\left.\begin{aligned} &\text{For Block: } F_{Total,y} = T_A - W = m a_y \\ &\text{For Pulley A: } F_{Total,y} = 2T_B - T_A = m_A a_{yA} \\ &\text{For Pulley B: } F_{Total,y} = T_C - 2T_B = m_B a_{yB} \end{aligned}\right\}.$$

Let the block be pulled at constant velocity: $a_y = 0$

Let pulley A and B have zero mass (ideal pulleys)

$$\Rightarrow m_A = m_B = 0$$

$$\Rightarrow \left\{\begin{aligned} T_A - mg &= 0 \\ 2T_B - T_A &= 0 \\ T_C - 2T_B &= 0 \end{aligned}\right\} \Rightarrow \begin{aligned} T_A &= mg \\ T_A &= 2T_B \\ T_C &= 2T_B \end{aligned}$$

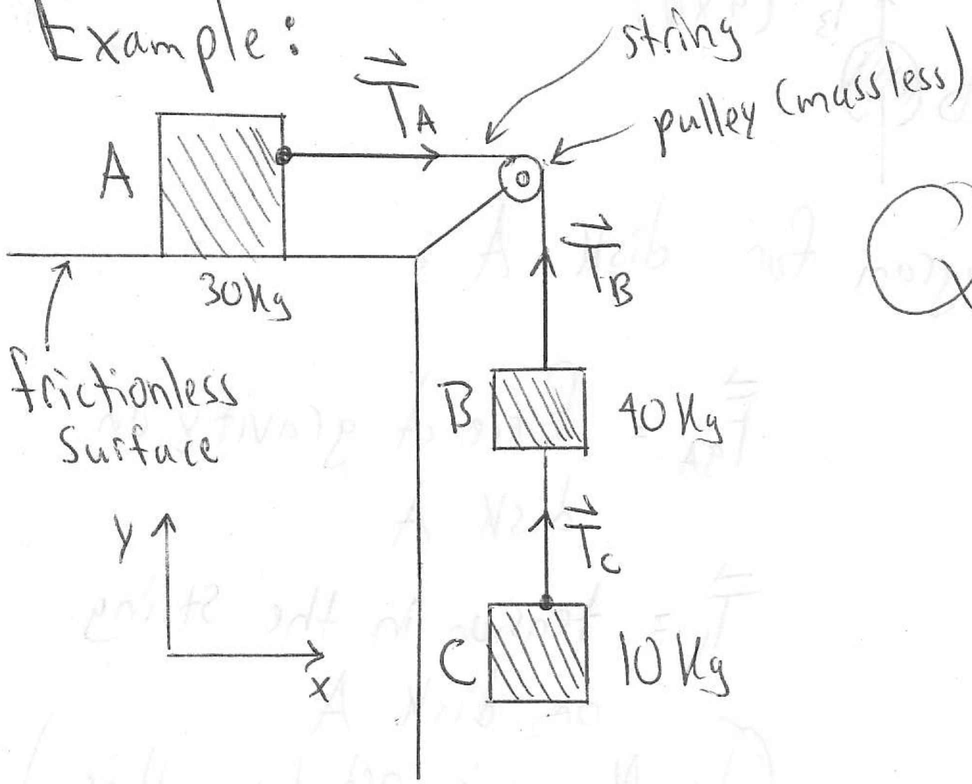
The person pulls with force $= T_B$

But $T_B = \frac{1}{2} T_A = \frac{1}{2} mg = \frac{1}{2}$ (weight of block)

This two pulley system allows a person to lift a block with a force that is only ½ the weight of the block.

