Physics 686 **Homework 1** **Due February 3, 2004**

Problem 1. Consider a theory with a charge +1 complex scalar field ϕ^+ and its complex conjugate field $\phi^- = (\phi^+)^*$ coupled to the photon, with the Lagrangian:

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + D_\mu\phi^- D^\mu\phi^+ - m^2\phi^+\phi^-.$$

Here $F^{\mu\nu}$ is the usual Maxwell field strength for the photon field A^μ, and D_μ is the covariant derivative:

$$\begin{aligned} D_\mu\phi^+ &= (\partial_\mu + i|e|A_\mu)\phi^+, \\ D_\mu\phi^- &= (\partial_\mu - i|e|A_\mu)\phi^-. \end{aligned}$$

(a) Write down all of the Feynman rules for this theory. (The case of a three-particle interaction vertex with a spacetime derivative acting on a field can be treated by replacing the derivative with i times the momentum of the field it acts on. See page 163 in the notes for a similar case.)
(b) Work out the kinematics for the process $\gamma\gamma \to \phi^+\phi^-$ in the center-of-momentum frame. (This means: assign 4-momenta to each initial-state and final-state particle; compute all possible dot products of these 4-momenta in terms of the Mandelstam variables s, t, u, and m; and write t, u in terms of s, m, and the angle θ of the final state ϕ^+ particle.) Do not assume that m is small.
(c) Draw the three (3) tree-level Feynman diagrams for this process, and write down the corresponding reduced matrix element $\mathcal{M}$, using your Feynman rules.
(d) Compute the differential cross section $d\sigma/d(\cos\theta)$. Assume that the initial state photon spins are both unpolarized. (In the Real World, a photon-photon collider could be part of a linear collider program; it would have photons in polarized spin states, however!)
(e) Integrate the differential cross section to get the total cross section. (Use $x = \cos\theta$ as your integration variable.)
(f) How does the total cross-section behave for $s \gg m^2$?

Problem 2. Repeat parts (b)–(f) of the previous problem, but now for the standard QED process $\gamma\gamma \to e^+e^-$, summing over the final-state spins. Do not neglect the electron mass. You may either do the computation of the spin-summed reduced matrix element

from scratch, or by using equation (6.245) and the Crossing Symmetry theorem in the notes. Compare your answers to the previous problem.

<u>Problem 3.</u> (a) Consider a generic Lagrangian density $\mathcal{L}$ depending on generic fields Φ_i subject to the global symmetry transformations

$$\delta_{\epsilon^a}\Phi_i = i\epsilon^a T_i^{aj}\Phi_j,$$

satisfying

$$\delta\mathcal{L} = \epsilon^a(C_a + \partial_\mu K_a^\mu).$$

If $C_a = 0$, then the theory is invariant under the global symmetry; that is the case we studied in class. Suppose that the global symmetry is explicitly broken, so that $C_a \neq 0$. Show that the Noether current

$$J_a^\mu = -iT_i^{aj}\Phi_j\frac{\partial\mathcal{L}}{\partial(\partial_\mu\Phi_i)} + K^\mu$$

satisfies

$$\partial_\mu J_a^\mu = ?$$

when the equations of motion are satisfied, and identify the right hand side.
(b) In class, we discussed the case of chiral symmetries of fermions, governed by the Lagrangian:

$$\mathcal{L} = i\overline{\Psi}^i\gamma^\mu\partial_\mu\Psi_i - \overline{\Psi}^i m_i{}^j\Psi_j \tag{1}$$

where $i = 1, \ldots, N$. If the mass matrix $m_i{}^j$ is absent, then this Lagrangian is invariant under $SU(N)_V$ and $U(1)_V$ transformations:

$$\begin{aligned}\delta_{\epsilon_V^a}\Psi_{Li} &= i\epsilon_V^a T_i^{aj}\Psi_{Lj}\\ \delta_{\epsilon_V^a}\Psi_{Ri} &= i\epsilon_V^a T_i^{aj}\Psi_{Rj}\end{aligned}$$

and $SU(N)_A$ and $U(1)_A$ transformations:

$$\begin{aligned}\delta_{\epsilon_A^a}\Psi_{Li} &= i\epsilon_A^a T_i^{aj}\Psi_{Lj}\\ \delta_{\epsilon_A^a}\Psi_{Ri} &= -i\epsilon_A^a T_i^{aj}\Psi_{Rj}.\end{aligned}$$

Write each of these in terms of the full Dirac spinor, using the matrix γ_5, so in the form:

$$\begin{aligned}\delta_{\epsilon_V^a}\Psi_i &= i\epsilon_V^a T_i^{aj}(?)\Psi_j \\ \delta_{\epsilon_A^a}\Psi_i &= i\epsilon_A^a T_i^{aj}(?)\Psi_j.\end{aligned}$$

Write down the corresponding transformations of the fields $\overline{\Psi}^i$.

(c) Find the Noether currents for the $SU(N)_V$, $U(1)_V$, $SU(N)_A$, and $U(1)_A$ symmetry transformations. [In doing this, remember that fermion fields anti-commute. To be consistent, this means that the operation of taking a derivative with respect to a fermionic field must also acquire a minus sign when it moves past a fermionic field! In other words, if A, B are any fermionic objects, or more generally any Grassman objects, then:

$$\frac{\partial}{\partial A}(AB) = B,$$

but

$$\frac{\partial}{\partial A}(BA) = -B.$$

Also, you will want to watch out for total derivative terms.]

(d) What is $\partial_\mu J^\mu$ for the Noether currents you identified in part (c), when the equations of motion are satisfied, but including the general mass matrix m?

(e) Find the Noether charges Q_{Va} and Q_{Aa} for the $SU(N)_V$, $U(1)_V$, $SU(N)_A$, and $U(1)_A$ symmetry transformations. Write them in terms of conjugate momentum fields for the Dirac fermions [see equations (4.104) and (4.107) in the notes], eliminating the $\overline{\Psi}^i$ fields. Use these expressions to compute

$$\begin{aligned}\left[Q_{Va}, \Psi_i(\vec{\mathbf{y}})\right] &= ?, \\ \left[Q_{Aa}, \Psi_i(\vec{\mathbf{y}})\right] &= ?\end{aligned}$$

(note these are commutators, not anticommutators!) and comment on the results.

<u>Problem 4.</u> There are 18 parameters in the Standard Model of particle physics, not counting gravity or neutrino mass effects. Name them all, and say roughly what is known experimentally about each. (I'll get you started; the top Yukawa coupling is known to be approximately 1...)