Physics 686 **Homework 4** **Due March 3, 2004**

Problem 1. (a) Prove equation (3.12) in the Primer notes by brute force. (Write out each side explicitly in terms of the 1,2 components of all of the spinors involved, first for $\alpha = 1$, and then for $\alpha = 2$.)

(b) Use the same method to prove the Fierz identity:

$$(\psi^\dagger \overline{\sigma}^\mu \chi)(\xi^\dagger \overline{\sigma}_\mu \eta) = N(\psi^\dagger \xi^\dagger)(\chi\eta)$$

where N is an integer that you will identify.

Problem 2. Consider the massless, non-interacting Wess-Zumino model for a chiral supermultiplet, as discussed in class, with the Lagrangian:

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + i\psi^\dagger \overline{\sigma}^\mu \partial_\mu \psi + F^* F$$

and the SUSY transformation given by eqs. (3.3), (3.15) and (3.17) of the Primer notes.

(a) For two different SUSY transformation parameters ϵ_1 and ϵ_2, compute

$$\delta_{\epsilon_2}(\delta_{\epsilon_1} X)$$

for each of $X = \phi, \phi^*, \psi, \psi^\dagger, F, F^*$.

(b) Show explicitly (this means do not omit details, or argue "by analogy...") that equation (3.18) is true for each X.

(c) In the Primer notes, the variation of $\mathcal{L}$ is claimed to be a total derivative:

$$\delta\mathcal{L} = \partial_\mu K^\mu.$$

Verify this explicitly, and show that K^μ can be written as

$$K^\mu \;=\; N_1\, \epsilon\sigma^\mu\psi^\dagger\, F \;+\; N_2\, \epsilon\psi\, \partial^\mu\phi^* \;+\; N_3\, \epsilon\sigma^\nu\overline{\sigma}^\mu\psi\, \partial_\nu\phi^* \;+\; N_4\, \epsilon^\dagger\psi^\dagger\, \partial^\mu\phi$$

where N_1, N_2, N_3 and N_4 are (possibly complex) numbers that you will find. [Hints: each of the N_i has magnitude 1. You may find equation (3.7) to be useful.]

(d) Use the result of part (c) to prove that equation (3.19) implies equation (3.20).

(e) Now suppose that we add a superpotential

$$W = \frac{m}{2}\phi^2 + \frac{y}{6}\phi^3.$$

Write out the full Lagrangian $\mathcal{L}$ without eliminating the auxiliary field. Find the supercurrent and its Hermitian conjugate in the interacting theory. [Hint: it is different from the supercurrent in the non-interacting case.] Now eliminate the auxiliary field by its equation of motion, and rewrite the full Lagrangian and the supercurrent and its Hermitian conjugate.