NORTHERN ILLINOIS UNIVERSITY

PHYSICS DEPARTMENT

Physics 210 – Mechanics & Heat Fall 2021

Lab #10

Lab Writeup Due: Tue/Wed/Thu, Nov. 9/10/11, 2021
Read Serway & Vuille: Chapter 10.3-10.5, Lecture Notes #6b, #8

Rolling

Apparatus

In this experiment a sphere, disk, and cylinder are rolled down an inclined plane with a raised guide to keep it on the track. Two photogates are positioned over the track to measure the velocity of each object at the position of each photogate. Each photogate only records the elapsed time between the when the object enters and leaves the photogate. The experimenter must determine the width d of the object as seen by the photogate detector to determine the velocity through each photogate.

Theory

Velocity is the time rate of change of position of an object. If the width, d, of an object and the time, Δt, it takes to pass a point are both known, the average velocity is

$$v_{ave} = \frac{d}{\Delta t} \tag{1}$$

Angular velocity is the time rate of change of the angle of a rotating object, measured in radians per second (rad/s). For an object that rolls without slipping the angular velocity is related to its linear velocity as

$$\omega = \frac{v}{R} \approx \frac{d}{R\Delta t} \tag{2}$$

The resistance of an object to a force (the inertia of an object) is caused by the object's mass. The resistance of an object to a torque (a force that causes a rotation) is caused by the object's moment of inertia (which is related to the object's mass and how far the mass is from the axis of rotation: $I \sim mR^2$). This is why when a figure skater on ice brings in her arms she rotates faster—her moment of inertial is smaller because more of her mass is closer to her body (her effective radius decreases). Decreasing the moment of inertia makes it easier for her to rotate (her rotational inertial decreases).

Objects in motion possess kinetic energy K. If the object is rolling it has kinetic energy due to the forward motion of its center of mass, K_{CM}, and its rotation, K_{rot}. Translational kinetic energy is based on the mass and velocity, $K_{CM} = \frac{1}{2} m v_{CM}^2$. Rotational kinetic energy about the center of mass is based on the moment of inertia and angular velocity, $K_{rot} = \frac{1}{2} I_{CM} \omega^2$.

$$K = \frac{1}{2} m v_{CM}^2 + \frac{1}{2} I_{CM} \omega^2 \tag{3}$$

Using Eq. (2) we can convert velocity to angular velocity:

$$K = \frac{1}{2} m R^2 \omega^2 + \frac{1}{2} I_{CM} \omega^2 = \frac{1}{2}\left(I_{CM} + mR^2\right)\omega^2 = \frac{1}{2} I \omega^2 \tag{4}$$

Notice that we have derived the parallel-axis theorem: $I = I_{CM} + Mh^2$ where, in the situation for this lab, the object rolls about an axis at the point where it touches the ramp, thus $h = R$.

As a disk rolls down a slope the gravitational potential energy, $U_g = mgh$, is converted into kinetic energy and overcomes the work done by friction, W_{f_k} (which results in thermal energy being generated). If the initial and final angular velocities are ω_i and ω_f, then the relationship for the conservation of energy is:

$$K_i + U_i = K_f + U_f + \Delta E_{thermal}$$

$$\frac{1}{2} I \omega_i^2 + mgh_i = \frac{1}{2} I \omega_f^2 + mgh_f + \Delta E_{thermal} \tag{5}$$

This equation [and using Eq. (4)] can be rearranged to solve for the moment of inertia about the center of mass, I_{CM}. Rolling friction is relatively small, so neglecting the thermal energy it generates gives:

$$I_{CM} = \frac{2mg(h_f - h_i)}{\omega_i^2 - \omega_f^2} - mR^2 \tag{6}$$

Data Collection

(1) Measure the **mass** and **diameter** of each object (use the Vernier caliper to measure the diameter). Record this in your logbook.

(2) Measure the coefficient of static friction, f_s, for the solid cylinder (see your PreLab) for the current inclination angle of the ramp. Determine if any *sliding* will occur (*note*: we only want rolling [without *sliding*] to occur). If necessary, adjust the inclination angle of the ramp so that *sliding* does not occur. Protractors are available in the lab (or you can measure the angle using the method you used in your ***Inclined Plane lab***). Assume that your measured value of f_s is valid for all the objects: sphere, solid cylinder, hollow cylinder.

(3) Measure the height of the track at each photogate. This gives you h_f and h_i. Also measure the distance between the photogates. This give you L.

(4) Place the sphere at the upper photogate so it just begins to interrupt the beam as indicated by the red light. Place a ruler with its flat side on the track so that it stops the sphere just before it enters the photogate and causes the red light to come on (the photogate should be adjusted just high enough for the ruler to slide under it). Mark the position of the ruler on the track (such as with tape) just before the red light comes on, and allow the sphere to roll through the gate, recording the distance d_i^{sphere} until the sphere no longer interrupts the photogate beam (the red light goes off). Do the same for the disk and the cylinder to get d_i^{disk} and $d_i^{cylinder}$. This gives you the width d of each object as seen by the photogate detector.

(5) Repeat Step (3) for the lower photogate and record the distances d_f^{sphere}, d_f^{disk}, and $d_f^{cylinder}$ in your logbook.

(6) Start the Logger Pro program, release the sphere from rest as close to the photogate as possible without starting it. Record the times measured by the computer (both in an Excel spreadsheet and in your lab notebook). Make certain to label which pair of times are for the upper and lower photogate.

