Problem 1. Consider the process of photon-photon scattering to produce an electron-positron pair:

\[ \gamma \gamma \rightarrow e^- e^+ \]

Work in the center-of-momentum frame, assign the initial-state photons 4-momenta \( p_a \) and \( p_b \), and call their energies \( E = \sqrt{s}/4 \). Assign the final-state electron and positron 4-momenta \( k_1 \) and \( k_2 \), and let \( \theta \) be the angle between one of the initial-state photons and the \( e^- \). Call the mass of the electron \( m \).

(a) Draw the Feynman diagram(s) which contribute to the reduced matrix element for this process at order \( e^2 \).

(b) Write down a complete expression for the reduced matrix element for the process, but you don’t have to simplify it.

(c) At this point, you have a choice. (Choose wisely!) You can either compute the spin summed and averaged reduced matrix element squared, \( \frac{1}{4} \sum_{\text{spins}} |M|^2 \), by hand, or you can compute it using crossing symmetry from results given in the notes.

(d) From the answer to part (c), compute the differential cross-section \( d\sigma/d(cos\theta) \).

(e) Find the total cross-section for the limit \( 4m^2/s \rightarrow 0 \), and for the limit of small but non-vanishing \( s/4m^2 - 1 \), and for \( s/4m^2 \rightarrow 0 \).

Hint: you will likely find the following definite integrals to be useful.

\[
\int_{-1}^{1} dx \frac{1}{1 - a^2 x^2} = \frac{1}{a} \ln \left( \frac{1 + a}{1 - a} \right),
\]

\[
\int_{-1}^{1} dx \frac{1}{(1 - a^2 x^2)^2} = \frac{1}{1 - a^2} + \frac{1}{2a} \ln \left( \frac{1 + a}{1 - a} \right).
\]

Problem 2. Draw the following Feynman diagrams in QED. In each case, write down consistent expressions for the reduced matrix elements in terms of clearly defined momenta and spins, but you do not need to simplify or evaluate them.

(a) All tree-level diagrams contributing to \( e^- e^+ \rightarrow \mu^+ \mu^- \gamma \)

(b) All one-loop diagrams contributing to \( e^- e^+ \rightarrow \mu^+ \mu^- \). [Here, you do not need to write down the reduced matrix elements for the subset of diagrams that just involve corrections to external legs. They have to be handled differently, by a method that we haven’t gone over in class.]

(c) A representative diagram contributing to \( \gamma \gamma \rightarrow \gamma \gamma \)

(d) All diagrams contributing to \( e^- \mu^+ \rightarrow \mu^- e^+ \)