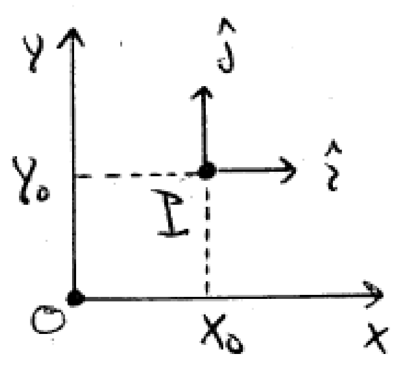
Vectors

Unit #3

A quantity that is just a numerical value is called a **Scalar Quantity**.

A quantity that is a number and a direction is called a **Vector Quantity**.

We need a formalism for describing the direction of an object. For a two-dimensional Cartesian coordinate system, this is typically done with unit vectors $\hat{i}$ and $\hat{j}$ (pronounced "i-hat" and "j-hat").



$\hat{i}$ and $\hat{j}$ = dimensionless unit vectors of magnitude 1

$$|\hat{i}| = |\hat{j}| = 1$$

All unit vectors do is just point in a particular direction:

$\hat{i} \Rightarrow$ points in the x-direction

$\hat{j} \Rightarrow$ points in the y-direction

The displacement of point P from the origin is then:

$$\text{displacement} = \vec{r} = x_0\hat{i} + y_0\hat{j}$$

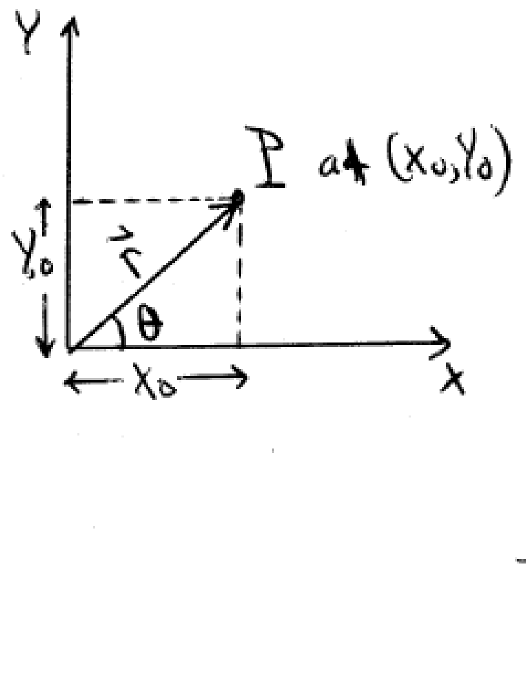
$$\text{magnitude of } \vec{r} = |\vec{r}| = |x_0\hat{i} + y_0\hat{j}|$$

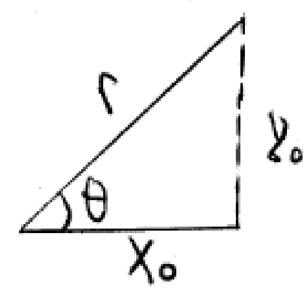
$$= \sqrt{x_0^2 + y_0^2} \equiv r$$

$\Rightarrow$ a scalar — note, no arrow on top

Thus, $\left\{ \begin{array}{l} \vec{r} = \text{a vector} \\ r = \text{a scalar} = \text{magnitude of } \vec{r} \end{array} \right\}$

Graphically:





By Pythagorean's Theorem

$$r = \sqrt{x_0^2 + y_0^2}$$

$$\tan\theta = \frac{y_0}{x_0}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{y_0}{x_0}\right)$$

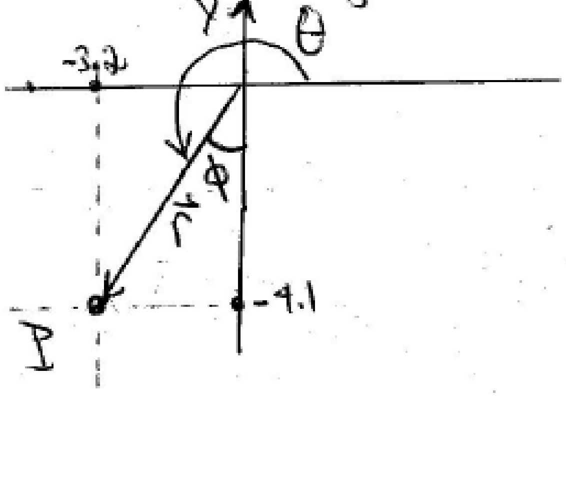
Thus, there are two ways to describe the vector $\vec{r}$:

