N-Body Simulation Code Documintation

June 2, 2017

1 Introduction

The code "n.body.sim.fox" is written in COSY INFINITY 9.1. It uses Picard iteration method to solve the N-Body problem of particles interacting via Coulomb potential. The particles can be relativistic, and there can be external electric and/or magnetic fields. The default mass and charge are set to the mass and charge of a proton. For other particles, f is the mass factor, and n is the charge factor with respect to proton's. The system of differential equations that the code solve is:

$$\frac{d\hat{Y}_i}{d\hat{t}} = \begin{bmatrix} \frac{\hat{p}_{x_i}}{\sqrt{f_i^2+\hat{p}_{x_i}^2+\hat{p}_{y_i}^2+\hat{p}_{z_i}^2}} \\ \frac{\hat{p}_{y_i}}{\sqrt{f_i^2+\hat{p}_{x_i}^2+\hat{p}_{y_i}^2+\hat{p}_{z_i}^2}} \\ \frac{\hat{p}_{z_i}}{\sqrt{f_i^2+\hat{p}_{x_i}^2+\hat{p}_{y_i}^2+\hat{p}_{z_i}^2}} \\ \frac{q\,n_i}{mc^2}\left[\frac{q}{4\pi\epsilon_0}\sum_{\substack{j=1\\ j\neq i}}^{N}\frac{n_j\,(x_i-x_j)}{\gamma\left[(x_i-x_j)^2+(y_i-y_j)^2+\gamma^2\,(z_i-z_j)^2\right]^{3/2}}+E_{x_i}+c\,(\hat{v}_{y_i}B_{z_i}-\hat{v}_{z_i}B_{y_i})\right] \\ \frac{q\,n_i}{mc^2}\left[\frac{q}{4\pi\epsilon_0}\sum_{\substack{j=1\\ j\neq i}}^{N}\frac{n_j\,(y_i-y_j)}{\gamma\left[(x_i-x_j)^2+(y_i-y_j)^2+\gamma^2\,(z_i-z_j)^2\right]^{3/2}}+E_{y_i}+c\,(\hat{v}_{z_i}B_{x_i}-\hat{v}_{x_i}B_{z_i})\right] \\ \frac{q\,n_i}{mc^2}\left[\frac{q}{4\pi\epsilon_0}\sum_{\substack{j=1\\ j\neq i}}^{N}\frac{n_j\gamma\,(z_i-z_j)}{\left[(x_i-x_j)^2+(y_i-y_j)^2+\gamma^2\,(z_i-z_j)^2\right]^{3/2}}+E_{z_i}+c\,(\hat{v}_{x_i}B_{y_i}-\hat{v}_{y_i}B_{x_i})\right] \end{bmatrix}$$

where: $\hat{Y}_i=(x_i,y_i,z_i,\hat{p}_{x_i},\hat{p}_{y_i},\hat{p}_{z_i})\,,\quad \hat{t}=\frac{t}{c},\quad f_i=\frac{m_i}{m},\quad n_i=\frac{q_i}{q},\quad i=1,2,\ldots,NP$

$$\hat{p}_{x_i} = \frac{p_{x_i}}{mc}, \quad \hat{p}_{y_i} = \frac{p_{y_i}}{mc}, \quad \hat{p}_{z_i} = \frac{p_{z_i}}{mc}$$

$$\hat{v}_{x_i} = \frac{\hat{p}_{x_i}}{\sqrt{f_i^2 + \hat{p}_{x_i}^2 + \hat{p}_{y_i}^2 + \hat{p}_{z_i}^2}}, \quad \hat{v}_{y_i} = \frac{\hat{p}_{y_i}}{\sqrt{f_i^2 + \hat{p}_{x_i}^2 + \hat{p}_{y_i}^2 + \hat{p}_{z_i}^2}}, \quad \hat{v}_{z_i} = \frac{\hat{p}_{z_i}}{\sqrt{f_i^2 + \hat{p}_{x_i}^2 + \hat{p}_{y_i}^2 + \hat{p}_{z_i}^2}}$$

2 Description

All the necessary variables are defined in the beginning of the code. Then, the physics constants are set, and the DA vectors are initialized. The code calculates the initial momentum and energy of the system, and it calculates the final momentum and energy of the system at the end of the simulation.

2.1 Input and Output

The input can be set manually in the code, or they can be read from a data file. The output can be written to the screen or to a data file.

