Magnetoresistance Anisotropy of a One-Dimensional Superconducting Niobium Strip

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We investigated confinement effects on the resistive anisotropy of a superconducting niobium strip with a rectangular cross section. When its transverse dimensions are comparable to the superconducting coherence length, the angle dependent magnetoresistances at a fixed temperature can be scaled as $R(\theta, H) = R(H/H_{c\theta})$ where $H_{c\theta} = H_{c0}(\cos^2\theta + \gamma^{-2}\sin^2\theta)^{-1/2}$ is the angular dependent critical field, γ is the width to thickness ratio, and H_{c0} is the critical field in the thickness direction at $\theta = 0°$. The results can be understood in terms of the anisotropic diamagnetic energy for a given field in a one-dimensional superconductor.

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London theory [1] predicts that properties of a superconductor can be characterized by the penetration depth λ and that size effect should be observed in a superconductor with characteristic dimension comparable to λ (Ref. [2]). For example, the critical field H_{cw} in the long axis direction for a superconducting wire with diameter $D \sim \lambda$ can be $8\lambda/D$ times larger than the bulk thermodynamic critical field H_{cb} (Refs. [2–4]). The critical field $H_{c\parallel}$ of a superconducting thin film in parallel fields can also be increased by a factor of $2\sqrt{6}\lambda/d$ from its bulk value H_{cb}, when its thickness d is comparable to λ (Refs. [5,6]). The enhancement occurs because the penetration of the magnetic field leads to an incomplete flux exclusion, thereby reducing the diamagnetic energy for a given magnetic field [5,6]. Such confinement effects can introduce anisotropic behavior to a bulk isotropic superconductor when its dimensions are reduced to the order of λ for nonspherical geometries. Two-dimensional (2D) superconducting thin films have been intensively investigated for confinement-induced anisotropy due to their ease of fabrication. For example, the angular dependence of the critical field $H_{c\theta}$ of a 2D thin film follows the well-known Tinkham formula [5,6]

$$\frac{H_{c\theta}\cos\theta}{H_{c\perp}} + \left(\frac{H_{c\theta}\sin\theta}{H_{c\parallel}}\right)^2 = 1, \quad (1)$$

where $H_{c\perp}$ and $H_{c\parallel}$ are the critical fields for the magnetic field perpendicular and parallel to the film surface, respectively, and θ is the angle between the magnetic field and the axis normal to the film.

Recent advances in nanofabrication technologies have made it possible to produce mesoscopic superconductors in a more controlled way [7–12], enabling the pursuit of superconducting properties in desired confinement geometries. Among those studied, quasi-one-dimensional (1D) superconductors have attracted intense attention. They provide unique platforms to explore the phenomena of thermal and quantum phase slips [10–14]. However, exploration of their confinement-induced anisotropic properties is long overdue. In this Letter we investigate the resistive anisotropy of a 1D superconducting strip patterned out of a 2D thin film. We find that the angle dependent magnetoresistance of a 1D strip depends only on the reduced magnetic field. The angular dependence of the critical field is found to differ from that of a 2D thin film and can be derived from Ginzburg-Landau theory with an angle dependent diamagnetic energy for a given magnetic field.

We carried out experiments on superconducting niobium strips patterned out of 100 nm thick niobium films with focused-ion-beam (FIB) milling (FEI Nova 600). A strip with a desired width and four contacts for four-probe transport measurements was inscribed out of the film by milling grooves of 150–200 nm wide using FIB machining (30 keV Ga^+, 10–20 nm beam diameter) to separate it from the rest of the film. The scanning electron microscopy (SEM) image presented in the inset of Fig. 1 (sample *A*) shows that the strip has a smooth surface and sharp edges. Two strips with widths $w = 150$ nm (sample *A*) and 217 nm (sample *B*) were investigated with dc four-probe transport measurements using a constant current mode with a current density of $\sim 2 \times 10^4$ A/cm^2. A criterion of $0.5R_N$, where R_N is the normal state resistance, was used to define the critical temperature T_c and the critical field H_c. The zero-field critical temperature T_{c0} for samples *A* and *B* is 7.038 and 7.641 K, respectively. We also measured one unpatterned reference film for comparison and its T_{c0} is 8.850 K. Angular dependencies of the resistance and the critical field were obtained by placing the sample on a precision, stepper-controlled rotator with a rotation range of $-10°$ to $370°$ and an angular resolution of $0.05°$ (see upper inset of Fig. 1 for the definition of θ). The magnetic field is always perpendicular to the current I which flows along the long axis of the strip.