Method #1: $\boxed{\vec{r} = X_o\hat{i} + Y_o\hat{j}}$ $\Rightarrow$ $\vec{r}$ expressed in terms of unit vectors

Method #2: $\vec{r}$ = magnitude $\sqrt{X_o^2 + Y_o^2}$
at an angle $\theta = \tan^{-1}\left(\frac{Y_o}{X_o}\right)$
relative to the x-axis

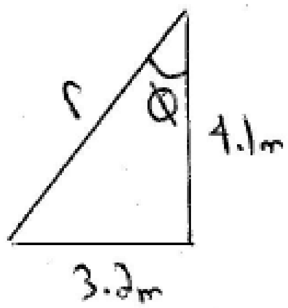
Both ways of expressing $\vec{r}$ are equivalent. For the rest of this course (and the rest of your life) all vectors are to be expressed in terms of unit vectors or in terms of a "magnitude <u>and</u> an angle θ relative to an axis."

Example: Point I is located at $x = -3.2m$, $y = -4.1m$.
(a) Express displacement vector $\vec{r}$ in terms of unit vectors
(b) Express displacement vector $\vec{r}$ in terms of a magnitude and an angle



(a) $\vec{r} = \left(-3.2\hat{i} - 4.1\hat{j}\right) m$

(b) $|\vec{r}| = \sqrt{(-3.2m)^2 + (-4.1m)^2}$
$= 5.20096145m$



$$\tan \Phi = \frac{3.2m}{4.1m}$$

$$\Rightarrow \Phi = \tan^{-1}\left(\frac{3.2}{4.1}\right) = 37.97160376^\circ$$

$$\Rightarrow \theta = 270^\circ - \Phi = 270^\circ - 37.97160376^\circ$$

$$= 232.0283962^\circ$$

Then $\boxed{\vec{r} \approx 5.2m \text{ at an angle } \theta \approx 232^\circ \text{ relative to the positive x-axis}}$

expressed with the proper number of significant figures

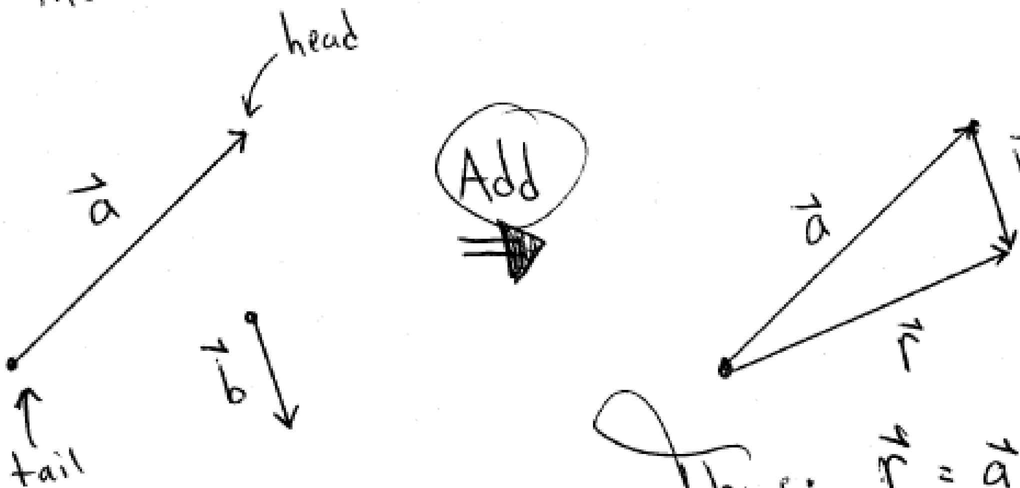
Adding Vectors:

