Fun with relativistic kinematics!

**Problem 1.** Using data from the “Review of Particle Properties” and/or the lecture notes, find numerical values for the mean distance traveled by a muon, a neutral pion, a charged pion, a tau lepton, a $K^+$ meson, a $D^+$ meson, and a $B^+$ meson produced in high-energy physics experiments, for the following values of the particle energy: 10 GeV, 100 GeV, 1000 GeV.

**Problem 2.** What is the threshold energy for the initial-state proton, for the reaction $p^+ n \rightarrow p^+ p^+ \pi^-$, assuming the neutron is initially at rest?

**Problem 3.** Consider a particle of mass $M$, which undergoes a 2-body decay to particles $a, b$ of masses $m_a$ and $m_b$. You are encouraged to set the speed of light $c$ to be 1 (this is a choice of units). Assuming that all motion is along the $z$ direction, find the four-momenta of the particles $a, b$ for:

- (a) the special case $m_b = m_a$.
- (b) the special case $m_b = 0$.
- (c) the general case. Write your answer in the simplest form. [Hint: write your answer in terms of $\lambda \equiv M^4 + m_a^4 + m_b^4 - 2m_a^2M^2 - 2m_b^2M^2 - 2m_a^2m_b^2$.] Check that your answer agrees with the special cases in parts (a) and (b).

**Problem 4.** Consider the elastic scattering of a muon neutrino off of an electron: $\nu_\mu e^- \rightarrow \nu_\mu e^-$. Suppose that in the lab frame the electron is initially at rest and the neutrino has energy $E$, and treat the neutrino as massless.

- (a) Write expressions for the 4-vectors of the particles in the center-of-momentum frame.
- (b) In the lab frame, what is the maximum angle of the scattered electron with respect to the incoming neutrino?