We measured the magnetoresistance $R(\theta, H)$ at a fixed temperature in two ways. In the first method, we chose a

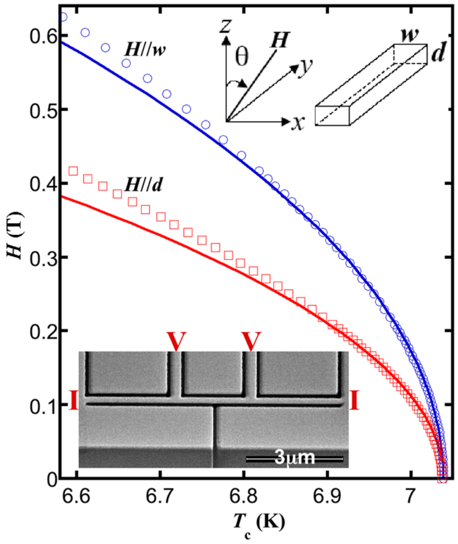

FIG. 1 (color online). Magnetic field versus temperature (H-T) phase diagrams of sample A. The symbols are experimental data and solid lines represent the parabolic fits (see text). Lower inset is a SEM micrograph of the strip and upper inset presents schematic of the width $\boldsymbol{w}$ and thickness $\boldsymbol{d}$ directions, the coordinates and the angle θ for the field direction. $\theta = 0°$ and $90°$ corresponds to $\boldsymbol{H} \parallel \boldsymbol{d}$ and $\boldsymbol{H} \parallel \boldsymbol{w}$, respectively.

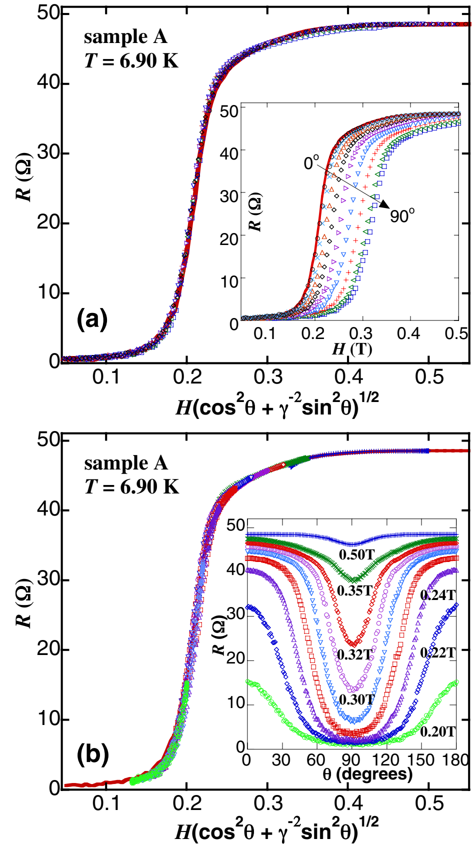

FIG. 2 (color online). Scaling behavior of the anisotropic resistance of sample A at 6.90 K. (a) for $R(H)$ curves at various field directions and (b) for $R(\theta)$ curves in various fixed fields. The main panels present the data of R versus $H(\cos^2\theta + \gamma^{-2}\sin^2\theta)^{1/2}$ with $\gamma = 1.52$, and the original data are presented in the insets.

particular applied magnetic field angle θ with respect to the sample and swept the field H_θ (where H_θ denotes the applied field in the direction of angle θ). In the second method, we chose a particular magnetic field value H and rotated the field angle θ with respect to the sample. The data obtained with the first and second methods for sample A at a fixed temperature (6.90 K) close to T_{c0} are given in the insets of Figs. 2(a) and 2(b), respectively. They indicate that the resistance of the niobium strip in the superconducting state is anisotropic, with higher dissipation for magnetic fields applied at $\theta = 0°$. Figure 2(a) also shows that the critical field is angle dependent and increases as the magnetic field tilts towards the plane ($\theta = 90°$). Although this behavior is qualitatively consistent with that expected from Eq. (1), the critical field of a strip has a different angular dependence from that of a thin film, as revealed below.

The main panel of Fig. 2(a) presents the essential finding of our research: $R(H_\theta)$ curves obtained for finite θ can be scaled to that obtained at $\theta = 0°$ by multiplying the field with a factor of $(\cos^2\theta + \gamma^{-2}\sin^2\theta)^{1/2}$, where $\gamma = 1.52$. Hence, the resistance of the strip has the scaling behavior $R(\theta, H) = R(H_\theta \varepsilon / H_{c0})$ with $\varepsilon = (\cos^2\theta + \gamma^{-2}\sin^2\theta)^{1/2}$ where H_{c0} is the critical field at $\theta = 0°$ and γ is close to the width to thickness ratio of the strip.