To add, sum the components of each vector:

$$\vec{a} + \vec{b} = (a_x\hat{i} + a_y\hat{j}) + (b_x\hat{i} + b_y\hat{j})$$

$$= (a_x + b_x)\hat{i} + (a_y + b_y)\hat{j}$$

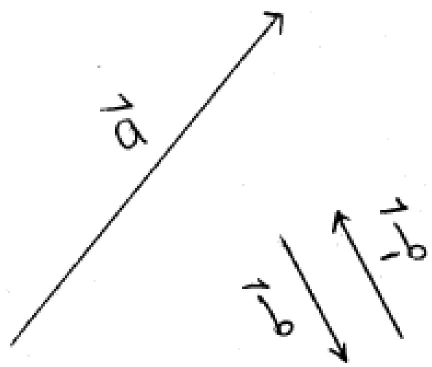
To graphically add vectors, we parallel transport one vector so that the tail of one vector is at the head of the other:



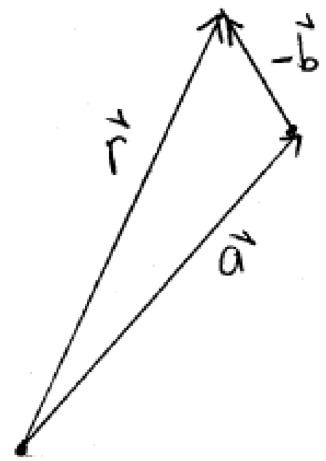
Thus: $\vec{r} = \vec{a} + \vec{b}$

$=$ sum (or resultant) vector

To subtract, set one vector to be antiparallel to itself (point in the opposite direction), then parallel transport it so that its tail is at the head of the other vector:

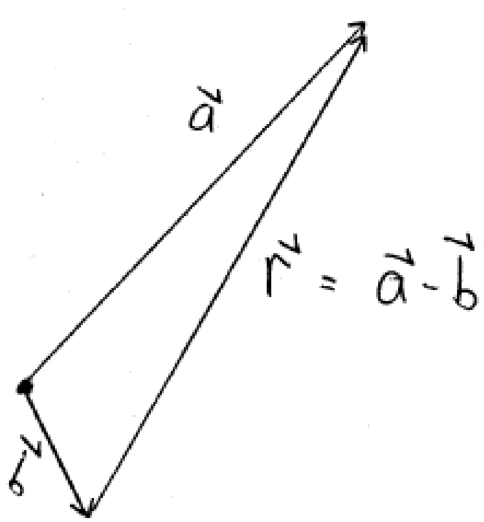


Subtract
$\vec{r} = \vec{a} - \vec{b}$
$\Longrightarrow$
$\vec{r} = \vec{a} + (-\vec{b})$

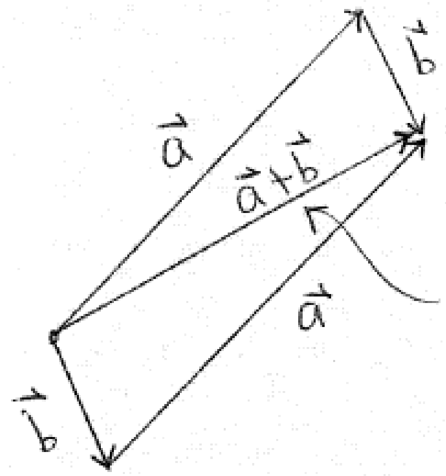


$$\begin{aligned} \vec{r} &= \vec{a} - \vec{b} \\ &= \vec{a} + (-\vec{b}) \end{aligned}$$

Alternatively, we can place the tails of vectors $\vec{a}$ and $\vec{b}$ together to subtract them