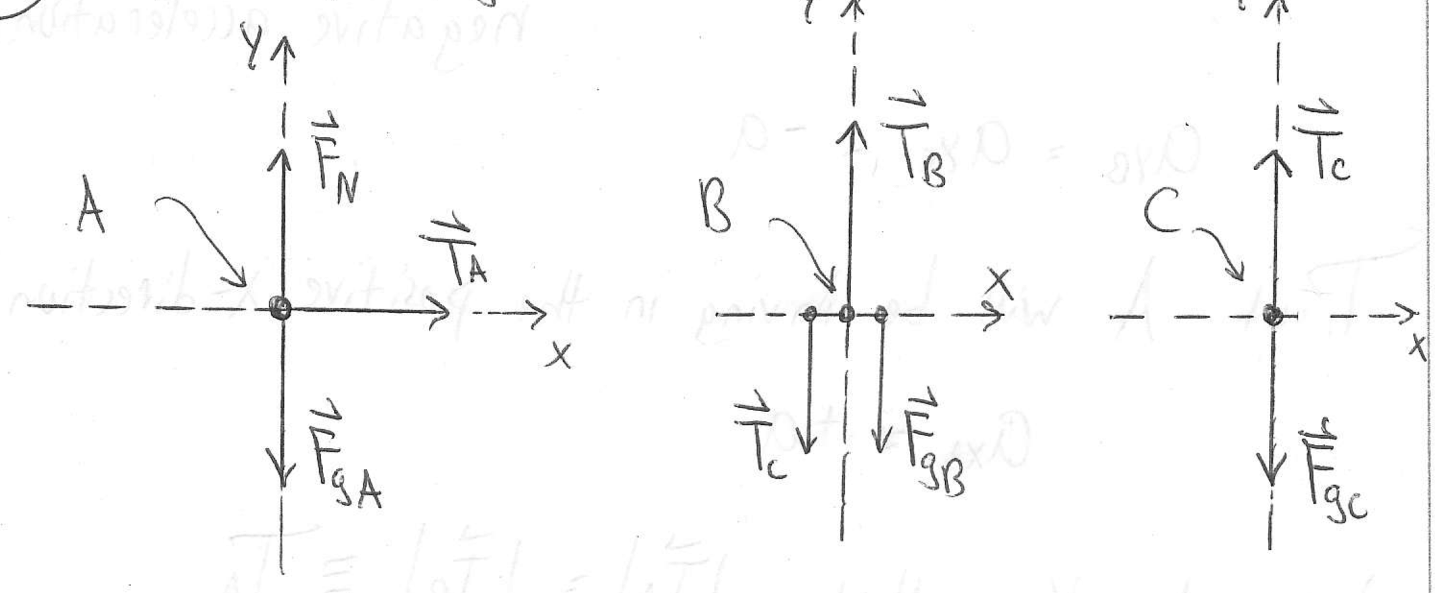
This is why pulleys are so useful → 200 pound objects feel like they weigh only 100 pounds if you use pulleys.

Example:




Question: What is $T_C$?

(A) Free body diagrams:




(B) Newton's 2nd law:

$(a_{YA} = 0)$

For mass A: $\begin{cases} F_{Total,YA} = F_N - F_{gA} = m_A \cancel{a_{YA}}^{\,0} \\ F_{Total,XA} = T_A = m_A a_{XA} \end{cases}$

For mass B: $F_{Total,Y_B} = T_B - T_c - F_{gB} = m_B a_{YB}$

For mass C: $F_{Total,Y_c} = T_c - F_{gc} = m_c a_{Yc}$

Note: Since all the masses are connected together, they will all have the same acceleration

$$|\vec{a}_{XA}| = |\vec{a}_{YB}| = |\vec{a}_{Yc}| \equiv a$$

Let B and C be falling ⇒ they will have negative accelerations

$$a_{yB} = a_{yC} = -a$$

But A will be moving in the positive x-direction:

$$a_{xA} = +a$$

We also know that $|\vec{T}_A| = |\vec{T}_B| \equiv T_A$ since it is the same string

Newton's 2nd law reduces to:

$$\left.\begin{cases} ① & F_N - m_A g = 0 \\ ② & T_A = m_A a \\ ③ & T_A - T_c - m_B g = -m_B a \\ ④ & T_c - m_c g = -m_c a \end{cases}\right\}$$

There are 4 unknowns ($F_N, T_A, T_c, a$) and 4 equations

Thus, this set of simultaneous equations has a unique solution for each unknown.

Now, what was the problem asking for again? I forgot after all this! → Yes, we are to find $T_c$.

$1^{st}$ Substitute $T_A$ in Eq. ②: $T_A = m_A a$ into Eq. ③

$$T_A - T_c - m_B g = -m_B a$$

$$⑤ \left\{ m_A a - T_c - m_B g = -m_B a \right\}$$

Solve for "a" in Eq. ④: $T_c - m_c g = -m_c a$

$$\Rightarrow a = -\frac{(T_c - m_c g)}{m_c}$$

Substitute this value of "a" into Eq. ⑤

Eq. ⑤: $m_A a - T_c - m_B g = -m_B a$

$$a(m_A + m_B) = T_c + m_B g$$

$$-\frac{(T_c - m_c g)}{m_c}(m_A + m_B) = T_c + m_B g$$

$$-T_c(m_A + m_B) + m_c g(m_A + m_B) = m_c(T_c + m_B g)$$

$$T_c(m_c + m_A + m_B) = m_c g(m_A + m_B) - m_c m_B g$$

$$T_c(m_A + m_B + m_c) = m_c m_A g + \cancel{m_c m_B g} - \cancel{m_c m_B g}$$

cancels out

$$\Rightarrow T_c = \left(\frac{m_A m_c}{m_A + m_B + m_c}\right) g$$

$$T_C = \left[\frac{(30\,kg)(10\,kg)}{(30+40+10)\,kg}\right](9.8\,m/sec^2) = 36.75\,N \approx 40\,N$$

We can also find out what $T_A$ and "a" are through a little more algebra as well.