Similar to a thin film in parallel fields, the density of Cooper pairs $|\psi|^2$ of a strip at temperatures close to T_{c0} should be proportional to $1 - (H/H_c)^2$, where $|\psi|$ is the Ginzburg-Landau order parameter [6]. This means that the field dependence of the dissipation $R(H)$ should be a function of H/H_c. Consequently, the scaling behavior observed in Fig. 2(a) implies an angle dependence of the critical field

$$H_{c\theta} = H_{c0}/\varepsilon = \frac{H_{c0}}{(\cos^2\theta + \gamma^{-2}\sin^2\theta)^{1/2}}. \qquad (2)$$

That is, the angular dependence of the resistance originates *solely* from the angle dependent critical field. This can be further verified by the scaling behavior of the $R(\theta)$ curves obtained at various fixed magnetic field with a smaller angle interval. The main panel of Fig. 2(b) demonstrates that all the $R(\theta)$ curves from the inset of Fig. 2(b) can be scaled onto the $R(H_0)$ curve at $\theta = 0°$ at the same temperature if the data are plotted as R versus $H(\cos^2\theta + \gamma^{-2}\sin^2\theta)^{1/2}$.

The above scaling behavior for the $R(\theta, H)$ curves indicates that the resistances obtained at different angles but at the same reduced field $H_\theta/H_{c\theta}$ should be equivalent. The variation of the resistance $R(\theta, H)$ with angle θ for a fixed field H is due to the change in the reduced field $H_\theta/H_{c\theta}$ since the critical field $H_{c\theta}$ depends on θ. Thus, we can use the $R(H_\theta)$ curve at a specific angle to derive the

$R(\theta)$ curve at a fixed field. For example, the value at a field H_0 of the $R(H_0)$ curve at $\theta = 0°$ should be the same as that of the $R(\theta)$ curve at $\theta = \arccos\{[\gamma^2(H_0/H)^2 - 1]/(\gamma^2 - 1)\}^{1/2}$ obtained at a fixed field H since in both cases the reduced fields are the same, i.e., $H_0/H_{c0} = H_\theta/H_{c\theta}$. Obviously, the reduced field at $H_0 = H/\gamma$ and $\theta = 0°$ is equal to that at H and $\theta = 90°$. This means that the $R(H_0)$ curve at fields H_0 between H/γ and H can be used to derive the $R(\theta)$ curve obtained at a fixed field H between $\theta = 90°$ to $0°$ if the above conversion from H_0 to θ is applied. We confirmed this scaling by comparing the $R(\theta)$ curves derived from the $R(H_0)$ curve at $\theta = 0°$ with the directly measured curves. The results for sample A are presented in Fig. 3(a) for data obtained at a fixed temperature (6.90 K) and various fixed magnetic fields H and Fig. 3(b) for those at a fixed field (0.4 T) and various temperatures. For the derivation of $R(\theta)$ from the $R(H_0)$ curve at $\theta = 0°$ we used $\gamma = 1.52$ which is the ratio of the critical fields at $\theta = 90°$ and $0°$. It is very close to the nominal value of $w/d = 1.50$. As revealed below, this is because the critical fields at $\theta = 90°$ and $0°$ are inversely proportional to d and w, respectively.

The scaling behavior of the data in Figs. 2 and 3 implies that the angular dependence of the critical field for a superconducting strip should follow Eq. (2). It is different from Eq. (1) and indicates that the reduction of the dimension of a superconductor from 2D to 1D will affect its anisotropy. In order to confirm this observation we obtained the critical fields at a fixed temperature and various angles for both the reference 2D film and the wire strips. The data for the reference film and sample A obtained at the same reduced temperature ($T/T_{c0} = 0.98$) are given in the inset of Fig. 4(b), and additional data for the strips are presented in the main panel of Fig. 4(a). The striking feature of the data is a lack of a cusp at $\theta = 90°$ for the strips compared to the reference film which follows the Tinkham formula Eq. (1) nicely. Instead, the data for the strips are consistent with the relationship derived from the scaling behavior of $R(\theta, H)$, i.e., Eq. (2). For comparison, we also present as a dashed curve in Fig. 4(a), the expected anisotropy of a strip (sample A at 6.90 K) based on Eq. (1). It deviates significantly from the experimental results and demonstrates the effect of confinement on the anisotropy of a superconductor.

