

$B=0$ spin splitting

Spin degeneracy

spatial inversion symmetry:

$$E(\mathbf{k}, \uparrow) = E(-\mathbf{k}, \uparrow)$$

time inversion symmetry (Kramers):

$$E(\mathbf{k}, \uparrow) = E(-\mathbf{k}, \downarrow)$$

$$E(\mathbf{k}, \uparrow) = E(\mathbf{k}, \downarrow)$$

Inversion asymmetry

\Rightarrow

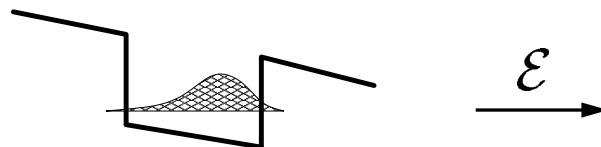
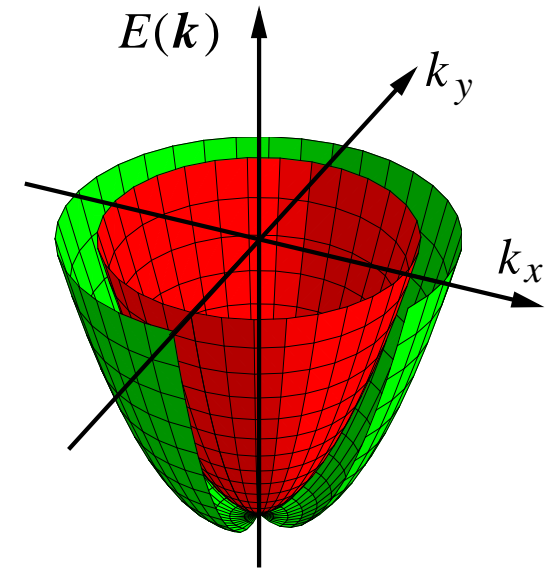
$$E(\mathbf{k}, \uparrow) \neq E(\mathbf{k}, \downarrow)$$

\Rightarrow two energy surfaces $E_{\pm}(\mathbf{k})$

Caused by:

spin-orbit interaction

+ {
bulk inversion asymmetry, e.g., zinc blende structure
structure inversion asymmetry, e.g., gate



Contributions to $B=0$ spin splitting in 2D electron systems

(1) Bulk Inversion Asymmetry (BIA)

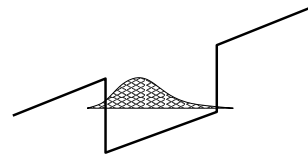
zinc blende structure \Rightarrow Dresselhaus term

$$\hat{H}_D = \eta [k_x (k_y^2 - k_z^2) \sigma_x + \text{cp}]$$

(2) Structure Inversion Asymmetry (SIA)

\Rightarrow Rashba term

$$\hat{H}_R = \alpha (\nabla V) \times \mathbf{k} \cdot \boldsymbol{\sigma}$$



(3) Interface Inversion Asymmetry (IIA)

$$\hat{H}_I = \pm \Upsilon [k_z, \delta(z - z_i)] (k_x \sigma_x - k_y \sigma_y)$$

Spin Splitting and Effective Magnetic Field

$$\hat{H}_{SO} = \mathcal{B}(\mathbf{k}) \cdot \boldsymbol{\sigma}$$

with $\mathcal{B}(\mathbf{k}) = \mathcal{B}_D(\mathbf{k}) + \mathcal{B}_R(\mathbf{k}) + \mathcal{B}_I(\mathbf{k})$