(7) Repeat the measurement from the same starting point a total of six times. Find the average final angular velocity ω_f (*note*: $\omega_i = 0$ since the object is released from rest). Record this calculation in your lab notebook.

(8) Repeat Step (6) for the disk and the cylinder.

Data Analysis

Make certain Part (6) below is done before leaving lab

(1) Complete the following sentence: To ensure rolling occurs without sliding, the ramp must be adjusted so that the solid cylinder (with its flat base on the ramp) ________________ (*complete the statement*).

(2) Find I_{CM} for the sphere, disk, and cylinder using Eq. (6).

(3) Using the table at the end of this lab writeup, calculate I_{CM} for the sphere, disk, and cylinder.

(4) Computer the *percent error* of I_{CM} for each object:

$$\%Error = \left| \frac{Experimental - Theoretical}{Theoretical} \right| \times 100$$

(5) Comment on the accuracy of your measurements and why there may be a discrepancy.

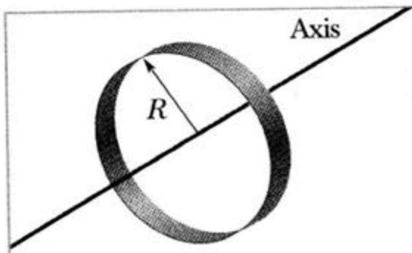
(6) Which one of the objects is the fastest? (That is, which one makes it to the bottom of the ramp first?)

(7) *Using the wooded ramp located at the front of the lab class*, test your prediction by racing the sphere, disk, and cylinder (2 at a time) to see which one is fastest. Did all go according to your prediction?

(8) Give a simple explanation as to why the winning object reached the bottom of the ramp first.

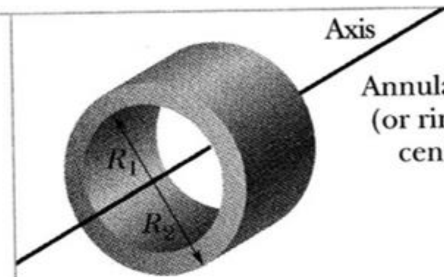
TABLE 10-2

Some Rotational Inertias



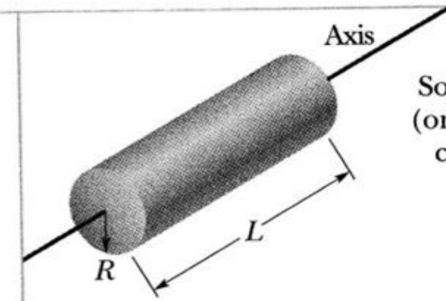
Hoop about central axis

$I = MR^2$ (*a*)



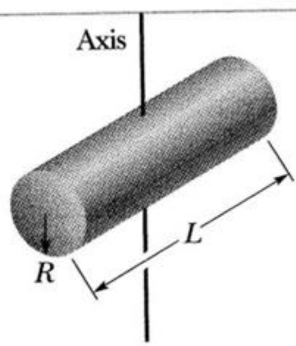
Annular cylinder (or ring) about central axis

$I = \frac{1}{2}M(R_1{}^2 + R_2{}^2)$ (*b*)



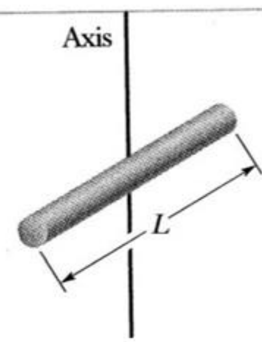
Solid cylinder (or disk) about central axis

$I = \frac{1}{2}MR^2$ (*c*)



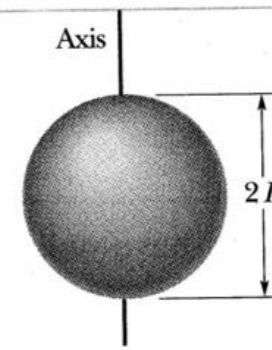
Solid cylinder (or disk) about central diameter

$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$ (*d*)



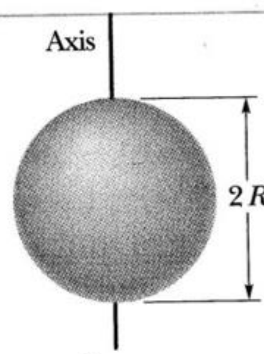
Thin rod about axis through center perpendicular to length

$I = \frac{1}{12}ML^2$ (*e*)



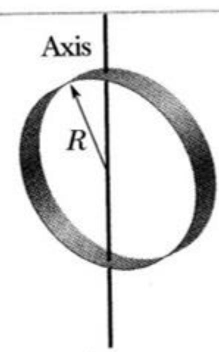
Solid sphere about any diameter

$I = \frac{2}{5}MR^2$ (*f*)



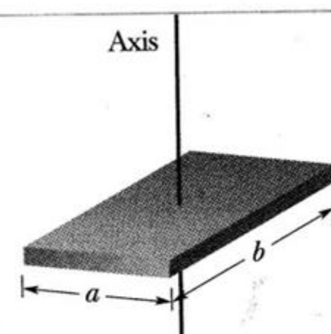
Thin spherical shell about any diameter

$I = \frac{2}{3}MR^2$ (*g*)



Hoop about any diameter

$I = \frac{1}{2}MR^2$ (*h*)



Slab about perpendicular axis through center

$I = \frac{1}{12}M(a^2 + b^2)$ (*i*)