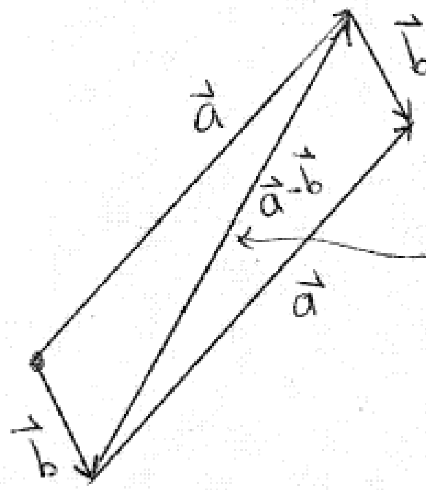


A useful nemonic is to try to form a parallelogram:



this diagonal is the sum

$\vec{r} = \vec{a} + \vec{b}$



this diagonal is the difference

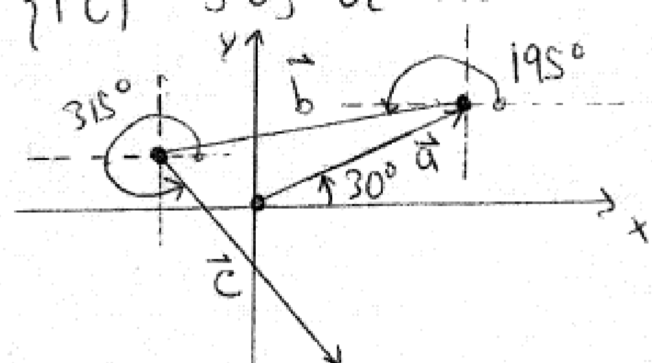
$\vec{r} = \vec{a} - \vec{b}$

Example: Add the three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$, where

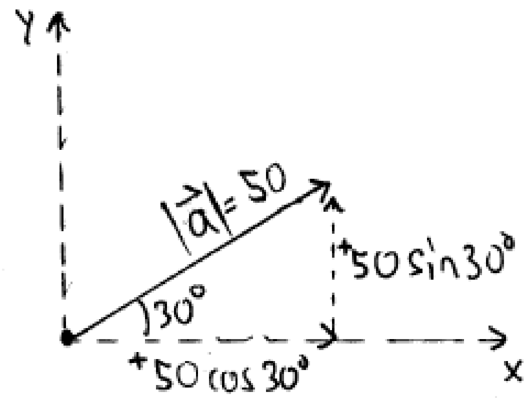
for $\vec{a} = \{|\vec{a}| = 50,\ \theta_a = 30°$ relative to positive x-axis$\}$

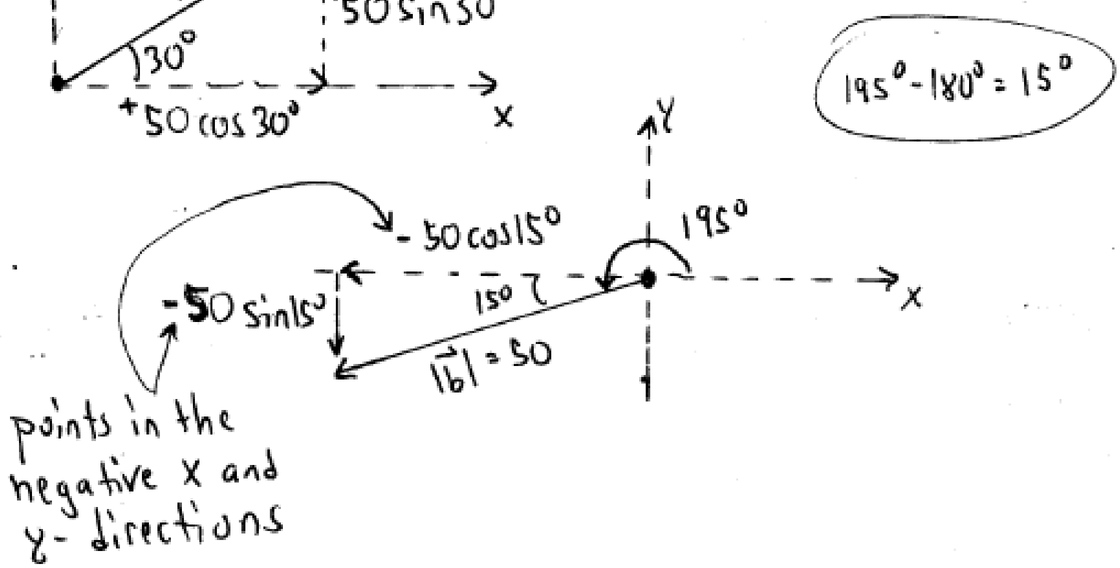
$\vec{b} = \{|\vec{b}| = 50,\ \theta_b = 195°$ " " " "$\}$

