Reading assignment: sections 3.8, 3.9, 4.1, 4.2, and 4.3 of Carroll.

Problem 1. Do exercise 8 on pages 148-149 of Carroll.

Problem 2. Consider the three dimensional cylinder, with independent coordinates $(z, \theta, \phi)$, and metric:

$$ds^2 = dz^2 + A^2(d\theta^2 + \sin^2 \theta d\phi^2)$$

where $A$ is a constant. Find the Riemann tensor, the Ricci tensor, and the Ricci scalar. Show that $R_{\rho\sigma\mu\nu} \neq Kg_{\rho[\mu}g_{\nu]\sigma}$ for any constant $K$. Hence, this is not a maximally symmetric space; compare to equation (3.191) and the surrounding discussion in Carroll. This is despite the fact that the Ricci scalar curvature is constant everywhere. (In fact, it is possible to show the even stronger statement that the Riemann tensor is covariantly constant, $\nabla_\kappa R_{\rho\sigma\mu\nu} = 0$, but you don’t need to do that.) How many independent Killing vector fields do you think this metric has? You don’t need to find them from scratch, but you ought to be able to write them down from other results that we’ve seen. In words, what isometries do they correspond to?

Problem 3. Do exercise 13 on page 149 of Carroll.