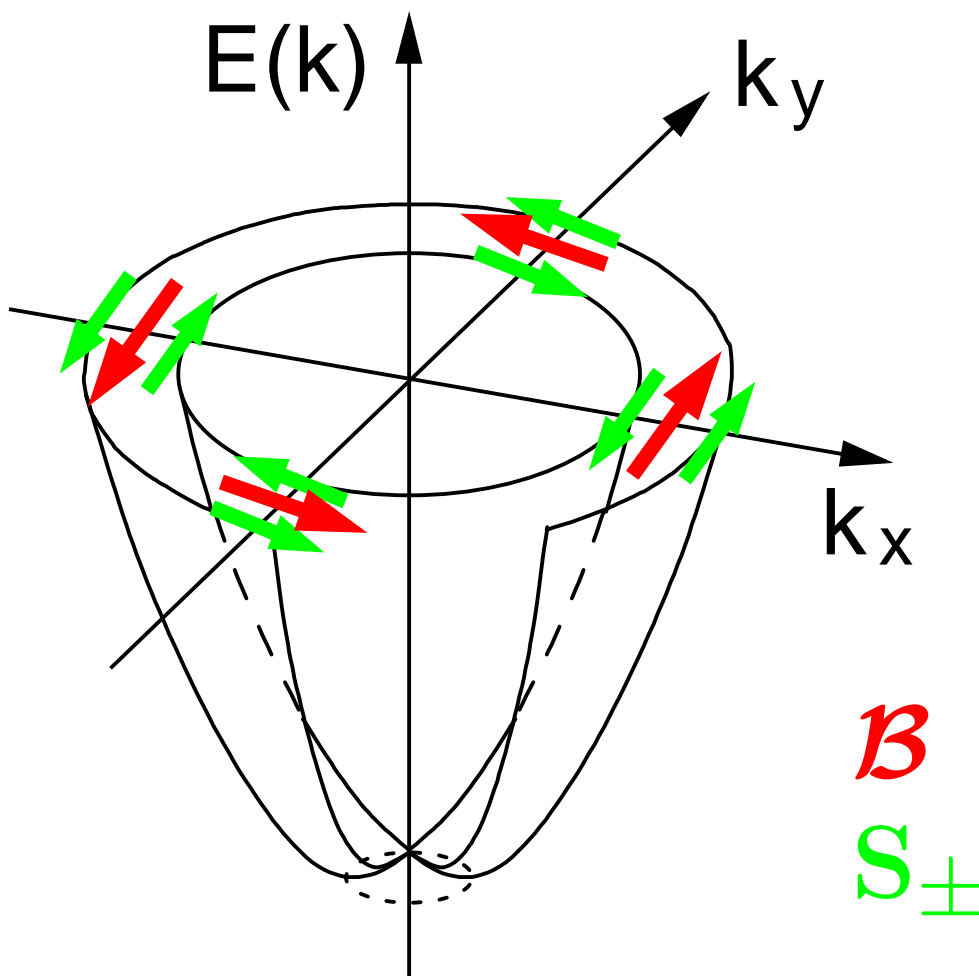
spin orientation at \mathbf{k} (anti) parallel to $\mathcal{B}(\mathbf{k})$

Rashba Spin Splitting

$$H_R = \alpha \mathcal{E} \times \mathbf{k} \cdot \boldsymbol{\sigma} = \alpha \mathcal{E}_z \begin{pmatrix} 0 & ik_- \\ -ik_+ & 0 \end{pmatrix}$$

$$k_{\pm} = k_x \pm ik_y$$

$$\Delta E(k_{\parallel}) = \alpha \mathcal{E}_z k_{\parallel}$$



Dresselhaus Spin Splitting

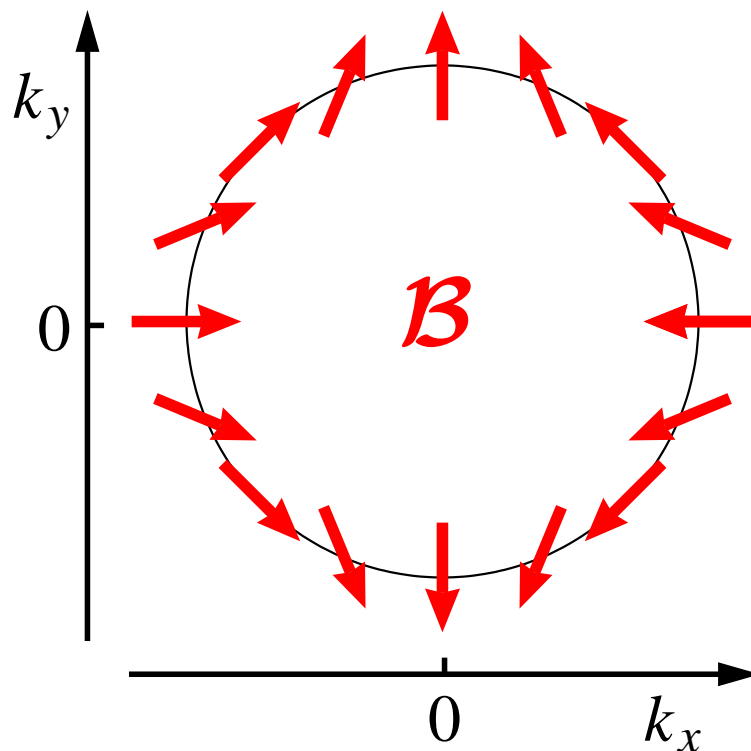
$$2\text{D: } k_z \rightarrow \langle k_z \rangle = 0$$