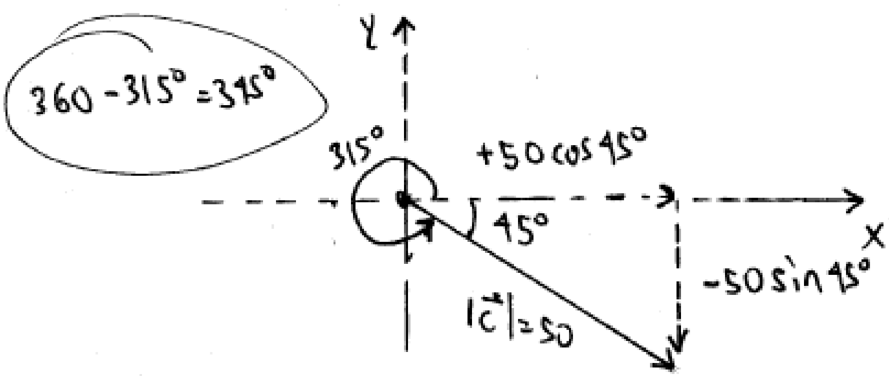
$\vec{c} = \{|\vec{c}| = 50,\ \theta_c = 315°$ " " " "$\}$



Examine each vector individually:







resultant vector

Add components in the x-direction: $\vec{a} + \vec{b} + \vec{c} = \vec{r}$

$r_x = a_x + b_x + c_x = 50\cos 30° - 50\cos 15° + 50\cos 45° = 30.36$

$r_y = a_y + b_y + c_y = 50\sin 30° - 50\sin 15° - 50\sin 45° = -23.296$

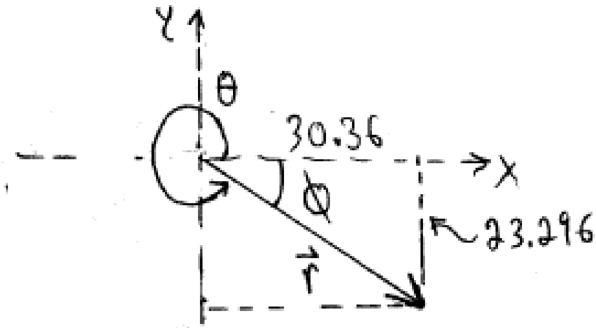
There are 3 ways to express $\vec{r}$ as a vector:

Method #1

$$\vec{r} = 30.36\,\hat{i} - 23.296\,\hat{j}$$ } Done!

Method #2 The magnitude of $\vec{r}$ is

$$|\vec{r}| = r = \sqrt{r_x^2 + r_y^2} = \sqrt{(30.36)^2 + (23.296)^2} = 38.268$$



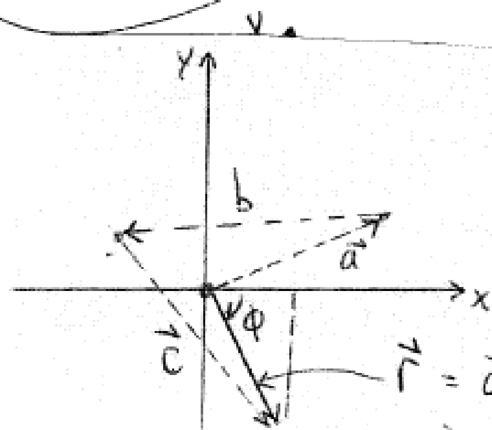
$$\tan\phi = \frac{23.296}{30.36} = \frac{r_y}{r_x}$$

$$\phi = \tan^{-1}\left(\frac{23.296}{30.36}\right) = 37.50^\circ$$

$$\theta = 360^\circ - \phi = 322.50^\circ$$

$\Rightarrow$ $\vec{r}$ = magnitude 38.268 at 322.50° relative to the positive x-axis

Method #3 Graphically (must draw $\vec{r}$ with correct length)



$\vec{r} = \vec{a} + \vec{b} + \vec{c}$

note: resultant vector $\vec{r}$ points in the positive x and negative y-direction