INPUT

NP $\rightarrow$ Number of Particles
NITER $\rightarrow$ Number of Time Steps
DELT $\rightarrow$ Time Step Size
ORDER $\rightarrow$ Number of Picard Iterations
$\hat{Y}\left(x, y, z, \hat{p}_x, \hat{p}_y, \hat{p}_z, f, n\right) \rightarrow$ Initial Conditions of Each Particle at $t = 0$

OUTPUT

$\hat{Y}\left(t\right) \rightarrow \left\{x\left(t\right), y\left(t\right), z\left(t\right), \hat{p}_x\left(t\right), \hat{p}_y\left(t\right), \hat{p}_z\left(t\right)\right\}$ for Each Particle at Each Time Step

2.2 Loops

2.2.1 Main Loop

The main loop is the "J" loop, which is the time stepper loop. It loops over the number of time steps and evaluates $\hat{Y}\left(t\right)$ at each time step. The loop performs the following operations:

- Calculates the average $\hat{p}_z$ of the system in order to evaluate γ as $\hat{p}_z = \sqrt{\gamma^2 - 1}$
- Starts Picard iteration loop
- Evaluates $\hat{Y}\left(t\right)$ for each particle
- Forwards the lab time used in electric and magnetic fields calculations

2.2.2 Picard Iteration Loop

It is the "L" loop from 1 to the specific order we want to truncate Taylor series at. To perform Picard iterations, the loop does the following:

- Calculates the system of the differential equations $\frac{d\hat{Y}(t)}{d\hat{t}}$ for each particle
- Integrates theses equations iteratively up to the chosen order in accordance to Picard iteration method

3 Examples

3.1 Example1

This is an example of two protons with the initial conditions:

$$\hat{Y}_1(0) = \{0, 0, 0, 0, 0, 0, 1, 1\}$$

$$\hat{Y}_2(0) = \{5 \times 10^{-14}, 0, -5 \times 10^{-13}, 0, 0, 0.02, 1, 1\}$$

The number of time steps is 400 steps with a step size of 10^{-13} seconds, and the order of integration is 5. The outputs are written to the file "data.txt". The final coordinates of the particles are:

$$\hat{Y}_1(t) = \{-4.2 \times 10^{-14}, 0, 2.1 \times 10^{-14}, -2.99 \times 10^{-3}, 0, 5.8 \times 10^{-4}, 1, 1\}$$

$$\hat{Y}_2(t) = \{9.2 \times 10^{-14}, 0, 2.8 \times 10^{-13}, 2.99 \times 10^{-3}, 0, 1.9 \times 10^{-2}, 1, 1\}$$

The trajectories of theses two protons are shown in Figure1.

3.2 Example2

In this example, there are 100 particles. The inputs of these particles is imported from the files "tightn.ssv" and "massAndcharge100.ssv". The number of time steps is 90 steps with a step size of 10^{-5} seconds, and the order of integration is 2. The outputs are written to the files: "x_SA.dat", "y_SA.dat", "z_SA.dat", "p_x_SA.dat", "p_y_SA.dat", and "p_z_SA.dat".

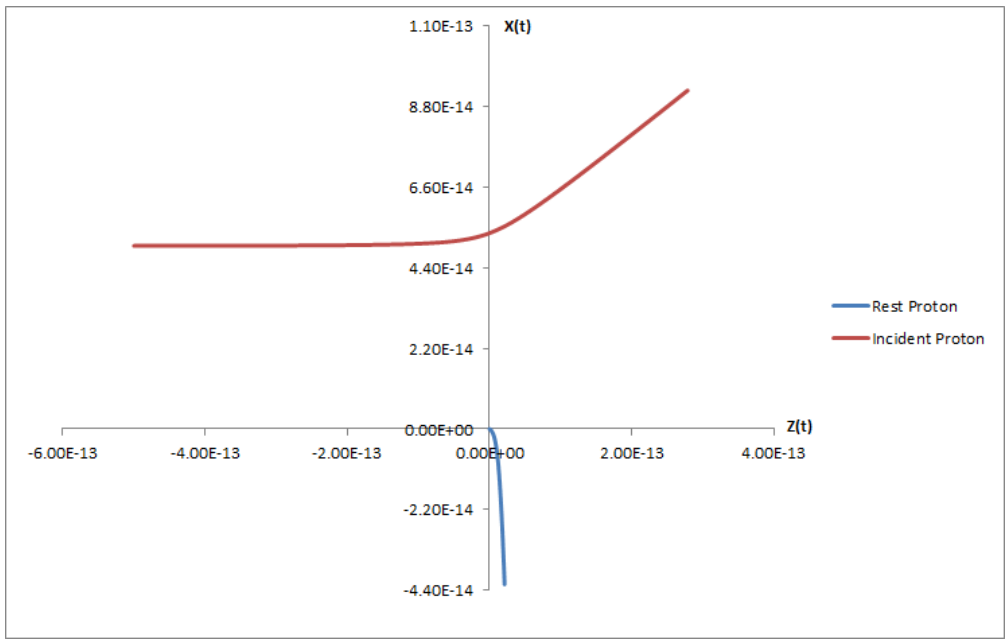

Figure 1: The protons trajectories as integrated in Example1