Physics 686 Homework Set 6 **Due Monday, October 18, 2021**

Problem 1. Consider the scalar $\lambda\phi^4$ theory discussed in section 4.4. Draw all Feynman diagrams corresponding to $\phi\phi \to \phi\phi\phi\phi$ scattering, to order λ^2. You should find 10 distinct diagrams, which can be organized into two distinct classes. Clearly label the particle 4-momenta for each external particle and internal propagator line in your diagram. Use Feynman rules to write down an expression for the reduced matrix element for the process. (You do *not* need to find the cross-section.)

Problem 2. Consider the process of antimuon scattering off of an electron:

$$\mu^+ e^- \to \mu^+ e^-$$

You may assume m_e is negligible, but keep m_μ. Work in the center-of-momentum frame. Assign the incoming electron and antimuon 4-momenta p_a and p_b, and call the magnitude of their 3-momenta P. Assign the final-state electron and antimuon 4-momenta k_1 and k_2, and note that the magnitude of their 3-momenta is also P. (In working this problem, do *not* use crossing symmetry. For the purposes of this homework problem, that is considered cheating. If you don't know what crossing symmetry is yet, that's fine.)

(a) Draw the Feynman diagram(s) which contribute to the reduced matrix element for this process at order e^2.

(b) Define θ to be angle between the initial-state e^- and the final state e^- directions in the center-of-momentum frame. Work out all of the following quantities in terms of m_μ, P, and $\cos\theta$:

$$\begin{aligned}
&p_a^2, \qquad p_b^2, \qquad k_1^2, \qquad k_2^2, \\
&(p_a \cdot p_b), \qquad (p_a \cdot k_1), \qquad (p_a \cdot k_2), \\
&(p_b \cdot k_1), \qquad (p_b \cdot k_2), \qquad (k_1 \cdot k_2), \\
&s = (p_a + p_b)^2, \qquad t = (p_a - k_1)^2, \qquad u = (p_a - k_2)^2.
\end{aligned}$$

(c) Use the Feynman rules of QED to obtain the reduced matrix element $\mathcal{M}$.

(d) Take the complex square of the reduced matrix element you found. Sum over final state spins, and average over initial state spins, and simplify. Write the result in terms of Mandelstam variables s, t, u. Then, rewrite it in terms of P and the scattering angle θ.

(e) Find the differential cross section. Simplify your answer as much as possible.

(f) Now take $m_\mu \to 0$. What is the differential cross section? You should note something interesting for a particular value of $\cos\theta$.

(g) Consider the 16 possible helicity processes:

$$
\begin{aligned}
\mu_L^+ e_L^- &\to \mu_L^+ e_L^-; & \mu_L^+ e_L^- &\to \mu_R^+ e_L^-;\\
\mu_L^+ e_L^- &\to \mu_L^+ e_R^-; & \mu_L^+ e_L^- &\to \mu_R^+ e_R^-;\\
\mu_L^+ e_R^- &\to \mu_L^+ e_L^-; & \mu_L^+ e_R^- &\to \mu_R^+ e_L^-;\\
\mu_L^+ e_R^- &\to \mu_L^+ e_R^-; & \mu_L^+ e_R^- &\to \mu_R^+ e_R^-;\\
\mu_R^+ e_L^- &\to \mu_L^+ e_L^-; & \mu_R^+ e_L^- &\to \mu_R^+ e_L^-;\\
\mu_R^+ e_L^- &\to \mu_L^+ e_R^-; & \mu_R^+ e_L^- &\to \mu_R^+ e_R^-;\\
\mu_R^+ e_R^- &\to \mu_L^+ e_L^-; & \mu_R^+ e_R^- &\to \mu_R^+ e_L^-;\\
\mu_R^+ e_R^- &\to \mu_L^+ e_R^-; & \mu_R^+ e_R^- &\to \mu_R^+ e_R^-.
\end{aligned}
$$

Do not compute them. Instead, figure out which ones vanish by helicity conservation in the limit $m_e, m_\mu \ll \sqrt{s}$.