$$k_z^2 \rightarrow \langle k_z^2 \rangle \neq 0$$

(a) [001] surface

$$H_D = -\eta \langle k_z^2 \rangle \begin{pmatrix} 0 & k_+ \\ k_- & 0 \end{pmatrix} + \mathcal{O}(k_{\parallel}^3)$$

$$\Delta E(\mathbf{k}_{\parallel}) = \pm \eta \langle k_z^2 \rangle k_{\parallel} \pm \mathcal{O}(k_{\parallel}^3)$$

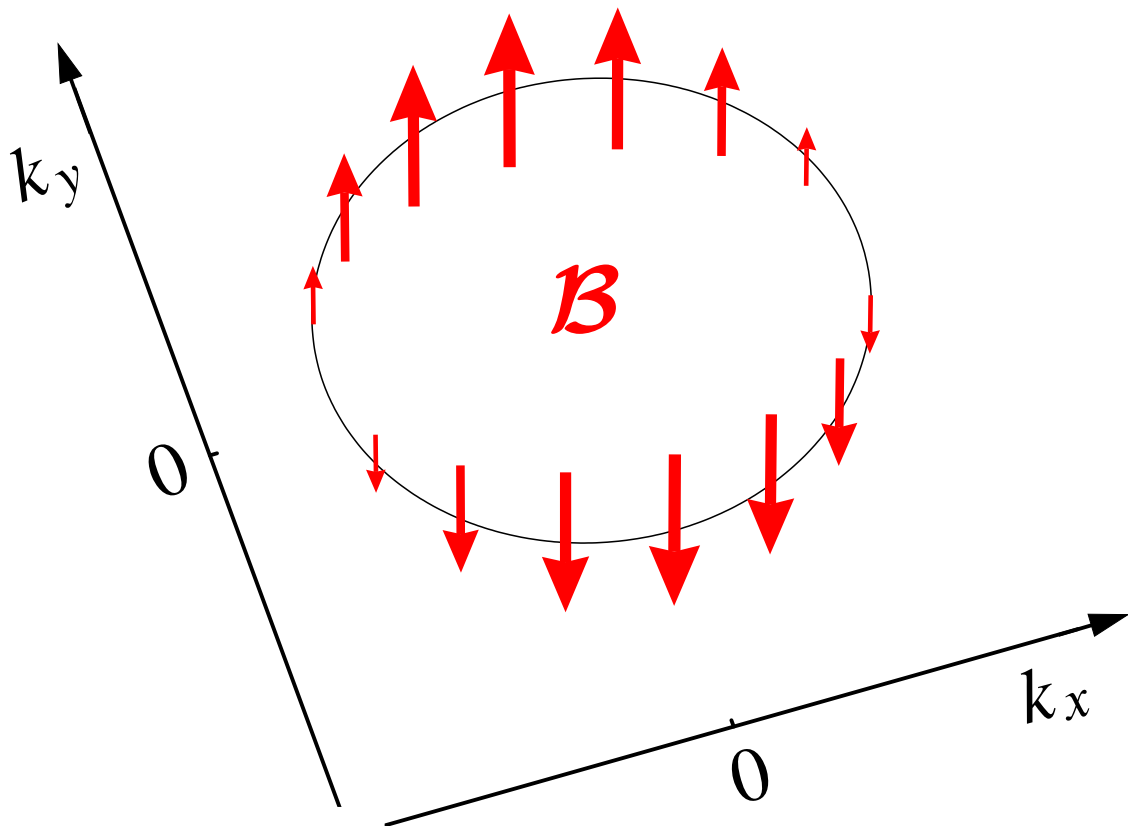


Dresselhaus Spin Splitting

(b) [110] surface

$$H_D = \eta \langle k_z^2 \rangle \begin{pmatrix} k_y & 0 \\ 0 & -k_y \end{pmatrix} + \mathcal{O}(k_{\parallel}^3)$$

$$\Delta E(\mathbf{k}_{\parallel}) \approx \pm \eta \langle k_z^2 \rangle k_y \pm \mathcal{O}(k_{\parallel}^3)$$



Effective magnetic field

Bulk Inversion Asymmetry (BIA)

Structure Inversion Asymmetry (SIA)

BIA and SIA

