Physics 686 **Homework 7** **Due April 7, 2004**

Problem 1. In class and in the Primer, we discussed the O'Raifeartaigh model, with the superpotential

$$W = -k\Phi_1 + m\Phi_2\Phi_3 + \frac{y}{2}\Phi_1\Phi_3^2,$$

with y, k, m each real and positive. We considered the case $m^2 > yk$. Now, consider the opposite case:

$$m^2 < yk.$$

(a) Prove that at the minimum of the potential, $F_3 = 0$ always.
(b) Find the values of ϕ_3, F_1, and F_2 at the minimum of the potential. (You should find that they are all non-zero.) What can you say about the values of ϕ_1 and ϕ_2 at the minimum? [Hint: there is a "flat direction".]
(c) Show that the value of the potential at its minimum is

$$V_{\rm min} = (n_1 ykm^2 + n_2 m^4)/y^2,$$

where n_1 and n_2 are non-zero integers that you will find.
(d) Find the part of the Lagrangian that involves the fermion fields ψ_1, ψ_2, ψ_3. In this expression, replace ϕ_3 by its VEV that you found in part (b). Write the resulting Lagrangian in the form:

$$\mathcal{L} = i\psi^{\dagger i}\overline{\sigma}^\mu\partial_\mu\psi_i + \frac{1}{2}[\psi_i M^{ij}\psi_j + \text{c.c.}]$$

where M is a 3×3 mass matrix.
(e) Square the mass matrix M found in part (d), and obtain the eigenvalues of M^2. For each eigenvalue m_i^2, find the corresponding eigenvector. Write these as normalized eigenvectors

$$\psi_i' = N_i{}^j\psi_j \tag{1}$$

with

$$\sum_j |N_i{}^j|^2 = 1.$$

for each i. This implies that N is a unitary matrix. [Here $\psi_{1,2,3}$ are the original fermion fields, and $\psi'_{1,2,3}$ are the squared-mass eigenstate fields. Four of the entries of M^2 should

vanish; the same is true of N.] Check your work by rewriting the fermion part of the Lagrangian as:

$$\mathcal{L} = i\psi^{\dagger i\prime}\overline{\sigma}^{\mu}\partial_{\mu}\psi'_i + \frac{1}{2}[\sum_i m_i\psi'_i\psi'_i + \text{c.c.}].$$

(f) What is the goldstino?

(g) Find the part of the Lagrangian involving only the scalars. In it, make the replacement:

$$\phi_3 = \langle\phi_3\rangle + \phi'_3,$$

where $\langle\phi_3\rangle$ is the VEV that you found in part (b), and then throw away the (scalar)^3 and (scalar)^4 parts of the potential. Write the resulting potential in the form:

$$V = V_{\text{min}} + (\,\phi_1^* \quad \phi_2^*\,)\,\mu^2\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} + a|\phi'_3|^2 + b(\phi'^2_3 + \phi'^{*2}_3),$$

where μ^2 is a 2×2 matrix that you will find, and a and b are squared mass quantities. (The part of the potential that is linear in the scalar fields should vanish.)

(h) Find the squared mass eigenvalues and eigenvectors for ϕ_1 and ϕ_2, using the matrix μ^2 found in part (g). [Hint: this should be remarkably similar to part (e). The correct eigenvectors are linear combinations of ϕ_1 and ϕ_2; there is no need to split them up into real and imaginary parts.]

(i) Write $\phi'_3 = (R_3 + iI_3)/\sqrt{2}$ where R_3 and I_3 are real scalar fields. Find squared-mass matrix for these fields, using the results of part (g). What are the squared-mass eigenvalues?

(j) Make a list of all of the real scalar and Weyl fermion squared masses in the theory, as found above, and check for evidence that supersymmetry was indeed broken. Finally, check that the sum rule, equation (6.13) in the Primer, is satisfied.