Final 12/10/2010

1. Derive the density of states for a free-electron model in one dimension.

2. Let us consider a one-dimensional chain of atoms. The hybridization matrix element between the atoms is alternating $t - \Delta t$ and $t + \Delta t$, where $\Delta t \ll t$ (this could be due to a displacement of every other atom, but we neglect the displacements and take the interatomic distance a). Take the on-site energies zero.
(a) Calculate the eigenenergies of this system.
(b) Plot the eigenenergies in the first Brillouin zone for $\Delta t \cong 0.1t$.
(c) What is the size of the gap at the edge of the first Brillouin zone?

3. Consider a (two-dimensional) centered rectangular lattice. Treat this system as a rectangular lattice with sides a_x and a_y and a basis. Determine the reciprocal lattice and the structure factors $S_{\mathbf{K}}$.

4. Consider a (two-dimensional) rectangular lattice with $a_x \cong 0.8a_y$. In reciprocal space, draw the first three Brillouin zones.

5. In Fig. 41 of the notes, it is shown that the Fermi surface for half filling is a square. What is the shape of the Fermi surface for very small fillings (for example, with a Fermi energy of $E_F/t \cong -3.8$? Explain.