Following the procedure used to derive the enhanced critical field of a superconducting thin film in parallel fields, the above anisotropy of the critical field can be obtained with Ginzburg-Landau theory in which the free energy density f is expanded as [5]

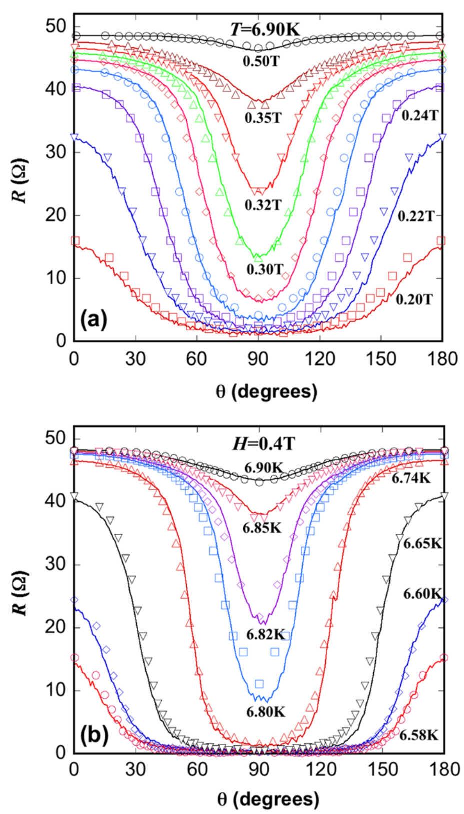

FIG. 3 (color online). Scaling behavior of the anisotropic resistance of sample A. (a) $R(\theta)$ curves at a fixed temperature (6.90 K) and in various fixed fields H. (b) $R(\theta)$ curves in a fixed field H (0.4 T) and at various fixed temperatures. The solid lines are directly measured data and symbols are values derived from the $R(H_0)$ curves obtained at $\theta = 0°$ (see text).

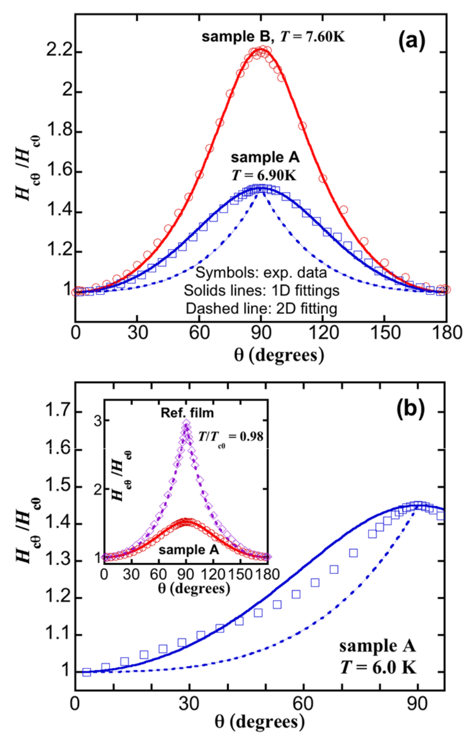

FIG. 4 (color online). Angle dependence of the critical field $H_{c\theta}$. (a) at 6.90 K for sample A and 7.60 K for sample B. (b) at 6.0 K for sample A. The inset of (b) compares $H_{c\theta}$ curves of sample A and the reference film at the same reduced temperature ($T/T_{c0} = 0.98$). $H_{c\theta}$ is normalized to H_{c0}, the value at $\theta = 0°$. The dashed lines and solid lines are fits with Eqs. (1) and (2) (for 2D and 1D), respectively.

$$f = f_{no} + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m^*}\left|\left(\frac{\hbar}{i}\nabla - \frac{e^*}{c}A\right)\psi\right|^2 + \frac{h^2}{8\pi}, \quad (3)$$

where $|\psi|$ is the Ginzburg-Landau order parameter, $\boldsymbol{A}$ is the vector potential, $\alpha = -2e^2H_{\rm cb}^2\lambda^2/mc^2$, $\beta = 16\pi e^4H_{cb}^2\lambda^4/m^2c^4$, λ is the effective penetration depth in zero field, $m^* = 2m$ and $e^* = 2e$ are the mass and charge of the Cooper pairs, h is the field in the superconductor, and $H_{\rm cb}$ is the bulk thermodynamic critical field. For the coordinate system displayed in the upper inset of Fig. 1, where x and z are measured from the strip's central line, y lies along the strip axis, and the magnetic field H is applied in the xz plane at an angle θ to the z axis, the vector potential can be chosen to be [6]

$$A_y = H(x\cos\theta - z\sin\theta). \quad (4)$$

Thus, the Gibbs free energy per unit length of the strip is

$$G = \iint\left(f - \frac{hH}{4\pi}\right)dxdz, \quad (5)$$

with the integral lower and upper limits of $-w/2$ and $w/2$ for x and $-d/2$ and $d/2$ for z, respectively. Substituting Eqs. (3) and (4) into Eq. (5) and minimizing the Gibbs free energy G with respect to $|\psi|^2$, we find

$$|\psi|^2 = -\frac{\alpha}{\beta}\left(1 - \frac{w^2H^2}{24\lambda^2H_{\rm cb}^2}(\cos^2\theta + \gamma^{-2}\sin^2\theta)\right), \quad (6)$$

with $\gamma = w/d$. Thus, the critical field $H_{c\theta}$ at which the strip becomes normal, i.e., $|\psi|^2 = 0$, is given by

$$H_{c\theta} = \frac{2\sqrt{6}H_{\rm cb}\lambda}{w(\cos^2\theta + \gamma^{-2}\sin^2\theta)^{1/2}}, \quad (7)$$

For the critical field applied along the z direction ($\theta = 0°$), the critical field $H_{c0} = 2\sqrt{6}H_{\rm cb}\lambda/w$. Thus, Eq. (7) is exactly the same as Eq. (2). Together with the fact that the supercurrent velocity $\boldsymbol{v}_s$ is proportional to the vector potential $\boldsymbol{A}$ in a narrow strip in which $|\psi|$ is uniform in space [6], this self-consistency indicates that the anisotropy of the critical field of a strip originates from the anisotropic diamagnetic energy in a confined geometry.

Since the temperature dependence of the bulk thermodynamic critical field and penetration depth at temperatures close to T_{c0} follow $H_{\rm cb} \sim (1 - T/T_{c0})$ and $\lambda \sim 1/(1 - T/T_{c0})^{1/2}$, Eq. (7) leads to $H_{c\theta} \sim (1 - T/T_{c0})^{1/2}$. This relation indeed describes the H-T phase diagrams of the strips near T_{c0}. Examples are given in Fig. 1 for sample A in magnetic fields aligned parallel to the x and z directions, respectively. The solid lines represent the parabolic fits, which follow the experimental data nicely at $T_x > 6.8$ K and $T_z > 6.9$ K for fields in the x and z directions, respectively. The zero temperature effective penetration depth $\lambda(0)$ of Nb is larger than 80 nm (Ref. [15]). Thus $\lambda(T)$ is larger than the transverse dimensions of the strip at $T > 5$ K. The deviation between the solid lines and experimental data at temperatures below T_x and T_z indicates that in a Nb strip the superconducting coherence length ξ replaces the penetration depth λ to become the characteristic length for observing 1D behavior Eq. (7). This is understandable since Nb is type II with $\xi(0)$ much smaller than $\lambda(0)$. Using the zero-temperature coherence length $\xi(0)$ of 9.8 nm obtained from the reference film, we find that the strip at the above temperature ranges behaves as a 1D superconductor, since $d/\xi(T_x) = 1.84$ and $w/\xi(T_z) = 2.1$. Vortices will form when the transverse dimensions of the strip are larger than $2\xi(T)$, leading to the failure of the assumption of a uniform $|\psi|$ in the x and z direction used in deriving Eq. (7). However, the increase in the rate of the number of vortices is proportional to $w - 2\xi$ (or $d - 2\xi$) (Ref. [6]). Thus, the scaling behavior can still work at temperatures beyond the 1D range where the appearance of vortices does not alter the properties of the strip significantly. In fact, as demonstrated by the data shown in Fig. 3(b), $R(H, \theta)$ scales at temperatures down to 6.58 K where $d/\xi = 2.55$ and $w/\xi = 3.83$. At lower temperatures the critical field $H_{c\theta}$ at 6.0 K [Fig. 4(b)] shows significant derivation from Eq. (7), though it describes the data better than the thin film formula Eq. (1).

In summary, the magnetoresistance anisotropy of a 1D superconducting niobium strip is found to originate solely from the angle dependent critical field caused by the confinement-induced anisotropic diamagnetic energy.

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